

On the uncertainty of ground borne noise and vibration assessment

Grau, Loïc¹

Morphosense

18 Allée du Lac Saint André, 73370 Le Bourget du Lac, France

Laulagnet, Bernard²

INSA Lyon, Laboratoire Vibrations et Acoustique

20 Avenue Albert Einstein, 69100 Villeurbanne, France

ABSTRACT

For the past decades, railway vibration risk assessment has been a subject of concern especially due to the vicinity between railway infrastructures and buildings which create a source of annoyance for residents. These risk assessments often use modelling where the quantity of input data is important. In this paper, we introduce a study of the influence of the modelling uncertainty due to input data on the calculation of the ground borne noise and vibration in buildings. Monte Carlo methods are particularly well suited for this kind of study and consist in the generation of a certain number of simulations where the input data are estimated according to a specific law of probability. This method makes it possible to identify the most sensitive parameters of a model. The objective is to provide a minimum uncertainty on the input data to ensure a sufficiently accurate impact study. It should be noted that the ground must not only be characterized with methods adapted to the impact studies but also have a significant precision which will be detailed.

Keywords: Railway, Ground borne noise and vibration, Risk assessment, uncertainty

I-INCE Classification of Subject Number: 43

<http://i-ince.org/files/data/classification.pdf>

1. INTRODUCTION

This paper is focused on method to quantify the effect of the data input uncertainty on a railway ground borne noise and vibration assessment. Since railway ground borne noise and vibration assessment becomes more and more requested for the design of new railway infrastructure, the extension of line or the construction of building closed to the lines, it becomes necessary to bound the input data uncertainty in order to get in an acceptable output range level uncertainty.

¹ loic.grau@morphosense.com

² bernard.laulagnet@insa-lyon.fr

The source of railway vibration has been widely studied by D. Thompson in the so called “TWINS” model [1]. Many numerical and experimental studies show that it represents a good approximation of the force contact at the wheel/ rail interaction [8]. The ground structure interaction has been model by different methods with different degree of precision. The FEM/BEM modelling allows to consider many geometrical cases which requires long computation time [7]. Analytical and semi analytical modelling allows on the other hand to model only simple geometry with fast computation time [2]. All these models are based on the same ground modelling (electrodynamics model) which is the propagation path of waves coming from the pass-by train. Finally, the building model is often based on database measurement (RIVAS project). Except the classical FEM software, there is a BEM/FEM model called MEFISSTO [6] which can consider the building function transfer with a lot of details.

This paper aims at making a synthesis and combine of all this model to get what is of interest of all railway assessment which is the ground borne noise and vibration in buildings. We focus our study on the influence of the ground uncertainty input data on the ground borne noise and vibration.

2. GROUND BORNE NOISE AND VIBRATION MODELLING FOR RISK ASSESMENT

In this section, we present models used for the simulation of ground borne noise and vibration risk assessment. This model which goes from the excitation source to the reception building is the one used for the Monte Carlo simulation presented in section 3.

2.1 Vehicle model

The source of vibration is the rolling stock with the roughness. It must be said that the vibration due to moving load due to the passage of the train is of second order compared to those due to the roughness in the frequency range of interest.

2.1.1 Rolling stocks

In the frequency range of interest, it is assumed that the rolling stock deformation is negligible. The vehicle is therefore considered as a stack of four masses (coach, bogie, axle and wheel) linked together by three sets of damped springs (secondary suspension, primary suspension and rubber ring) as shown in Figure 1.

