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Uncertainties and interdependence of porous parameters using Bayesian model-based analysis

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ABSTRACT

Traditional modeling for sound propagation in porous media has been done knowing the macroscopic physical parameters intrinsic to the material. Using Bayesian model-based analysis the uncertainties of these parameters and their effect on the accuracy of the selected model can be studied. The Horoshenkov Model [Horoshenkov, *et al.*, JASA vol 139 (2016)], or similarly Miki Model [Miki, JASJ, vol 11, pp.25-28 (1990)], can be used to estimate the mean pore size and porosity of the material under test from measurements of surface impedance or absorption coefficient using glass beads of known diameter and layer thickness as an experimental model. The accuracy of these models can be compared using quantitative measures such as mean, variance and the interdependence of the estimated parameters. This paper discusses the error and likelihood of several models using the experimentally measured data sets.

Keywords: Porous, Horoshenkov, Bayesian
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1. INTRODUCTION

Porous materials in acoustics may be used to design sound absorbers for a variety of science and engineering applications. While some parameters of porous materials may be measured directly others can only be calculated by measuring the acoustic characteristics of the samples and inverting the parameters from these measurements, often done in a standard impedance tube. Starting with the Miki model [1] these materials could be modeled by three parameters: porosity(ϕ), tortuosity(α_∞), and flow resistivity(σ_f) along with sample thickness(d), whereas the Horoshenkov model [2] requires four parameters. This paper applies Bayesian model-based analysis to inversely inference these parameters from experimentally measured normal incident surface impedances. The samples are made of glass beads of varied diameters and thicknesses, and are well investigated in the literature. Bayesian model-based analysis has recently applied to analyze porous materials. [3–5] This work sheds light on uncertainties and interdependence of key non-acoustical parameters of porous media estimated using Bayesian model-based framework, particularly Bayesian parameter estimation. Bayesian parameter estimation is considered as the first level of inference, a thorough description on the parameter estimation has recently fully exposed by Xiang and Fackler. [6]

2. TWO MODELS OF POROUS MEDIA

This section briefly states the models under investigation within this work.

2.1 Modeling Porous Media using Miki's Model

The propogation coefficient, γ, and characteristic impedance, Z_c, are then given by Equations 1 and 2.

$$\underline{\gamma}(\omega) = \frac{\omega\sqrt{\alpha_\infty}}{c}[0.16\Psi^a + i(1 + 0.109\Psi^a)], \tag{1}$$

and

$$\underline{Z}_c(\omega) = \rho c\frac{\sqrt{\alpha_\infty}}{\phi}[1 + 0.07\Psi^b - 0.107i\Psi^b], \tag{2}$$

with

$$\Psi = \frac{f\alpha_\infty}{\phi\sigma_f}, \tag{3}$$

where coefficient $a = -0.618$, and coefficient $b = -0.632$. $\omega = 2\pi f$ is circular frequency with f being the frequency. ρ and c are the density of air and speed of sound in air, respectively, and $i = \sqrt{-1}$. An under bar of a variable, e.g. $\underline{Z}_c$ explicitly represents a complex-valued function. The surface impedance can then be expressed as

$$\underline{Z}_s(\omega) = \underline{Z}_c(\omega)\coth(\underline{\gamma}\, d). \tag{4}$$

Miki model [1] represents a simple predictive model which contains three parameters, namely porosity, tortuosity, and flow resistivity. Horoshenkov et al. [2] proposed a model that requires 4 parameters but is based on the assumption of non-uniform pores that vary with radius in depth, which allows for fitting a log-normal distribution to a variety of pore geometries.

2.2 Modeling Porous Media using Horoshenkov's Model

Horoshenkov's porous media model depends on 4 parameters: porosity (ϕ), tortuosity, (α_∞), mean pore size ($\bar{s}$), and standard deviation in mean pore size (σ_s). The intrinsic properties can be calculated from the complex compressibility, $\tilde{C}_x$, and dynamic density, $\tilde{\rho}_x$, given by Equations 5 and 6.

$$\underline{\gamma}(\omega) = \omega\sqrt{\tilde{\rho}_x(\omega)\tilde{C}_x(\omega)}, \tag{5}$$

and

$$\underline{Z}_c(\omega) = \sqrt{\tilde{\rho}_x(\omega)/\tilde{C}_x(\omega)}. \tag{6}$$

