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Bayesian inference for layered porous material analysis

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ABSTRACT

In noise control applications, porous materials are often inhomogeneous with depth. For better understanding of sound absorbing mechanism, the depth-dependent inhomogeneity of the porous media is approximated by multilayered porous absorbers of finite-thickness layers with each layer being considered as homogeneous. This work applies the Bayesian probabilistic inference to analyze multilayer porous materials, developing a method to determine simultaneously the number of constituent layers and the physical properties of each layer. The Bayesian analysis is based on experimental measurements of the acoustic surface impedance of a potentially multilayered material sample with multilayer porous acoustic propagation models. The number of layers considered in the propagation model is varied so as to formulate a finite set of different porous materials models, and Bayesian model selection is applied in estimating the model with a parsimonious number of layers present in the sample. Once the number of layers has been determined, Bayesian parameter estimation inversely estimates the physical properties of each layer.

Keywords: Absorptive materials, porous materials
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1. INTRODUCTION

This paper presents a recent effort using two levels of Bayesian inference to analyse porous materials in multiple layers. This paper is heavily replying on the most recent publication by the authors [1]. Recent publications have also reported on Bayesian methods of parameter estimation applied for the characterization of single-layered porous materials [2-3]. In these two papers, Bayesian analysis is exploited to inverse the physical parameters of porous materials from acoustical measurements in an impedance tube. The current study investigates an extension to these existing studies in such a way that the Bayesian framework includes a porous model selection. It represents a higher level of Bayesian inference to estimate the number of layers present in material samples under investigation, beyond parameter estimation, a lower level of Bayesian inference.

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Furthermore, the prior probabilities for inverted parameters are assigned to be broad, uninformative distributions so that the inverted parameter values are objectively estimated, predominantly from the experimentally measured acoustic data. The approach is a model-based method, besides the parameter estimation, Bayesian model selection has also carried out using a model of potentially multiple layers. The model selection estimates the number of layers in the measured data. The following discussions begin first with the model formulation, followed by the formulation of two levels of Bayesian inference. The results are then demonstrated.

2. MULTI-LAYER MODEL FORMULATION

There exist a number of porous material models [4,5], the current work employs Miki generalized empirical model [6] that describes a single layer homogeneous porous material by its acoustic propagation coefficient as

$$\gamma(f) = \frac{2\pi f\sqrt{\alpha_\infty}}{c_0}\left\{0.160\left(\frac{f}{\sigma_e}\right)^{-0.618} + i\left[1+0.109\left(\frac{f}{\sigma_e}\right)^{-0.618}\right]\right\}, \qquad (1)$$

and its characteristic impedance as

$$Z_c(f) = \rho_0 c_0 \frac{\sqrt{\alpha_\infty}}{\phi}\left[1+0.070\left(\frac{f}{\sigma_e}\right)^{-0.632} - i\,0.107\left(\frac{f}{\sigma_e}\right)^{-0.632}\right], \qquad (2)$$

with the effective flow resistivity,

$$\sigma_e = \frac{\phi}{\alpha_\infty}\,\sigma_f\ , \qquad (3)$$

where ϕ is porosity, α_∞ is tortuosity, σ_f is flow resistivity, ρ_0, c_0 are the air density and speed of sound, and $i = \sqrt{-1}$. Miki model describes a porous material comprising cylindrical tubes oriented at an arbitrary angle to the surface normal.

Potential multi-layered configurations employ the transfer matrix method for modeling one layer equivalent-fluid materials of thickness *d* [5] by a matrix

$$\mathbf{T}_{\mathrm{eq}} = \begin{bmatrix} \cosh(\gamma d) & \sinh(\gamma d) Z_c \\ \sinh(\gamma d)/Z_c & \cosh(\gamma d) \end{bmatrix}, \qquad (4)$$

where γ is the propagation coefficient of the equivalent fluid as given in Equation 1, and Z_c is the characteristic impedance as given in Equation 2. When this layer of the material is terminated by a rigid backing, the sound pressure, p , and the normal particle velocity, v_x , along x-direction can be determined by

$$\begin{bmatrix} p \\ v_x \end{bmatrix}_{x=0} = \mathbf{T}_{\text{eq}} \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \tag{5}$$

where the subscript $x = 0$ indicates the front material surface. For multiple *N* equivalent-fluid porous layers, a chain of square transfer matrices replaces the single transfer matrix in Equation 4 for each distinct layer. Equation (5) is extended, leading to

$$\begin{bmatrix} p \\ v_x \end{bmatrix}_{x=0} = \mathbf{T}_{\text{eq}}^{(1)} \cdot \mathbf{T}_{\text{eq}}^{(2)} \cdots \cdot \mathbf{T}_{\text{eq}}^{(N)} \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \tag{6}$$

where the superscript *(n)* denotes the transfer matrix for *n*th equivalent-fluid layer determined by Equation 4. The equivalent-fluid layers (and corresponding transfer matrices) are stacked with layer 1 being the front, its surface is defined at $x = 0$ and layer *N* is adjacent to the rigid backing. Sound pressure, p, and the normal particle velocity, v_x, along x-direction at the surface of the 1st layer material are used to determine the complex-valued surface impedance,

$$Z_{\text{s}} = \frac{p}{v_x}. \tag{7}$$

This surface impedance of multiple layer structure is used as 'model' in Bayesian model-based inference in the following. The acoustical data experimentally measured using the transfer function method in an impedance tube are in form of normal incident surface impedance, which are used for the inversion by the two-levels Bayesian inference.

