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Design of a ground mitigation system by means of mass network at the ground top surface

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ABSTRACT
For the past decades, railway vibration risk assessment has been a subject of concern especially due to the vicinity between railway infrastructures and buildings which create a source of annoyance for residents. In this context, mitigation system must be developed at different level of propagation. This paper presents a solution for vibrations reduction on the propagation path, the ground. A mass network coupled at the ground surface, inspired by sonic crystals in acoustics, create cut-off band. The influence of these masses' distribution as well as the mass and surface effects of the structures are presented for an ideal reduction. Ground characteristics have a direct effect on the performance of the system characterized by Rayleigh's velocity. This kind of solution is a variant of the vertical and horizontal wave barrier.
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1. INTRODUCTION

In recent decades, number of railway infrastructures grew up such as high-speed railways, trams and subways. All these infrastructures are more and more often placed in urban area with strong acoustic and vibratory requirements. These infrastructures are generally coupled to the ground and therefor at the origin of the propagation of waves in the ground. These waves generate the vibration of buildings near railway infrastructure and are a source of significant noise pollution for residents. In this context, one of the strongest issues is to reduce the vibrations transmitted to the ground. There are currently different systems to reduce the vibration from the train. Three major categories could be distinguished to mitigate railway vibrations. The first category is the mitigation at the source, i.e. in the vicinity of the wheel-rail interaction [1]. Without being exhaustive about all existing techniques, maintenance operations performed on the wheels of trains and rails may contribute to the reduction of vibrations. In addition, pads between the rail and the concrete slab can perform mitigation. The second category of mitigation is at the

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propagation path. Thus, trenches in the ground depth can be made near the track and thus reduce the propagation [2]. Horizontal Vibration Barrier system has recently been developed using techniques like those used in this paper [3]. A slab is placed at the ground surface and blocks the vibrations. The third category of mitigation is at the reception in the building where it is possible for example to isolate buildings with springs or pads.
The purpose of this article is to provide an analytical formulation of a network of masses at the ground top surface in order to mitigate ground vibration. It can be pointed out that this kind of system

2. PROBLEM FORMULATION

The problem of interest is a network of mass coupled to the ground at the top surface. For railway, a pass by train is represented as an uncoherent line of force. For sake of simplicity in this paper, the excitation is a surface force located at (x_0, y_0) with the amplitude F_0. The Figure 1 gives an overview of the problem.

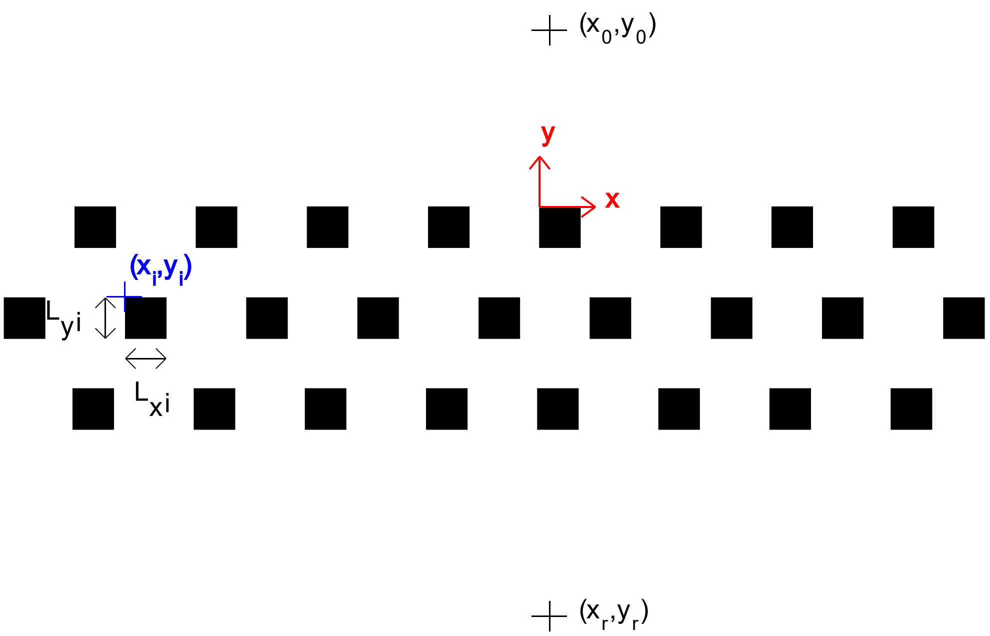

Figure 1 :Network of mass at the top ground surface

In the section 2.1, the ground modelling is presented and the section 2.2 presents the mass equation of motion. The section 2.3 presents the coupling between the ground and the mass.