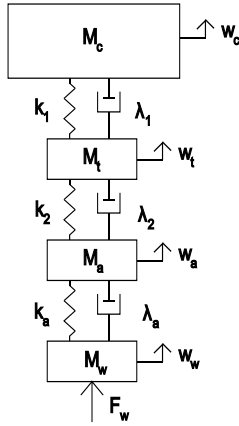


Figure 1 : Overview of the model for the rolling stock

The linear equations that govern the motion of the rolling stock are:

$$\left(\begin{bmatrix} k_1 & -k_1 & 0 & 0 \\ -k_1 & k_1 + k_2 & -k_2 & 0 \\ 0 & -k_2 & k_2 + k_a & -k_a \\ 0 & 0 & -k_a & k_a \end{bmatrix} - \omega^2 \begin{bmatrix} M_c & 0 & 0 & 0 \\ 0 & M_t & 0 & 0 \\ 0 & 0 & M_a & 0 \\ 0 & 0 & 0 & M_w \end{bmatrix} \right) \begin{bmatrix} w_c \\ w_t \\ w_a \\ w_w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ F_w \end{bmatrix}$$

Where M_c, M_t, M_a and M_w represents the mass of the coach, the bogie the axle and the wheel respectively and k_1, k_2 and k_a represents the secondary suspension, the primary suspension and rubber ring.

The mobility of the wheel is given by:

$$Y_w = \frac{w_w}{F_w}$$

2.1.2 Roughness

A spectrum of roughness is an estimation of the statistical properties of the roughness record over the length of measurement. A roughness spectrum represents the general geometrical condition of the running surface.

The roughness level, L_r , is expressed in decibels and given by the following equation:

$$L_r = 10 \cdot \log_{10} \left(\frac{r_{RMS}^2}{r_{ref}^2} \right)$$

Where r_{RMS} is the root mean square roughness level in μm and r_{ref} is the reference roughness; $1 \mu m$. This definition of roughness level applies to values measured either in the form of a wavelength.

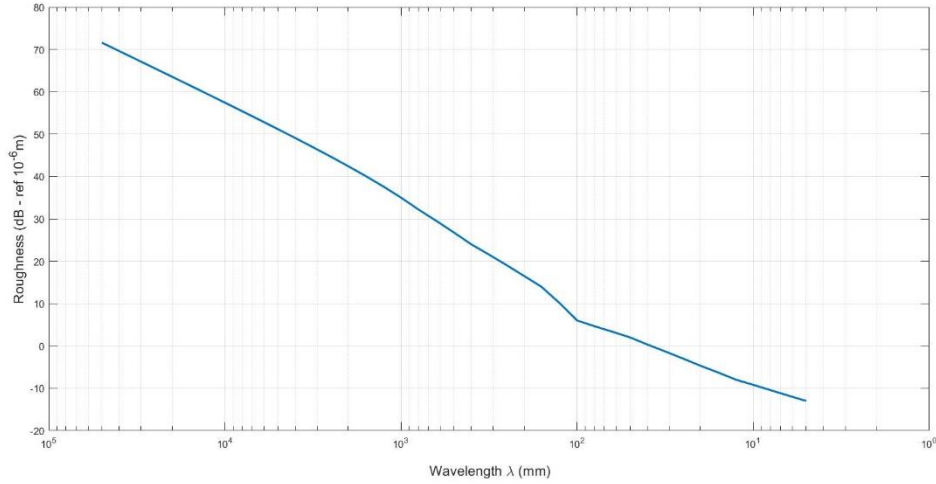


Figure 2 : An example roughness spectrum of guidance EN 15610 used in this paper

When the train runs at speed V over undulations of the surfaces with wevalength λ , these produce vibration at a frequency f given by:

$$f = \frac{\lambda}{V}$$

Typical wavelength of roughness relevant are between about 0.1 μm and 1 mm.

In the interaction model described above, the surface roughness is assumed to excite the wheel/rail system at the contact point. The contact filter for a circular contact is given by:

$$|H(k)|^2 = \frac{1}{1 + \frac{\pi}{4}(ka)^3}$$

2.2 Railway platform

The continuity of the displacement at the contact between the wheel and the rail is determined from the displacement of the track, the rolling stock and the contact spring (Hertz contact). Introducing the roughness, $r(\omega)$, which insure the contact, it comes the following relation:

$$w_r = r + w_w - w_c$$

Using this formula (TWINS Model [1]), it is possible to derive the force contact:

$$F_r = \frac{r}{Y_r + Y_w + Y_c}$$

Where Y_r , Y_w and Y_c is the mobility of the rail, the wheel and the contact.