The complex compressibility and and dynamic density can further be calculated by the Padé approximations in Equations 7 and 8.

$$\tilde{\rho}_x(\epsilon_\rho) \approx 1 + \epsilon_\rho^{-2}\tilde{F}_\rho(\epsilon_\rho), \tag{7}$$

$$\tilde{C}_x(\epsilon_c) \approx \frac{1}{\gamma P_0}(\gamma - \frac{\gamma - 1}{1 + \epsilon_c^{-2}\tilde{F}_c(\epsilon_c)}), \tag{8}$$

where

$$\tilde{F}_\rho(\epsilon_\rho) = \frac{1 + \theta_{\rho,3}\epsilon_\rho + \theta_{\rho,1}\epsilon_\rho}{1 + \theta_{\rho,3}\epsilon_\rho}, \tag{9}$$

and

$$\tilde{F}_c(\epsilon_c) = \frac{1 + \theta_{c,3}\epsilon_c + \theta_{c,1}\epsilon_c}{1 + \theta_{c,3}\epsilon_c}. \tag{10}$$

In the above equations

$$\epsilon_p = \sqrt{-i\omega\rho_0/\sigma_x}, \tag{11}$$

$$\epsilon_c = \sqrt{-i\omega\rho_0 N_{Pr}/\sigma'_x}, \tag{12}$$

$\theta_{\rho,1} = 1/3$, $\theta_{\rho,2} = e^{-1/2(\sigma_s log2)^2}/\sqrt{2}$, $\theta_{\rho,3} = \theta_{\rho,1}/\theta_{\rho,2}$, $\theta_{c,1} = 1/3$, $\theta_{c,2} = e^{3/2(\sigma_s log2)^2}/\sqrt{2}$, and $\theta_{c,3} = \theta_{c,1}/\theta_{c,2}$. N_{Pr} is the Prandtl number, γ is the ratio of specific heats, and P_0 is the ambient atmospheric pressure. The bulk flow resistivity is represented by σ_x, which can be predicted from the mean pore size and standard deviation in mean pore size [7] by Equation 13

$$\sigma_x = \frac{8\eta\alpha_\infty}{\bar{s}^2\phi}e^{6(\sigma_s log2)^2} \tag{13}$$

where η is the dynamic viscosity of air.

3. BAYESIAN INFERENCE FOR POROUS PARAMETER ESTIMATION

Bayesian inference problems are defined by the updating of prior information that determine the probability of a certain hypothesis to achieve parameter estimation and model selection. Using Bayesian estimation and the direct measures of acoustic characteristics the parameters of these porous materials can be inverted. Glass beads of known thickness are of interest because some of their intrinsic parameters can be measured directly making comparison of the predictive power of the models possible.

Fackler et al. [4] write the Bayes theorem for this parameters estimation problem in Equation 1.

$$p(\theta|D, M) = \frac{p(\theta|M)\, p(D|\theta, M)}{p(D|M)}, \tag{14}$$

where $p(\theta|D, M)$ represents the posterior information as a probability density function, $p(\theta|M)$ the prior information about the parameters, $p(D|\theta, M)$ the likelihood function, and $p(D|M)$ the Bayesian evidence. The likelihood function is given by the student t-distribution

$$\mathcal{L}(\theta) = p(D|\theta, M) = \frac{\Gamma(B/2)}{2}\left(\pi \sum_{b=1}^{B} E_b^2\right)^{-B/2}, \tag{15}$$

where Γ is the gamma function, B is the number of frequency points, and E_b^2 is the square error between both the real and imaginary parts of the data and model at each frequency point. Uniform distributions of the parameters in reasonable physical ranges are used as the prior probability distributions. These ranges are given by $p(\text{porosity}, \phi) = \text{Uniform}(0, 1)$, $p(\text{tortuosity}, \alpha_\infty) = \text{Uniform}(1, 5)$, $p(\text{mean pore size}, \bar{s}) = \text{Uniform}(.01, 1)$ mm, and $p(\text{standard deviation}, \sigma_s) = \text{Uniform}(0, 1)$.

4. RESULTS

The experimental setup provides a known thickness for the material under test, reducing the parameter estimation problem to the 4 remaining parameters in Horoshenkov's model. The inference is performed over 500,000 uniform random samples in a frequency range of 300Hz to 3kHz. Figure 1 shows the measured and modeled, real and imginary parts of the normalized surface impedance for glass beads of 2mm diameter in a hard-backed sample with thickness of 40 mm.