2.1 Ground modelling

The ground is modelled with Navier's equation which considers a continuous, homogeneous and isotropic elastic layer. In the absence of a body force, and assuming the motion is harmonic, one obtains the following vector equation for the displacement vector $\vec{u}$ in the layer:

$$\mu\nabla^2\,\vec{u} + (\mu+\lambda)\nabla(\nabla.\vec{u}) + \rho\omega^2\,\vec{u} = \vec{0} \qquad 1$$

where $\vec{u}^T = \{u_x, u_y, u_z\}$ is the vector of ground displacement, $\mu = \frac{E(1+j\eta)}{2(1-\nu)}$ and $\lambda = \frac{\nu E(1+j\eta)}{(1+\nu)(1-2\nu)}$ are the Lamé constants.

Using Helmholtz decomposition, the ground equation of motion can be expressed as 4 waves equation of motion, one for the dilatational wave and the three others for the shear wave. Then, the ground is a semi-infinite medium in z-direction and infinite in the direction x- and y-direction. A 2D spatial Fourier transform allows to solve the problem and to give a relation between the normal displacement and the normal stress at the ground Surface:

$$\tilde{u}_z(k_x, k_y, 0) = N(k_x, k_y)\,\tilde{\sigma}_{zz}(k_x, k_y, 0) \qquad 2$$

The term $N(k_x, k_y)$ depends on the number of horizontal layers. It accounts for the mechanical and geometrical properties of the ground.
The stress component σ_{xz}- and σ_{yz} are zero at the surface. Normal stress along the z axis is also zero everywhere on the surface z = 0 except under all mases and can be written as:

$$\begin{cases} \sigma_{xz}(x,y,0) = 0\ \forall(x,y) \in \mathbb{R}^2 \\ \sigma_{yz}(x,y,0) = 0\ \forall(x,y) \in \mathbb{R}^2 \\ \sigma_{zz}(x,y,0) = \begin{cases} \sigma_1\ \forall(x,y) \in S_1 \\ \vdots \\ \sigma_N\ \forall(x,y) \in S_N \\ F_0\forall(x,y) \in S_0 \\ 0\ \forall(x,y) \in \mathbb{R}^2 - \{S_i \cap \cdots \cap S_N\} \end{cases} \end{cases} \qquad 3$$

where σ_{xz} and σ_{yz} represent tangential stresses and σ_{zz} represents normal stress. σ_i represents the stress applied to the masses i by the ground and F_0 is the surface force applied at the ground top surface. The stress due to the mass and the force excitation is assumed to be constant which is an acceptable hypothesis in the frequency range of interest and in regards with the surface compared to the ground wavelength travelling at the top surface. The force surface allows to regularize the problem which is singular for a punctual force.

2.2 Coupled mass at the ground surface

In regards with the surface of the masses and the frequency range of interest, it is assumed that the mass deformation is negligible. Each mass is located at the position (x_i, y_i) and its surface is given by $S_i = L_{x_i} L_{y_i}$. There are N masses coupled to the ground. The equation of motion for the mass i is given by:

$$-\rho_i h_i \omega^2 w_i = \sigma_i \quad \forall i \in [\![1, N]\!] \qquad 4$$

Where ρ_i and h_i is the mass density and thickness of the mass i respectively and σ_i is the coupling stress applied by the ground to the mass i.

In section 2.1, we shown that we need to know the ground stress at the top surface in the Fourier domain. The stress vanishes everywhere expect under each mass. So, we need to express the stress in the Fourier domain of each mass.