In the section 2.1.1, it has been shown how to estimate the mobility of the wheel as well as the roughness used this paper. In this section, we focus

Contact

There are several different models used for the contact between the wheel and the rail but a simple and realistic is the Hertz model. The contact stiffness due to the elastic

deformation creating a contact zone. The contact area is supposed elliptical with semi-axes a and b . If the contact area is circular:

$$a = \sigma_1 \left(\frac{3F_0 R_0}{2E} \right)^{\frac{1}{3}}$$

The interaction is modelled by a contact spring of stiffness K_H :

$$w_c = \frac{F}{K_H}$$

Where the contact spring mobility is defined as:

$$Y_c = \frac{1}{K_H} = \frac{\xi}{2} \left(\frac{2}{3E^2 F_0 R_0} \right)^{\frac{1}{3}}$$

With E is the Young modulus of both bodies assumed to have the same material properties, F_0 is the mass applied by the rolling stock to the rail at the contact and R_0 is the effective radius of curvature of the surfaces in contact.

2.2.1 Rail-sleeper- platform

A classical railway track is composed of a rail laid upon pads that protect the sleeper from high impact forces. The sleepers are laid upon the platform (ballast or concrete). The support of the rail is assumed continuous.

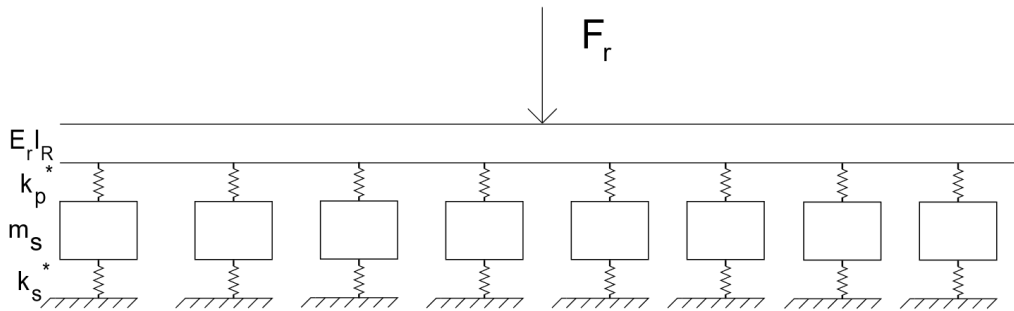


Figure 3 : Beam on two layer

The rail is described as a Timoshenko beam and is described with the deflection w_r and rotation ϕ_r of the cross section. The equation of motion for a Timoshenko beam on elastic foundation $s(\omega)$ per unit length is:

$$E_r I_r \frac{d^4 w_r(x)}{dx^4} + s w_r(x) - m_r \omega^2 w_r(x) = F \delta(x)$$

where E_r , I_r and m_r are the Young modulus, second moment of area and density of the rail. F denotes the punctual force applied to the rail at the position $x = 0$.

The term $s(\omega)$ depends on kind of foundation which is in this case a spring pad and a sleeper mass per unit length and the platform. The stiffness can be written as:

$$s(\omega) = \frac{k_p(k_s(\omega) - \omega^2 m_s)}{(k_p + k_s(\omega) - \omega^2 m_s)}$$

Where k_p and m_s are the stiffness of the pad and the density of the sleeper per unit length. The term $k_s(\omega)$ correspond to the stiffness of the platform which is discuss in section 2.2.2.

The solution for the transfer mobility can be found using the Fourier transform which is given by:

$$Y_r = \frac{w_r(x)}{F} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{E_r I_r k^4 + s(\omega) - m_r \omega^2} e^{-ikx} dk$$

where k is the wavenumber.

2.2.2 Ground platform interaction and Ground propagation

In this section, the railway platform is modelled and Figure 1 gives an overview of the problem under consideration. The railway platform is assumed to be a semi-infinite slab coupled to the ground and excited by a punctual force.

