

An improved optimization algorithm to search for an optimal solution of a muffler design problem

Lee, Jin Woo¹

Department of Mechanical Engineering, Ajou University
206, World cup-ro, Yeongtong-gu, Suwon, Republic of Korea

Oh, Kee Seung²

Structural and Multidisciplinary System Design Group, Queen's University
35 Fifth Field Company Lane, Kingston, Ontario, Canada

ABSTRACT

In this work, an auxiliary algorithm is proposed to assist a gradient-based optimizer in searching for an optimal solution of a muffler design problem formulated by topology optimization. The proposed algorithm escapes an objective function from one of local minima and pushes it toward a global optimum. Its key idea is adjusting a sign of sensitivity, the first derivative of an objective function with respect to a design variable, by multiplying it with 1 or -1. In an acoustical topology optimization problem formulated for muffler design, the absolute value of an acoustic pressure at the outlet is selected as an objective function, and the number of allowed finite elements for partitions inside the muffler is constrained. The formulated muffler design problem is solved for several design conditions, and the numerical results support the validity of the proposed auxiliary algorithm.

Keywords: Mufflers, Topology optimization, Auxiliary algorithm, Global optimum

I-INCE Classification of Subject Number: 34

1. INTRODUCTION

A muffler is used in an exhaust system of a vehicle to attenuate noise of an exhausted gas generated from the engine. The noise attenuation performance of a muffler is evaluated by a transmission loss (TL) value at a target frequency, which should be maximized for low noise exhausted gas. Generally, the positions and shapes of the partitions inside the muffler should be optimally determined for a high TL value at a target frequency. A high TL value is identical to the low magnitude of acoustic pressure at the outlet¹. Therefore, if the magnitude of acoustic pressure at the outlet is selected as an objective function in a muffler design problem, even a single target frequency problem is formulated as a multi-objective optimization problem. Unfortunately, however, it is hard to obtain a satisfactory result in the multi-objective optimization problem because one of the sub-objectives often converges to the local minima. A new method to overcome this issue in a muffler design problem including a multi-objective function is required.

¹ jinwoolee@ajou.ac.kr

² keeseung.oh@queensu.ca

Since the topology optimization method was reported for the compliance minimization structural problem², it has been applied to various acoustic device design problems. Lee and Kim³ optimally designed a partition layout inside the muffler by using topology optimization for high TL values at several target frequencies. The eigenfrequency controlling problem for a double cavity was reported by using topology optimization⁴. Kook et al.⁵ solved the acoustical topology optimization problem for Zwicker's loudness model, and Du and Olhoff⁶ minimized sound radiation for vibrating structures by using the topology optimization method. In addition, acoustical topology optimization problems considering flow, thermal and structural characteristics⁷⁻⁹ were formulated and solved. However, a research result to figure out the aforementioned local minima issue in the acoustic topology optimization problem has not been reported yet.

In the study, an auxiliary algorithm to avoid the local minima is proposed and a multi-objective topology optimization problem is formulated for muffler design. The magnitude of the acoustic pressure at the outlet is selected as the objective function, and the partition volume is constrained. The proposed algorithm is validated by comparing the results obtained with/without using the algorithm.

2. TOPOLOGY OPTIMIZATION PROBLEM FORMULATION



Fig. 1 – 2-dimensional muffler for acoustical topology optimization problem

Fig. 1 shows the half of a 2-dimensional concentric expansion chamber muffler which has an inlet, an expansion chamber, and an outlet. The Helmholtz equation in Equation 1 governs the acoustic pressure p inside the muffler.

$$\nabla \cdot \left(\frac{1}{\rho} \nabla p \right) + \frac{\omega^2}{B} p = 0, \quad (1)$$

where ρ and B denote the density and bulk modulus of the acoustic medium, respectively, and the symbol of ω is angular frequency ($\omega = 2\pi f$; f denotes frequency). In a multi-objective optimization problem, the sum of the squared magnitude of the outlet acoustic pressure at the multiple target frequencies (f_n) is selected as an objective function as in Equation 2, and each sub-objective function is defined as in Equation 3.