Considering a mass i of surface S_i located at (x_i, y_i), the stress in the z-direction in the Fourier domain is given by:

$$\tilde{\sigma}_{zz}(k_x, k_y, 0)_i = \int_{x_i}^{x_i+L_{x_i}} \int_{y_i}^{y_i+L_{y_i}} \sigma_i . e^{-j(k_x x + k_y y)} dx dy \qquad 5$$

Assuming a rigid mass, the stress σ_i is constant over the surface S_i. After integrating over the surface, it gives the analytical formula:

$$\tilde{\sigma}_{zz}(k_x, k_y, 0)_i = \sigma_i S_i e^{-j\left(k_x\left(x_i + \frac{L_{x_i}}{2}\right) + k_y\left(y_i + \frac{L_{y_i}}{2}\right)\right)} sinc\left(\frac{k_x L_{x_i}}{2}\right) sinc\left(\frac{k_y L_{y_i}}{2}\right) \qquad 6$$

Considering all the stress applied by masses and the punctual force, the normal stress in the z-direction at the ground top surface is given in the Fourier domain by:

$$\begin{aligned}\tilde{\sigma}_{zz}(k_x, k_y, 0) = {} & F_0 S_0 e^{-j(x_0 + \frac{k_x L_{x_0}}{2} + y_0 + \frac{k_y L_{y_0}}{2})} sinc\left(\frac{k_x L_{x_0}}{2}\right) sinc\left(\frac{k_y L_{y_0}}{2}\right) \\ & + \sum_i^N \tilde{\sigma}_{zz}(k_x, k_y, 0)_i \end{aligned} \qquad 7$$

Using equation 7, the normal displacement at the top surface is:

$$\begin{aligned}\tilde{u}_z(k_x, k_y, 0) = N(k_x, k_y) \Bigg(& F_0 S_0 e^{-j(x_0 + \frac{k_x L_{x_0}}{2} + y_0 + \frac{k_y L_{y_0}}{2})} sinc\left(\frac{k_x L_{x_0}}{2}\right) sinc\left(\frac{k_y L_{y_0}}{2}\right) \\ & + \sum_i^N \tilde{\sigma}_{zz}(k_x, k_y, 0)_i \Bigg) \end{aligned} \qquad 8$$

2.3 Resolution of the problem

The continuity of the displacement between the ground top surface and the mass is given for each mass j:

$$w_j = u_z(x, y, 0)\ \forall (x, y) \in S_j \tag{9}$$

Using the relation 9, it comes:

$$w_j = \frac{1}{4\pi^2} \iint_{-\infty}^{+\infty} N(k_x, k_y) \tilde{\sigma}_{zz}(k_x, k_y, 0) e^{j(k_x x + k_y y)} dk_x dk_y \tag{10}$$

By integrating over the surface S_j and replacing the normal stress by its expression 7, we have:

$$\begin{aligned} S_j w_j = \frac{1}{4\pi^2} \iint_{S_j} \iint_{-\infty}^{+\infty} N(k_x, k_y) \Bigg(& F_0 S_0 e^{-j(x_0 + \frac{k_x L_{x_0}}{2} + y_0 + \frac{k_y L_{y_0}}{2})} sinc\left(\frac{k_x L_{x_0}}{2}\right) sinc\left(\frac{k_y L_{y_0}}{2}\right) \\ & + \sum_i^N \sigma_i S_i e^{-j\left(k_x\left(x_i + \frac{L_{x_i}}{2}\right) + k_y\left(y_i + \frac{L_{y_i}}{2}\right)\right)} sinc\left(\frac{k_x L_{x_i}}{2}\right) sinc\left(\frac{k_y L_{y_i}}{2}\right)\Bigg) e^{j(k_x x + k_y y)} dk_x dk_y\, dS \end{aligned} \tag{11}$$

After calculation of the integration over S_j, the linear equation to solve is:

$$S_j w_j = F_j + \sum_i^N \sigma_i \gamma_{ij} \tag{12}$$

With
$F_j = \frac{1}{4\pi^2} \iint_{-\infty}^{+\infty} N(k_x, k_y) F_0 S_0 S_j e^{-jk_x\left(x_0 + \frac{L_{x_0}}{2} - \frac{L_{x_j}}{2} - x_j\right) - k_y\left(y_0 + \frac{L_{y_0}}{2} - \frac{L_{y_j}}{2} - y_j\right)} sinc\left(\frac{k_x L_{x_0}}{2}\right) sinc\left(\frac{k_y L_{y_0}}{2}\right) sinc\left(\frac{k_x L_{x_j}}{2}\right) sinc\left(\frac{k_y L_{y_j}}{2}\right) dk_x dk_y$
and
$\gamma_{ij} = \frac{1}{4\pi^2} \iint_{-\infty}^{+\infty} N(k_x, k_y) S_i S_j e^{-jk_x\left(x_i + \frac{L_{x_i}}{2} - \frac{L_{x_j}}{2} - x_j\right)} e^{-jk_y\left(y_i + \frac{L_{y_i}}{2} - \frac{L_{y_j}}{2} - y_j\right)} sinc\left(\frac{k_x L_{x_i}}{2}\right) sinc\left(\frac{k_y L_{y_i}}{2}\right) sinc\left(\frac{k_x L_{x_j}}{2}\right) sinc\left(\frac{k_y L_{y_j}}{2}\right) dk_x dk_y$