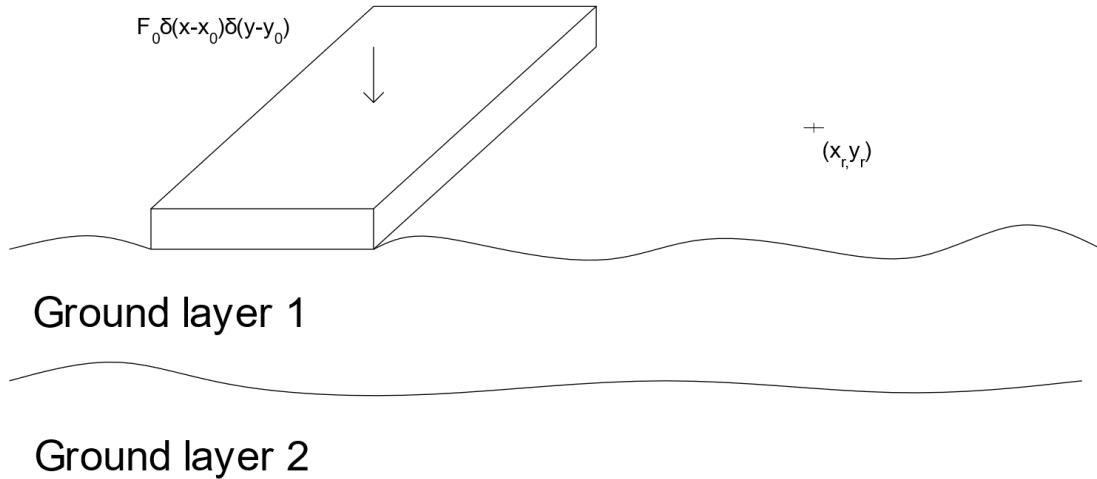


Figure 4 : Railway platform coupled to the a 2 layers ground

The semi-infinite slab is modelled using the Kirchhoff-Love hypothesis so we neglect the shear deformation and the rotary inertia. In the frequency domain, the equation of motion is:

$$D_s^* \nabla^4 w_s(x, y) - \rho_s h_s \omega^2 w_s(x, y) = F_0 \delta(x - x_0) \delta(y - y_0) + \sigma_s(x, y)$$

Where $D_s^* = D_s(1 + j\eta_s)$ is the complex flexural stiffness, F_0 is the amplitude of the force applied on the slab to the point $(x_0; y_0)$. $\sigma_s(x, y)$ represents the stress due to the ground/slab coupling.

The unknowns of the problem are the displacements of slab which can be expanded in series of slab mode and a Fourier transform:

$$w_s(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \sum_m a_m \phi_m(y) e^{jk_x x} dk_x$$

The ground is modelled with Navier's equation which considers a continuous, homogeneous and isotropic elastic medium:

$$\mu \nabla^2 \vec{u} + (\mu + \lambda) \nabla(\nabla \cdot \vec{u}) + \rho \omega^2 \vec{u} = \vec{0}$$

where $\vec{u}^T = \{u_x, u_y, u_z\}$ is the vector of ground displacement.

The ground is a semi-infinite medium in z-direction and infinite in the direction x- and y-direction. Tangential stresses along x- and y-direction with respect to the z are zero at the surface. Normal stress along the z axis is also zero everywhere on the surface $z = 0$ except under slabs. The expression of the ground displacement at the top surface in the Fourier domain can be put in the following form:

$$\tilde{u}_z(k_x, k_y, 0) = N(K_x, k_y) \tilde{\sigma}_{zz}(k_x, k_y, 0)$$

The linear system to evaluate is:

$$\sum_m ((D_s^* k_m^4 - \rho_s h_s \omega^2) a_m - F_m) \gamma_{mq} = S_q a_q$$

- Punctual mobility of the platform

It is now possible to give the expression of the modal amplitude by solving the linear system above.

$$k_s = \frac{w_s(x_0, y_0)}{F_0}$$

- Propagation from the platform at the ground top surface

We are also interested to determine the displacement at the ground surface. A pass by train is represented as a sum of uncorrelated force applied to the platform. The modal amplitude of the slabs allows to determine the ground displacement by carrying out an inverse 2D Fourier transform of expression:

$$u(x, y, 0) = \sqrt{\sum_i \left| \sum_m ((D_s^* k_m^4 - \rho_s h_s \omega^2) a_m - F_m^i) T_m(x, y) \right|^2}$$

Where the summation over i corresponds to the sum of uncorrelated force applied to the platform at the position (x_i, y_i) .

