MADRID
inter.noise 2019
June 16 - 19
NOISE CONTROL FOR A BETTER ENVIRONMENT

One Approach for Analyzing Natural Vibration of Submerged Cylindrical Shell Near a Rigid Wall

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ABSTRACT

When conducting experiments in a water tank, the tank wall, is expected to make a difference on test results. One approach is proposed to get clear the effect of rigid wall on vibration of typical structure, *i.e.*, finite cylindrical shell. The present method is verified by conducting comparative analysis on coupled modal frequencies from present method and the FEM. When the shell is close to rigid wall, the rigid effect will lead to obvious reduction of coupled modal frequency, and it could be finally neglected if the distance is large enough.

Keywords: Modal frequency, Rigid wall, Cylindrical shell
I-INCE Classification of Subject Number: 42

1. INTRODUCTION

During an underwater experiment for submerged structures in a water tank, the tank wall is expected to have an effect on test results. So it's very important to investigate the effect of tank wall on structural vibration. The impedance of tank wall is much larger compared with that of fluid medium, so the wall could be simplified to be rigid surface.

For the problem of structural vibration in bounded space, many works have been done. Seybert et al.[1-2] modified the form of Helmholtz integral equation to analyze acoustic radiation and scattering performance of structure in half-space. Gaunaurd [3] applied the image method to study acoustic scattering characteristics of a ball near a infinite plane. Ye et al. [4] applied the wave propagation approach to predict far-field

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sound pressure of a infinite long cylindrical shell submerged near rigid wall. Li et al.[5] analyzed the acoustic radiation problem of a infinite cylindrical shell submerged below the free surface. It should be noted that above works are mainly about vibration and acoustic radiation problem of infinite long cylindrical shell in half-space. Here, one approach is put forward to analyze the free vibration of a finite cylindrical shell submerged near a rigid wall.

2. THEORECTICLA DERIVATION

2.1 Margin Settings

Problem considered here is shown in Fig.1. *L, R, h* are shell length, mean radius and thickness. *E, μ,* ρ_s. are material elastic modulus, Poisson's ratio and density. The shell axis is parallel to the rigid surface, and the distance is *H*. A baffled shell model is applied here, and a cylindrical co-ordinate system (*r, θ, z*) is also applied.

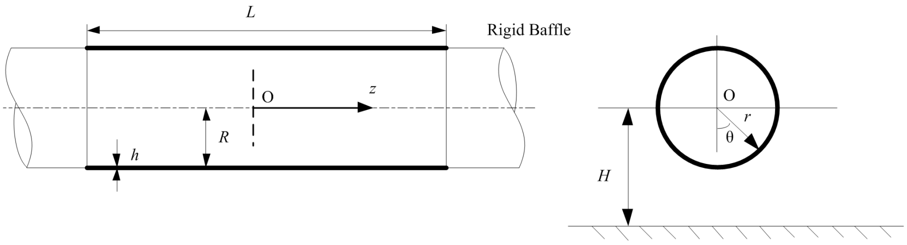

Fig.1. A cylindrical shell submerged in half-space bounded by a rigid surface

Motion of shell. Referring to the Flügge shell theory [6], motion equations of a submerged cylindrical shell near a rigid wall may be

$$[\mathbf{G}]\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{\left(1-\mu^2\right)R^2}{Eh}\begin{bmatrix} 0 \\ 0 \\ -p|_{r=R} \end{bmatrix} \tag{1}$$

where $[\mathbf{G}]$ is the differential operator matrix, the elements are described in Reference [6]. *u, v, w* represent the shell displacements in axial, circumferential and radial directions.

Motion of fluid. Here, the fluid is assumed to be acoustic medium, so sound pressure in fluid should be governed by the Helmholtz equation [7]

$$\nabla^2 p + k_f^2 p = 0 \tag{2}$$

where $k_f = \omega / c_f$, c_f is the sound speed in the acoustic medium. At the rigid wall, normal velocity of fluid should be zero, $\partial p / \partial n = 0$.

For problem considered here, the rigid wall will bound fluid and reflect acoustic wave from shell. So sound pressure in fluid contained two parts, radiated pressure from the shell and its reflections

$$p = p^r + p^i \tag{3}$$

where p^r, p^i are sound pressure from the shell and its image (in the view of image method).