3. TWO-LEVES OF BAYESIAN INFERENCE

This work intensively uses Bayes' theorem in two levels of inference. This section first pursues Bayesian model selection, followed by Bayesian parameter estimation. The acoustical data in form of vector are denoted as $\mathbf{D} = [Z_{\text{s}}]$. They contain experimentally measured surface impedance as function of frequency, while the surface impedance predicted using Equation 7 through Equations (4,6) are denoted as $\mathbf{M}_n$ for the predicted surface impedance of *n*-layers porous materials.

3.1 Bayesian model selection

The model selection applies Bayes' theorem to calculate the probability of the model, $\mathbf{M}_n$, given data, $\mathbf{D}$, over a set of *N* models,

$$p(\mathbf{M}_n \mid \mathbf{D}) = \frac{p(\mathbf{D} \mid \mathbf{M}_n) p(\mathbf{M}_n)}{p(\mathbf{D})}, \quad \text{for} \quad n = 1, \ldots, N, \tag{8}$$

where $p(\mathbf{D})$ is a constant once the data are collected. $p(\mathbf{D} \mid \mathbf{M}_n)$ is marginal likelihood, also termed '*evidence*'. Quantity $p(\mathbf{M}_n)$ is prior probability of model, $\mathbf{M}_n$. It encodes prior knowledge about the models. The key quantity here is the evidence, also denoted as $Z_n = p(\mathbf{D} \mid \mathbf{M}_n)$ for simplicity. When comparing two models, Bayes factor is used

$$\frac{p(\mathbf{M}_i \mid \mathbf{D})}{p(\mathbf{M}_j \mid \mathbf{D})} = \frac{Z_i}{Z_j} \frac{p(\mathbf{M}_i)}{p(\mathbf{M}_j)}, \quad (9)$$

This work assigns equal prior to all the models under test, so the Bayes factor becomes

$$\frac{p(\mathbf{M}_i \mid \mathbf{D})}{p(\mathbf{M}_j \mid \mathbf{D})} = \frac{Z_i}{Z_j}. \quad (10)$$

Eventually, the data prefer one of the models.

3.2 Bayesian parameter estimation

Once a suitable model is selected, the parameter estimation applies Bayes' theorem to determine the probability of the porous parameters, $\mathbf{\Theta} = [\phi, \alpha_\infty, \sigma_f]$ given the model, $\mathbf{M}_n$ and the data, $\mathbf{D}$, as

$$p(\mathbf{\Theta} \mid \mathbf{M}_n, \mathbf{D}) = \frac{p(\mathbf{D} \mid \mathbf{\Theta}, \mathbf{M}_n)\, p(\mathbf{\Theta} \mid \mathbf{M}_n)}{p(\mathbf{D} \mid \mathbf{M}_n)}, \quad (11)$$

where $p(\mathbf{D} \mid \mathbf{\Theta}, \mathbf{M}_n)$ is likelihood function. $p(\mathbf{\Theta} \mid \mathbf{M}_n)$ is prior probability of $\mathbf{\Theta}$ given the model, $\mathbf{M}_n$. It encodes prior knowledge about the parameters prior to the data analysis. Quantity, $p(\mathbf{D} \mid \mathbf{M}_n)$ is independent of parameter, $\mathbf{\Theta}$. In this context, it serves as a normalization constant. Note $p(\mathbf{D} \mid \mathbf{M}_n)$ is exactly the same as the marginal likelihood in Equations 8, 9, denoted as $Z_n = p(\mathbf{D} \mid \mathbf{M}_n)$. It plays a central role in the model selection. It is calculated by integral of the likelihood, $L(\mathbf{\Theta}) = p(\mathbf{D} \mid \mathbf{\Theta}, \mathbf{M}_n)$ multipled by the prior, $p(\mathbf{\Theta} \mid \mathbf{M}_n)$, over the entire parameter space. This work applies nested sampling for all the calculations of the probabilities. Consult Fackler *et al.* [1] for detailed exposition of the Bayesin formulation and nested sampling.