2.3 Building transfer

These transfers are not simulated and deduced from experimental data. Immission corresponds to ground-building vibration interactions and is also decomposed into three steps:

- ground to building foundation (transfer function TF2, usually an attenuation), leading to foundation vibration levels (Lv3):

$$TF2 = L_{foundation} - L_{freefield\ ground}$$

- building foundation to floor (transfer function TF3, usually an amplification due to the floor first resonant vibration modes):

$$TF3 = L_{floors} - L_{foundation}$$

- floor vibration to ground borne noise (transfer function TF4) corresponding to sound radiated by vibrating structures and leading to sound pressure levels in the room (Lp). The following frequency dependent transfer function is used, based on an energy approach [5]:

$$TF4 = 10 \cdot \log_{10}(\sigma) + 10 \cdot \log_{10}\left(\frac{4S}{A}\right)$$

where σ is the radiation efficiency of the floor, S its surface area and A the absorption area of the room; the reference for sound level is $2 \cdot 10^{-5}$ Pa and for velocity level $5 \cdot 10^{-8}$ m/s. A 3 dB constant is often added assuming both floor and ceiling are the main ground borne source in the room. Ground borne noise (room space average sound level) is estimated in a separate module from the floor space average velocity level according to the building acoustics theory.

The transfer function from the ground (free field) to the building is give, by the formula in decibel:

$$TF_{building} = TF2 + TF3 + TF4$$

3. MONTE CARLO SIMULATIONS

The Monte Carlo Method relies on repeated random sampling to solve problems that might be deterministic in principle. This method is used in many physical domains for many years [3,4]. This technique is particularly well suited for this type of simulation as the random processes involved in any event occurring.

The method allows investigation of the possible outcomes of a series of unpredictable situations in order to assess the impact, allowing for better decision making under uncertainty.

Probabilistic techniques are commonly used in simulations where millions of random processes (such as light scattering) govern the observed properties of certain parameters.

In this paper, the ground properties range are computed using a normal (Gaussian) function. The probability density of the normal distribution is:

$$f(x|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where μ is the mean of the distribution (and also its median and mode), σ is the standard deviation and σ^2 is the variance. Ground properties are sampled using a random number generator.

Since the model introduced in section 2 is not time consuming and takes less than 5 min to run, the Monte Carlo Method is well suited to investigate the influence of the input parameters uncertainties.

4. NUMERICAL RESULTS

In this section, we present results from the simulation using a Monte Carlo procedure. Using the modelling introduced in section 2, the vibration and acoustic level of interest in a railway vibration risk assessment is the ground borne vibration and noise defined as follow:

$$\begin{cases} L_{floor} = 10 \cdot \log_{10} \left(\frac{|u(x, y, 0)|^2}{u_{ref}^2} \right) + TF2 + TF3 \\ L_p = L_{floor} + TF4 \end{cases}$$

A set o 100 simulation has been done where we look at the mean and deviation of the vibration and acoustic level. In the following simulation, we only consider the uncertainty on the ground. The ground under consideration is a two layers ground with the following characteristics:

Characteristics	Layer 1	Layer 2
Shear celerity (m/s)	200	400
Dilatational celerity (m/s)	400	800
Mass density (Kg/m3)	2000	2000
Damping (%)	3	3
Thickness (m)	5	∞

Table 1: Ground Characteristics used in the simulation

This section focuses on the influence of the ground's uncertainties. The deviation applied to shear celerity, dilatational celerity, damping, density and thickness are 50m/s, 100m/s, 2%, 100 kg/m³ and 1m respectively. The Figure 5 gives the random value of the celerity used for the 100 simulations.