Using the wave propagation approach [8], shell displacement and sound pressure could be expanded in the wave propagation form

$$\begin{cases} u = \sum_{m=0}^{+\infty}\sum_{n=0}^{+\infty} U_{mn}\cos(n\theta)\sin\left(k_m z\right) \\ v = \sum_{m=0}^{+\infty}\sum_{n=0}^{+\infty} V_{mn}\sin\left(n\theta\right)\cos\left(k_m z\right) \\ w = \sum_{m=0}^{+\infty}\sum_{n=0}^{+\infty} W_{mn}\cos\left(n\theta\right)\cos\left(k_m z\right) \end{cases} \tag{4}$$

$$\begin{cases} p^r = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} p_{mn}^r H_n^{(2)}(k_r r)\cos(n\theta)\cos(k_m z) \\ p^i = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} p_{mn}^i H_n^{(2)}(k_r r^{'})\cos(n\theta^{'})\cos(k_m z) \end{cases} \tag{5}$$

where U_{mn}, V_{mn}, W_{mn} are displacement amplitudes of mode (*m, n*) in three directions. k_m is the axial wave-number. When the finite shell is simply-supported for both ends, $k_m = m\pi/L$ [9]. Harmonic motion is assumed for problem here. p_{mn}^r, p_{mn}^i are the sound pressure amplitudes from the shell and its image, $H_n^{(2)}()$ is the nth order Hankel function of the second kind, $k_r^2 = k_f^2 - k_m^2$.

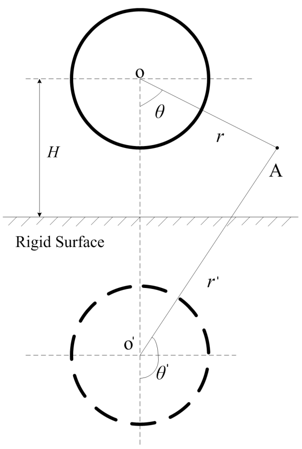

Fig.2. Schematic diagram for the image method

For simplicity of analysis, sound pressure from the image should be rewritten in the shell coordinate with the Graf additional theory [10]

$$H_n^{(2)}(k_r r^{'})\cos(n\theta^{'}) = \begin{cases} \sum_{a=-\infty} (-1)^n H_{n+a}^{(2)}(2k_r H) J_a(k_r r)\cos(a\theta) & r < 2H \\ \sum_{a=-\infty} (-1)^n J_{n+a}(2k_r H) H_a^{(2)}(k_r r)\cos(a\theta) & r > 2H \end{cases} \tag{6}$$

where $J_n()$ is the nth order Bessel function of the first kind.

Continuity condition at the fluid-shell interface require that

$$-\frac{1}{\mathrm{i}\rho_f \omega}\frac{\partial p^r}{\partial r}\bigg|_{r=R} = \frac{\partial w}{\partial t} \tag{7}$$

With analysis above, one may derive the sound-structure coupling equation for problem considered here

$$[\mathbf{T}]\begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{8}$$

where $[\mathbf{T}]$ is the coefficient matrix for mode (*m, n*), and its elements are

$T_{11} = \Omega^2 - \lambda^2 - n^2(1+K)(1-\mu)/2$, $T_{12} = -\lambda n(1+\mu)/2$, $T_{21} = T_{12}$,

$T_{13} = -[\mu\lambda + K\lambda^3 - n^2 K\lambda(1-\mu)/2]$, $T_{22} = \Omega^2 - n^2 - (1+3K)\lambda^2(1-\mu)/2$,

$T_{23} = -n[1 + K\lambda^2(3-\mu)/2]$, $T_{32} = -T_{23}$, $T_{31} = -T_{13}$, $T_{33} = 1 + K[1 - 2n^2 + (\lambda^2 + n^2)^2] - \Omega^2 + FL$.