Replacing σ_i by $\rho_i h_i \omega^2$, the linear equation 12 can be formulated in matrix format:

$$\left(\begin{bmatrix} S_1 & & (0) \\ & \ddots & \\ (0) & & S_N \end{bmatrix} + \omega^2 \begin{bmatrix} \rho_1 h_1 \gamma_{11} & \cdots & \rho_N h_N \gamma_{1N} \\ \vdots & \ddots & \cdots \\ \rho_1 h_1 \gamma_{N1} & \cdots & \rho_N h_N \gamma_{NN} \end{bmatrix} \right) \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix} = \begin{bmatrix} F_1 \\ \vdots \\ F_N \end{bmatrix} \tag{13}$$

Once we get the expression of w_i, the displacement at the ground top surface is given using equation 2 and 7:

$$u_z(x, y, 0) = F_0 T_0(x, y) + \sum_i^N (-\rho_i h_i \omega^2 w_i) T_i(x, y) \tag{14}$$

With $T_0(x, y) = \frac{1}{4\pi^2} \iint_{-\infty}^{+\infty} N(k_x, k_y) S_0 e^{-jk_x\left(\frac{L_{x_0}}{2} - x\right) - jk_y\left(\frac{L_{y_0}}{2} - y\right)} sinc\left(\frac{k_x L_{x_0}}{2}\right) sinc\left(\frac{k_y L_{y_0}}{2}\right) dk_x dk_y$ and $T_i(x, y) = \frac{1}{4\pi^2} \iint_{-\infty}^{+\infty} N(k_x, k_y) S_i e^{-jk_x\left(x_i + \frac{L_{x_i}}{2} - x\right)} e^{-jk_y\left(y_i + \frac{L_{y_i}}{2} - y\right)} sinc\left(\frac{k_x L_{x_i}}{2}\right) sinc\left(\frac{k_y L_{y_i}}{2}\right) dk_x dk_y$.

3. NUMERICAL RESULTS

In this section, numerical results of the problem presented in section 2 are introduced. Each mass is a concrete mass ($\rho = 2500Kg.m^3$) of surface 1m.1m and thickness 0.2m. The force is applied at 5m from the mass. The receiver is located on a surface area of 10m.5m which are averaged. The ground is a one-layer ground with the characteristics $c_s = 200m/s$, $c_p = 400m/s$, $\eta = 2\%$ and $\rho = 2000Kg/m^3$

Figure 2 gives an overview of a simulation.

We are interested in the insertion loss which correspond to the vibration level at the receiver with and without mass network. The definition of the insertion loss is given by:

$$IL(\omega) = \frac{\frac{1}{S}\iint_S |u_{z-with\ mass}(\omega)|^2 dS}{\frac{1}{S}\iint_S |u_{z-without\ mass}(\omega)|^2 dS} \qquad 15$$

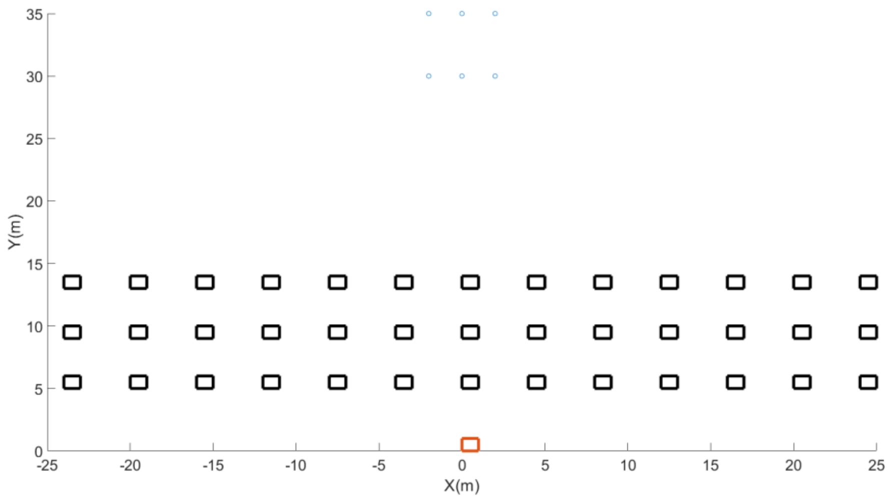

Figure 2 : overview of a simulation where the black box is the mass, the red box is the surface force and the blue circle is the receiver – case where $L_i = L_i = 1m$.