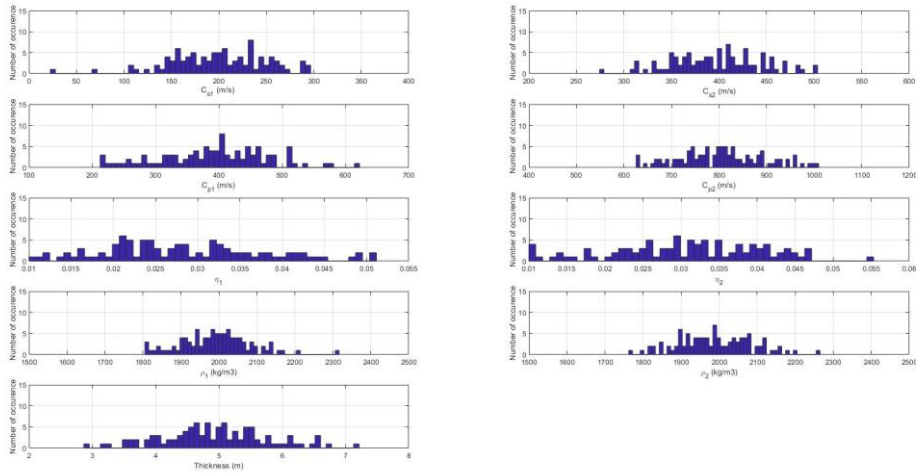


Figure 5 : Histogram of the ground properties for 100 simulations – case where all ground parameters are considered to follow a normal distribution

Figure 6 shows the ground borne noise and vibration in a building due to the pass by train. Because the function transfer from the floor vibration to the acoustics radiation in the room is constant, the deviation on the vibration level is identical as the deviation on the acoustics level. The deviation of the level per frequency goes up to $\pm 5\text{dB}$ and is 2.9dB in global level. Going into more details, it is possible to identify that the deviation at low frequency is mainly due to the uncertainty of the shear celerity while the deviation at high frequency is mainly due to the uncertainty of the damping. The ground density and the dilatational uncertainty are of second order between the shear celerity, the damping and the ground thickness.