$$L(f_n) = \sum_{n=1}^N w_n(f_n) L_n(f_n), \quad (2)$$

$$L_n = [Re\{p_{out}(f_n)\}]^2 + [Im\{p_{out}(f_n)\}]^2 \quad (3)$$

In Equation 2, w_n denotes an adaptive weighting factor, which is determined by utilizing former step sub-objective values (L_n^{old}) during the optimization process as expressed in Equation 4.

$$w_n(f_n) = L_n^{\text{old}}(f_n) / \sum_{n=1}^N L_n^{\text{old}}(f_n). \quad (4)$$

During the topology optimization process, the material properties (ρ and B) of each finite element are determined by using the carefully-selected interpolation functions in Equation 5. The design variable (χ_r) assigned to each finite element in a design domain changes between 0 and 1 during optimization process and determines the material properties of the associated finite element. The design variables are updated by using the MMA algorithm¹⁰ which is one of the gradient-based algorithms. When a design variable converges to ‘1’, the associated finite element becomes a rigid body element. In contrast, when it converges to ‘0’, it is filled with air.

$$\rho_r(\chi_r) = \left(1/\rho_{\text{air}} + \chi_r (1/\rho_{\text{rigid}} - 1/\rho_{\text{air}}) \right)^{-1}, \quad (5a)$$

$$B_r(\chi_r) = \left(1/B_{\text{air}} + \chi_r (1/B_{\text{rigid}} - 1/B_{\text{air}}) \right)^{-1}, \quad (5b)$$

where the subscripts ‘air’ and ‘rigid’ denote the air and acoustic rigid body elements, respectively.

The acoustical topology optimization problem for muffler design is formulated in Equations 6 and 7.

$$\min_{0 \leq \chi_r \leq 1} L(\chi_r; f_n), \quad (6)$$

$$\left(\sum_{r=1}^R \chi_r / R \right) \leq V_r, \quad (7)$$

where R and V_r denote the number of the total finite elements in a design domain and the volume ratio between allowed partitions and the design domain, respectively.

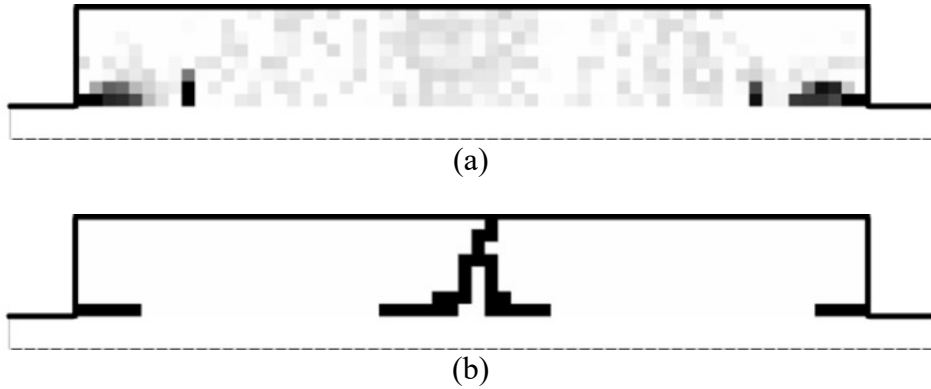


Fig. 2 – Optimal topologies: (a) Optimal muffler obtained by not using the auxiliary algorithm, (b) Optimal muffler obtained by using the auxiliary algorithm.