Figure 3 corresponds to the insertion loss in the case where the distance in x- and y-direction between each mass is 1m, 2m and 3m and are aligned. The insertion loss is almost negligible expect in a frequency range which correspond to a cut-frequency. For 1m, 2m and 3m, the maximum of insertion loss is at 70Hz, 50Hz and 40Hz respectively. This frequency depends on the wavelength travelling at the top surface. A destructive interference is created by the mass network. The wavelength of the shear wave is approximately 2;85m, 4m and 5m at 70Hz, 50Hz and 40Hz respectively which shows that the distance between masses needs to be half a wavelength to mitigate the corresponding frequency.

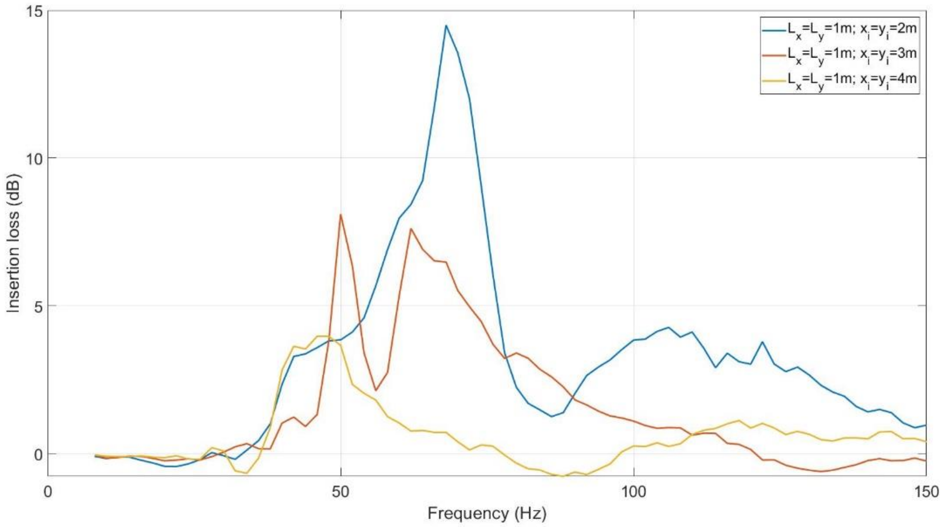

Figure 3 ; Insertion loss in the case of a parallel aligned network of mass for different inter distance

Figure 5 corresponds to the insertion loss where the second line is non-parallel to the first and third line in the X direction as shown in Figure 4. The difference of insertion loss between the aligned network of mass and the shifted network is almost the same which indicates that the distance between mass in the x-direction is the factor of main influence.

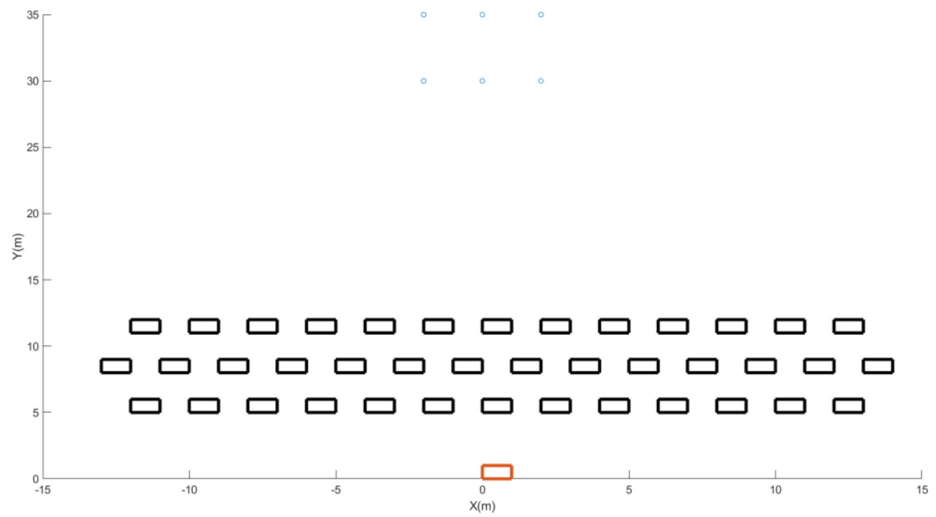

Figure 4 overview of a simulation where the black box is the mass, the red box is the surface force and the blue circle is the receiver – case where L_i=L_i=1m.