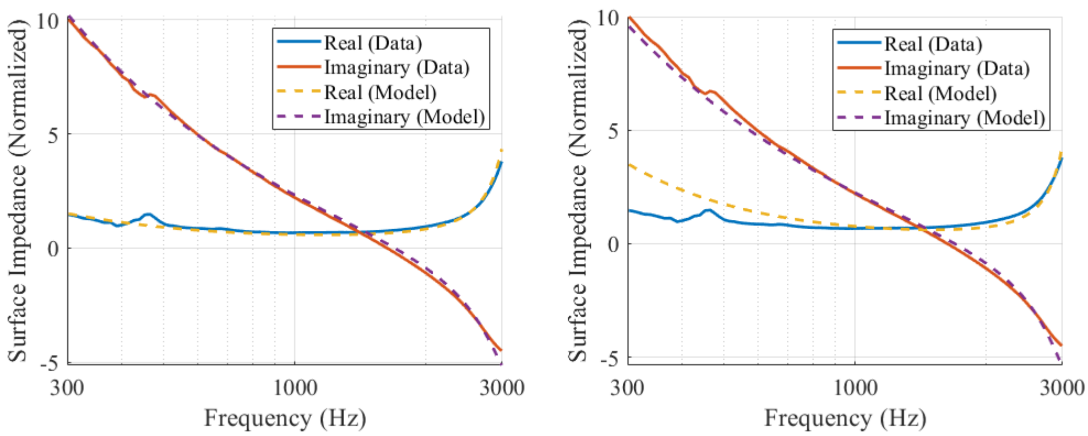

Figure 1: Modeled and measured normalized surface impedance for 40mm thick sample of 2mm diameter glass beads using Horoshenkov's model (a) and Miki's model (b)

Horoshenkov's model fits the experimental data better than Miki's model. Quantitatively, the likelihood function of each model measures quality of model fitting while the Bayesian evidence in Equation 15 is used to estimate a parsimonious number of parameters necessary to fully explain the acoustic behavior, shown by Table

3. The Bayesian information criterion can give us a method for ranking the porous models dependent on the peak of the likelihood [8]. Table 1 lists the mean estimated parameters and their variance, a measure of the uncertainty in the measurement which can be obtained through Bayesian inference from a single data set. There is a very low uncertainty in the estimation of the parameters below. Table 2 shows this information for the Miki model with the same data set.

Table 1: Estimated mean parameters and variance using Bayesian inference parameter estimation with Horoshenkov's model for porous materials with glass beads of 2mm diameter and 40mm sample thickness.

Parameter	Mean Value	Variance
Porosity	0.373	2.99×10^{-30}
Tortuosity	1.156	1.96×10^{-31}
Mean Pore Size	318 um	3.72×10^{-35}
Standard Deviation	0.410 ϕ	4.38×10^{-27}
Thickness	40 mm	0

Table 2: Estimated mean parameters and variance using Bayesian inference parameter estimation with Miki's model for porous materials with glass beads of 2mm diameter and 40mm sample thickness.

Parameter	Mean Value	Variance
Porosity	0.402	.02
Tortuosity	1.256	9.06e-7
Flow Resistivity	76.9 kNs/m^4	2.90e-7
Thickness	40 mm	0

Table 3: Maximum likelihood and Bayesian Information Criterion (BIC) from parameters estimated for Miki model in Table 1 and Horoshenkov model in Table 2

Model	Max Likelihood (deciBans)	BIC (deciBans)
Miki	478	885
Horoshekov	1897	3701

Figure 2 shows normalized posterior probability distributions for the parameters in the Horoshenkov model. These distributions provide information about how the parameters depend on each other within the model and each subplot also shows the relationship between every pair of parameters. The spread of the ellipse indicates the correlation between parameters and how fixing one parameter can change the inference of the other parameters. A positive correlation between porosity and tortuosity is recognizable, while little correlation between porosity and mean pore size. A large interdependence between mean pore size and standard deviation of the mean pore size in the small parameter ranges as defined. The size of the distributions indicates the uncertainty in estimation when performing Bayesian inference. Larger spreads present a larger challenge and variations when trying to estimate an accurate mean value for the parameter. This reflected by the greatest effect with the standard deviation in mean pore size.