3. OPTIMAL RESULTS

The topology optimization problem was solved at the target frequency of $f_t = 400$ Hz and 5% volume ratio ($V_r = 0.05$). Fig. 2 shows the optimal topologies

obtained with and without the proposed auxiliary algorithm. Fig. 3 compares the TL curves of two optimal topologies in Fig. 2.

$$TL(f_t) = 10 \log_{10} (|p_{in}|^2 / |p_{out}|^2), \quad (8a)$$

$$p_{in}(f_n) = \int_{\Gamma_{in}} p_0(f_n) d\Gamma_{in}, \quad (8b)$$

$$p_{out}(f_n) = \int_{\Gamma_{out}} p(f_n) d\Gamma_{out}, \quad (8c)$$

where p_0 denotes unit acoustic pressure.

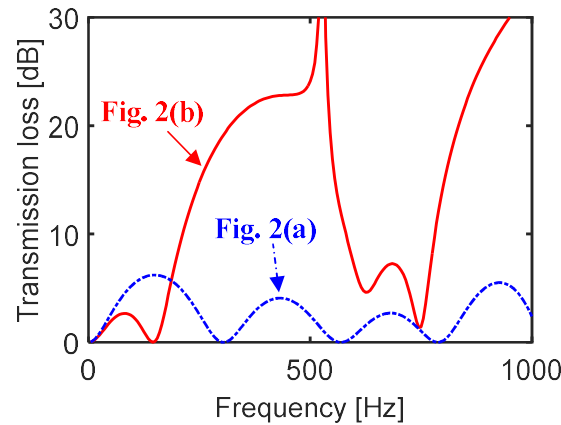


Fig. 3 – TL curve comparison for the two optimal topologies in Fig. 2(a) and 2(b).

4. CONCLUSIONS

In this paper, we proposed an auxiliary algorithm to avoid the local minima during the optimization process and applied it to a muffler design problem. Two optimal results with or without the proposed algorithm were compared. The comparison supported the validity of the proposed auxiliary algorithm.

5. ACKNOWLEDGEMENTS

This research was supported by the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (No.2016R1D1A1B03932357).

6. REFERENCES

1. Manohar L. Munjal, “Acoustics of ducts and mufflers with application to exhaust and ventilation system design”, John Wiley and Sons, New York, (1987)
2. M. P. Bendsøe, N. Kikuchi, “Generating optimal topologies in structural design using a homogenization method”, *Comput. Methods Appl. Mech. Eng.*, **71**(2), 197-224, (1988)
3. J. W. Lee and Y. Y. Kim, “Topology optimization of muffler internal partitions for improving acoustical attenuation performance”, *Int. J. Numer. Meth. Eng.*, **80**(4), 455-477, (2009)
4. J.W. Lee, Y.Y. Kim, “Optimal distribution of holes in a partition interfacing two cavities for controlling the eigenfrequencies by acoustical topology optimization”, *Comput. Methods Appl. Mech. Eng.*, **198**(27-29), 2175-2189, (2009)
5. J. KooK, K. Koo, J. Hyun, J.S. Jensen, S. Wang, “Acoustical topology optimization for Zwicker’s loudness model-Application to noise barriers”, *Comput. Methods Appl. Mech. Eng.*, **237-240**, 130-151, (2012)

6. J. Du, N. Olhoff, "Minimization of sound radiation from vibrating bi-material structures using topology optimization", *Struct. Multidisc. Optim.*, **33**(4-5), 305-321, (2007)
7. J.W. Lee, G.W. Jang, "Topology design of reactive mufflers for enhancing their acoustic attenuation performance and flow characteristics simultaneously", *Int. J. Numer. Methods Eng.*, **91**(5), 552-570, (2012)
8. K.S. Oh, J.W. Lee, "Topology optimization for enhancing the acoustical and thermal characteristics of acoustic devices simultaneously", *J. Sound Vib.*, **401**, 54-75, (2017)
9. H.G. Yoon, J.W. Lee, "Optimum core design to improve noise attenuation performance and stiffness of sandwich panels used for high-speed railway vehicles", *Struct. Multidisc. Optim.*, **55**(2), 723-738, (2017)
10. K. Svanberg, "The method of moving a symptotes: a new tool for structural optimization", *Int. J. Numer. Meth. Eng.*, **24**(2), 359-373, (1987)