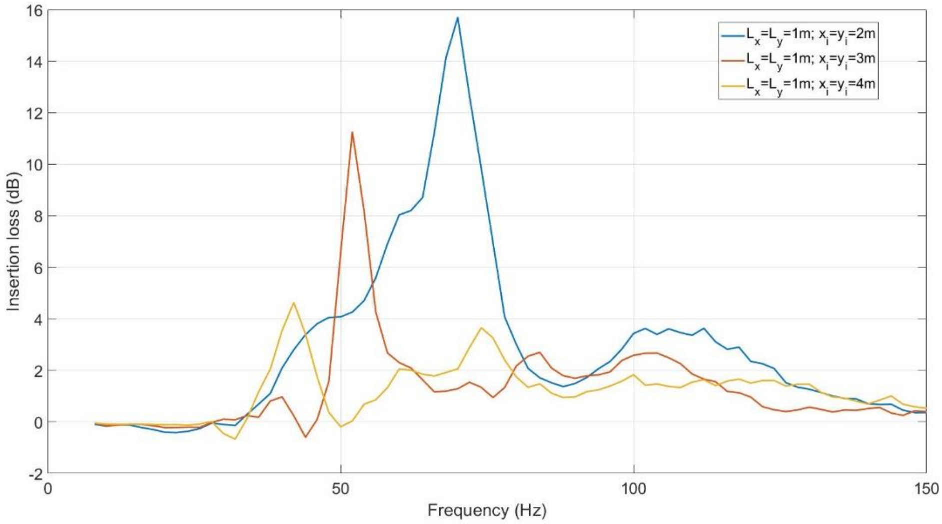

Figure 5 Insertion loss in the case of a parallel aligned network of mass for different inter distance with the line 2 shifted

Figure 6 correspond to a cartography of the insertion loss. It can be identifying the interference between masses.

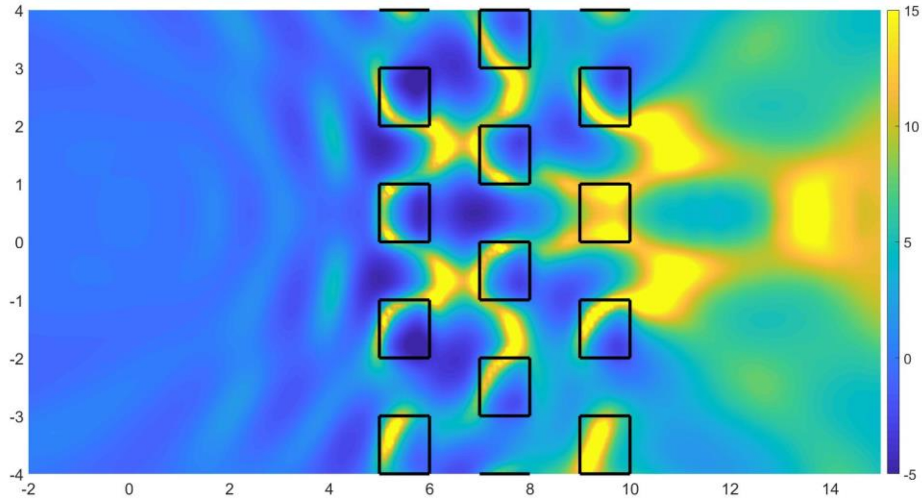

Figure 6 : Cartographie de la perte par insertion à la surface du sol - f=70Hz

Figure 7 corresponds to the insertion loss for different weight of masses. As expected, the more weight we are the more mitigation we get. However, the reduction of weight tends to shift the cut frequency to higher frequency. When the mass is sufficiently important, only the phenomenon of interference exist. The reduction of the mass creates tends to reduce this phenomenon and masses don't block the ground top surface.