4. EXPERIMENTAL RESULTS

Both Bayesian model selection and Bayesian parameter estimation are carried out using the experimentally measured data. When two layers porous materials[1], Melamine foam and Armafoam Sound, AFS 240 foam, are measured in a configuration of Melamine foam over AFS with the Melamine foam being the outer layer exposed to the normal incident sound waves. Figure 1 shows the Bayesian inferential results. The estimations have also explored when the layer thicknesses are taken as unknown. With both known and

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[1] Melamine form, Foam Techniques Limited, Wellingborough, Northamptonshire, UK and AFS 240 foam, Armafoam Sound, AFS; Armafoam Sound: Armacell UK Ltd, Oldham, Lancashire, UK.

unknown thickness, Bayesian inference is capable to yield inversion results in terms of the number of layers, and layer parameters. Table I lists all the parameters estimated.

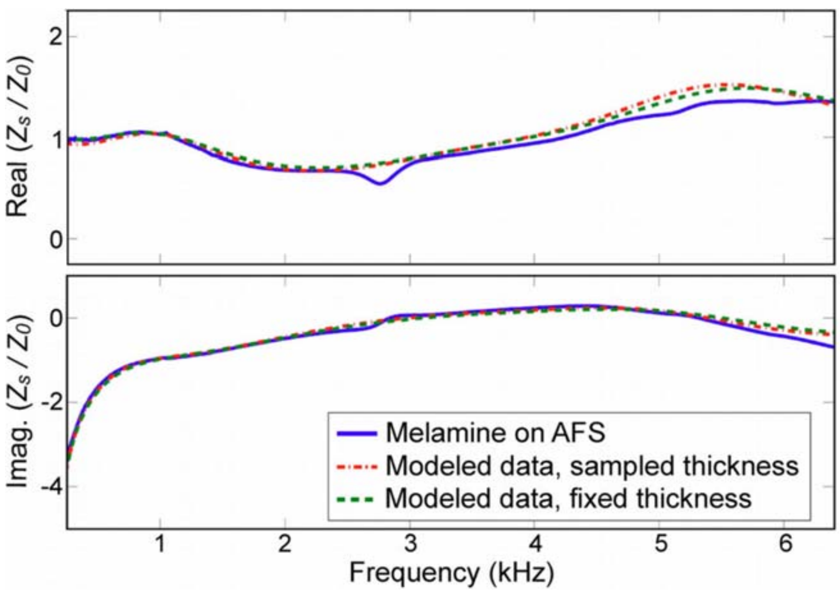

Figure 1. Comparison between experimentally measured normal incident, complex-valued surface impedance of multiple porous layers and the model prediction. Bayesian model selection determines two layers in the measured data, and the modelled data are determined through Bayesian parameter estimation.

5. CONCLUDING REMARKS

Two levels of Bayesian inference have been applied to the porous material inversion. Miki model is used to describe one layer among potentially multiple layers of porous materials. The normal incidence surface impedance experimentally measured in n impedance tube serves the experimental data. The model selection and porous parameter estimation are carried out within a unified Bayesian framework. Nested sampling is exploited to the both levels of inference. For the two layers porous materials, the Bayesian inference demonstrates its capability to infer inversely the number of layers and the physical parameters of foam materials. The detailed exposition of this analysis has recently been published in a peer-reviewed archival journal paper [1].

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TABLE I. Bayesian estimation of porous parameters (mean 6 standard deviation) using experimentally measured data in form of r the acoustic surface impedance, directly taken from a publication by the authors [1]. The materials used are the combination of melamine foam on AFS 240 foam. Two-layer setting is obtained using the Miki generalized model, containing three parameters. The layer thickness is estimated from measured acoustic data (top) and fixed at the known value (bottom). Directly measured porosity, ϕ , and flow resistivity, σ_f , from a round robin test [7] are also listed for ease of comparison.

Layer (sampled thickness)	Melamine	AFS 240
Layer thickness, d (cm)	2.60 ± 0.01	2.39 ± 0.26
Flow resistivity, σ_f (Ns/m^4)	7360 ± 140	$108\,200 \pm 8380$
(Directly measured), σ_f (Ns/m^4)	9900 ± 800	$141\,400 \pm 44\,000$
Porosity, ϕ	1.00 ± 0.00	0.96 ± 0.07
(Directly measured), ϕ	0.98 ± 0.01	0.80 ± 0.02
Tortuosity, α_∞	1.00 ± 0.00	5.07 ± 0.62
Layer (fixed thickness)	Melamine	AFS 240
Layer thickness (cm)	2.50	2.50
Flow resistivity, σ_f (Ns/m^4)	8050 ± 160	$108\,260 \pm 2{,}420$
(Directly measured), σ_f (Ns/m^4)	9900 ± 800	$141\,400 \pm 44\,000$
Porosity, ϕ	1.00 ± 0.00	0.93 ± 0.01
(Directly measured), ϕ	0.98 ± 0.01	0.80 ± 0.02
Tortuosity, α_∞	1.01 ± 0.01	4.16 ± 0.10

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