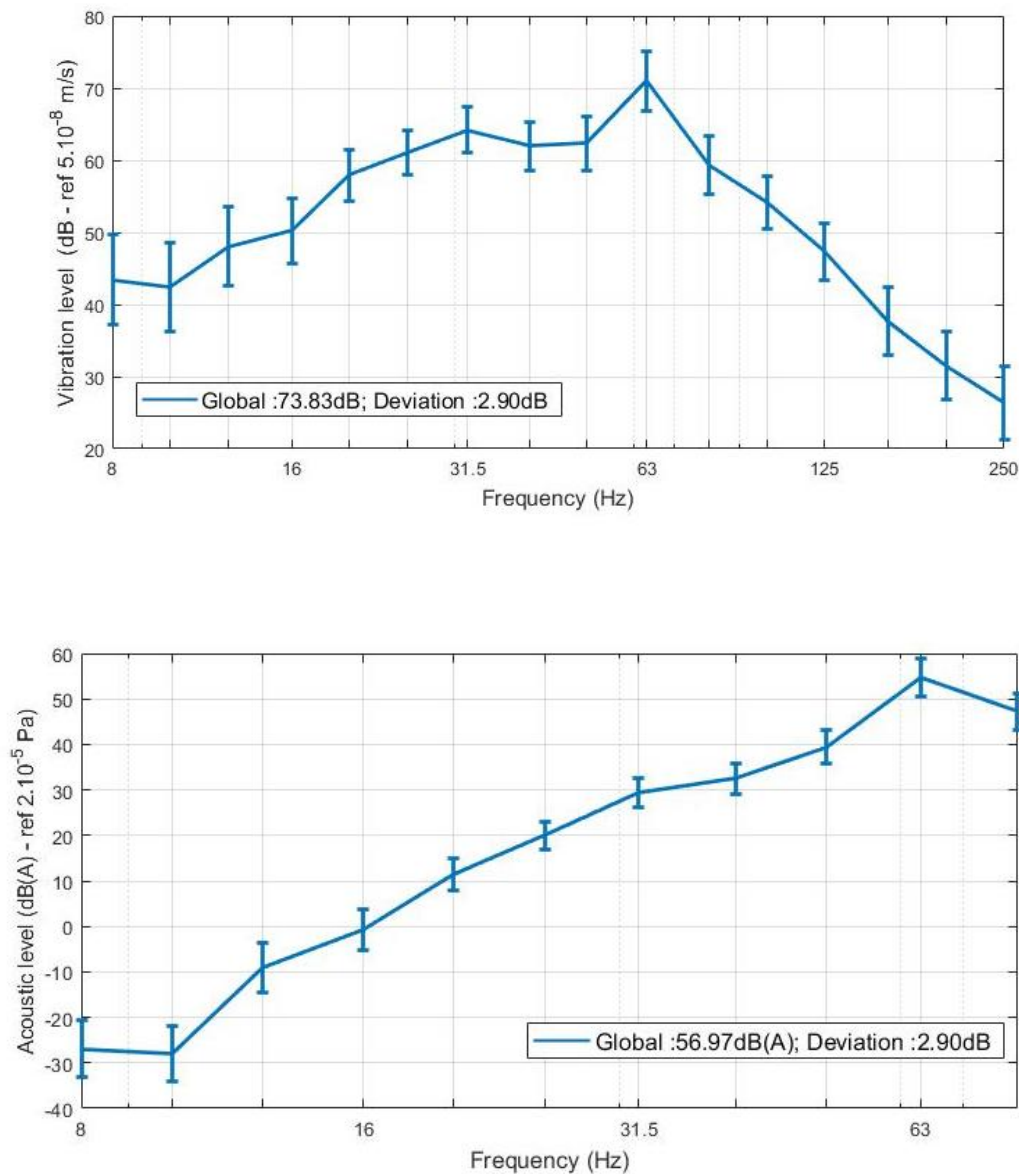


Figure 6 : Mean spectrum and deviation of the vibration level at the ground top surface (top left), the force level applied to the railway platform (top right), the ground borne vibration of the ceiling (bottom left) and the ground borne noise in a room of the building (bottom right)

5. CONCLUSIONS

This paper is a first attempt to introduce the uncertainty in a global model of railway ground borne noise and vibration. Since only the case of ground uncertainty has been considered, it already shows that for a two layers ground, the shear celerity needs to stay in a range of ± 50 m/s and the damping in a range of 1% to have global level lower than ± 3 dB. Further investigation aims at giving uncertainty for all input data in a railway ground borne noise and vibration assessment.

7. REFERENCES

1. David Thompson, “*Railway Noise and Vibration, Mechanisms, Modelling and Means of Control*”, Elsevier (2009)
2. Loïc Grau and Bernard Laulagnet, “*Ground cross-modal impedance as a tool for analyzing ground/plate interaction and ground wave propagation*”, The Journal of the Acoustical Society of America 137, 2901 (2015)
3. Albert Tarantola, “Inverse Problem Theory and methods for Model Parameter Estimation”,
4. Frédéric Marin, “*HERITAGE: A Monte Carlo code to evaluate the viability of interstellar travels using a multi-generational crew*”, Journal of the British Interplanetary Society, 70, 184, (2017)
5. Carl F. Eyring, “*Reverberation time in “dead” rooms*”, J. Acoust. Soc. Am, 1 217, (1930)
6. Philippe Jean, “*A 2.75D model for the prediction of noise inside buildings due to train traffic.*”, Acta Acustica united with Acustica, 104, (2018)
7. Michel Villot, Philippe Jean, Loïc Grau and Simon Bailhache, “*Predicting railway-induced ground-borne noise from the vibration of radiating building elements using power-based building acoustics theory.*”, International Journal of Rail Transportation, 6, (2018)
8. M. Villot, E. Augis, C. Guigou-Carter, P. Jean, P. Ropars, S. Bailhache and C. Callais, “*Vibration emission from railway lines in tunnel – characterization and prediction.*”, International Journal of Rail Transportation, 4, (2016)