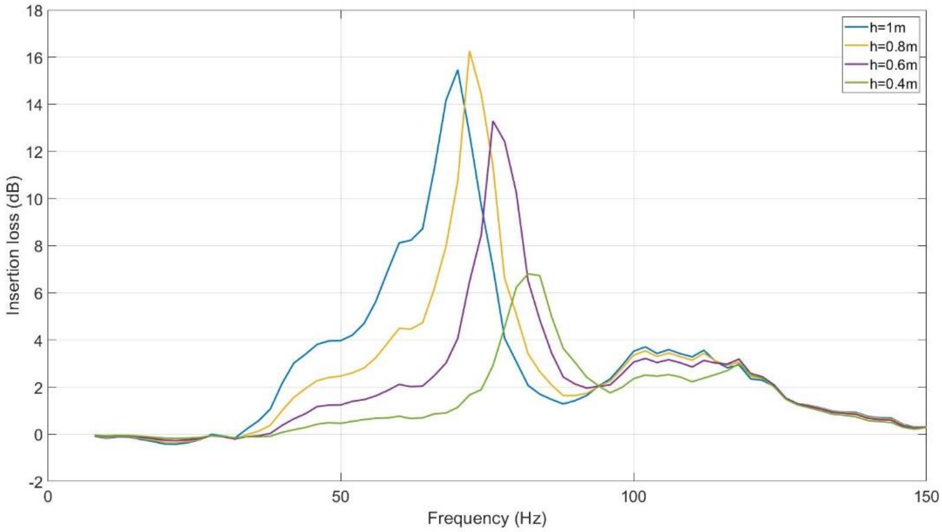

Figure 7 : Insertion loss for different weight of masses

Figure 8 corresponds to the insertion loss for different ground top surface. If we increase the dilatational celerity by a factor 1.5, the cut frequency slightly changes of few hertz to higher frequency. The increase of the shear celerity by a factor 1.5, the cut frequency is shifted from 70Hz to 110Hz. It shows that the shear celerity is the sizing parameter to investigate the cut frequency.

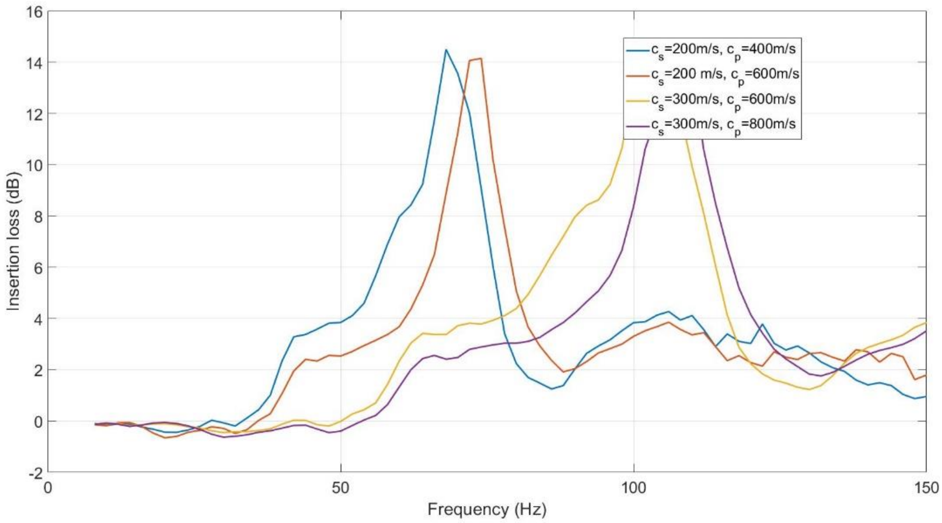

Figure 8 : Insertion loss for different ground characteristics

4. CONCLUSIONS

We have presented in this paper an analytical modelling of a network of masses at the ground top surface to mitigate vibration. It has been identifying a cut frequency which mitigate vibration up to 15dB in a specific frequency range. This cut frequency is due to an interference phenomenon from the network of masses.

6. REFERENCES

[1] D. Thompson, Railway Noise and Vibration. Elsevier Science (2008).

[2] L. Andersen and S.R.K. Nielsen, Reduction of ground vibration by means of barriers or soil improvement along a railway track, (2005) 701-716

[3] L. GRAU and B. LAULAGNET: Effect of horizontal wave barriers on ground vibration propagation. Journal of the Acoustical Society of America (2015) 1334-1345.