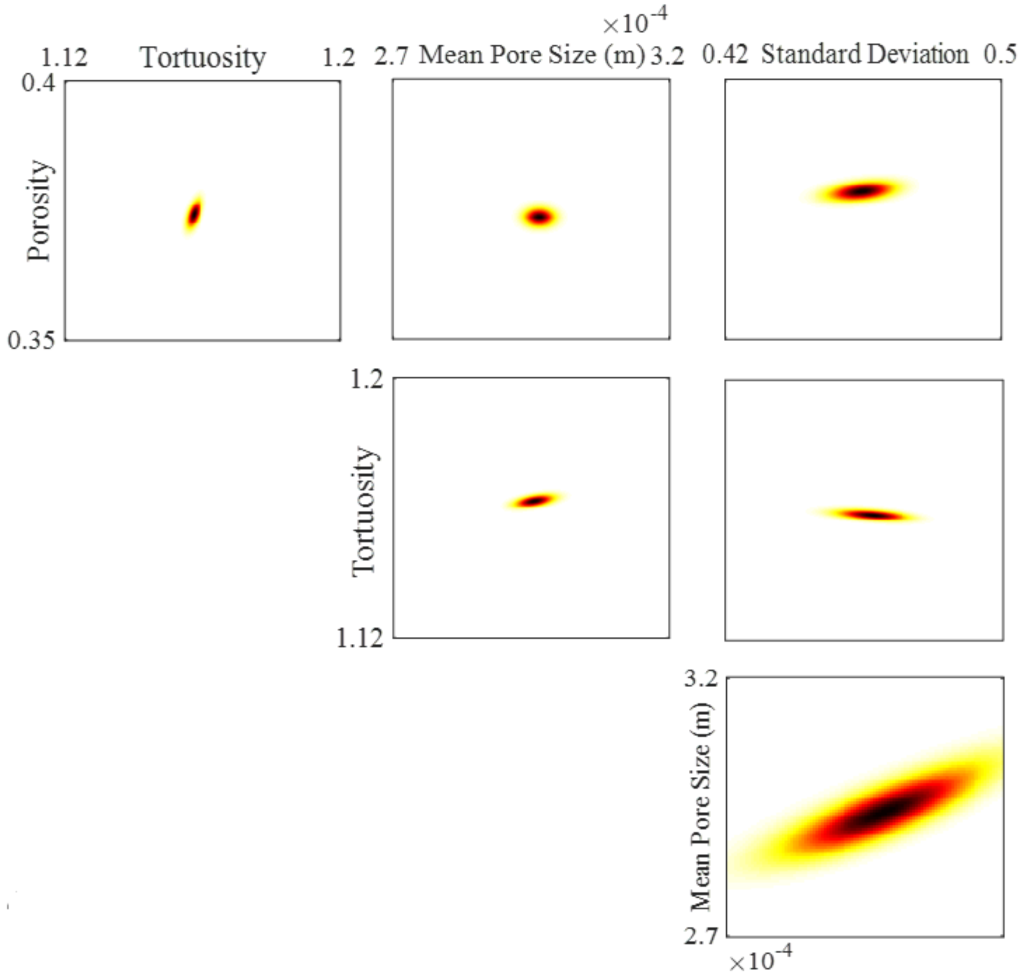

Figure 2: Normalized marginal posterior probability distribution over the parameter space using Horoshenkov's model

5. CONCLUDING REMARKS

Applying Bayesian parameter estimation to a data set of surface impedance measured from glass beads of known diameters in a rigid backing impedance tube. Horoshenkov's model [2] provides an accurate estimation of the intrinsic parameters of the porous materials under test. When compared to Miki model, Bayesian inference yields a better estimation of the mean values of the parameters and lower variation, resulting in better prediction of the acoustic behaviors of the porous material. These inverted parameters could be used to characterize the behaviors of unknown materials or design porous materials with desired absorption properties. Future work can incorporate more models in the evaluation of Bayesian evidence and explore more variables including frequency range and different acoustic measurements. Bayesian inference provides an effective method for inverting parameters of porous material from acoustic measurements. Horoshenkov's model can provide more accurate estimations than that using Miki model so far tested using glass beads.

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