$\Omega = \omega R\sqrt{\rho_s(1-\mu^2)/E}$ is the non-dimensional frequency, $\lambda = k_m R$. FL denotes the fluid loading term, $FL = FL^r + FL^i$. $FL^r = \rho_f \omega^2 \frac{R^2(1-\mu^2)}{Eh}\frac{H_n^{(2)}(k_r R)}{k_r H_n^{(2)\prime}(k_r R)}$,

$FL' = \rho_f \omega^2 \dfrac{R^2(1-\mu^2)}{Eh} \sum_{a=0}^{\infty} \dfrac{(-1)^a \rho_f \omega^2 \Delta J_n(k_r R)}{k_r H_a^{(2)\prime}(k_r R)}$, $\Delta = (-1)^{a+n} H_{a-n}^{(2)}(2k_r H) + (-1)^a H_{a+n}^{(2)}(2k_r H)$. FL_i reflect the effect of rigid wall on structural vibration. By solving the dispersion equation of Eq.(8), one can predict the natural frequencies of cylindrical shell submerged in fluid near a rigid wall.

3. NUMERICAL EXAMPLE

Numerical model. L=1.284m, R=0.18m, h=0.003m, ρ_s =7850 kg/m^3 , μ=0.3, E =206MPa, ρ_f=1025 kg/m^3 , c_f=1460 m/s.

Convergence analysis. To calculate FL, the subscript a is set to be $a = 0, 1, \cdots\cdots, A$. Convergence analysis is conducted in case of H=0.2m to ensure good accuracy, and the result are shown in Fig.3. it's noted that $A = 10$ is enough to ensure the convergence of present method.

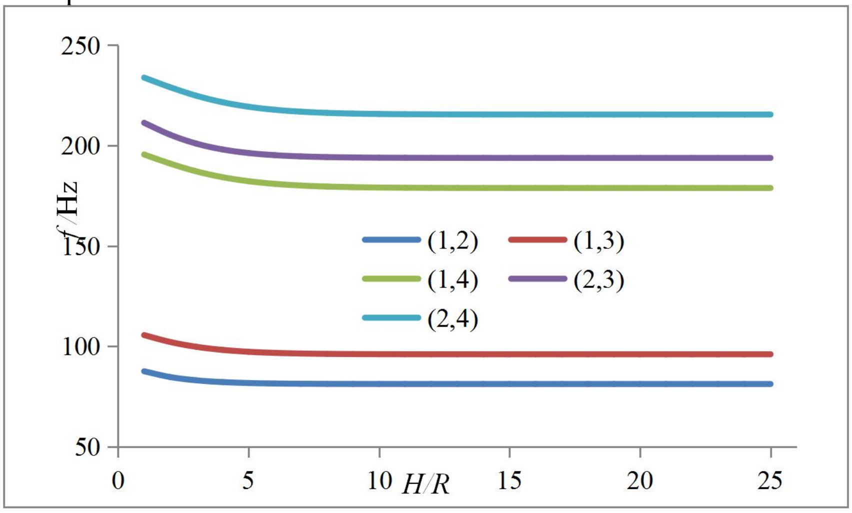

Fig.3. Convergence analysis of the present method

Validation. The theoretical results are compared with numerical results from the FEM with ANSYS, comparative results are shown in Table 1. Parameter $\eta_2 = (f_T - f_F)/f_F$ is defined to show the relative differences, while f_T , f_F are result from present approach and FEM. it is clear that coupled modal frequencies from the present method generally agree well with those from the FEM. The relative differences are relatively small, and the maximum is less than 3.7%. So the present method is reasonable to predict natural frequency of cylindrical shell submerged near rigid wall.

Table 1. Validation of the present method

Mode	H=0.15 m			H=0.2 m			H=0.5 m		
	f_T /Hz	f_F /Hz	η_2 /%	f_T /Hz	f_F /Hz	η_2 /%	f_T /Hz	f_F /Hz	η_2 /%
(1,2)	89.7	94.2	-4.8	97.9	98.3	-0.4	98.8	98.5	0.3
(2,2)	105.1	110.2	-4.6	109.2	111.5	-2.1	109.3	111.6	-2.0
(1,3)	196.5	207.7	-5.4	202.3	208.8	-3.1	202.4	209.0	-3.2
(2,3)	210.6	216.4	-2.6	216.9	219.6	-1.2	217.0	219.7	-1.2

Rigid wall effect. To make clear the effect of rigid wall on structural vibration, comparative analysis on coupled modal frequency in the cases of 'rigid wall' and 'Infinite field' is carried out, the results are shown in Fig.4.

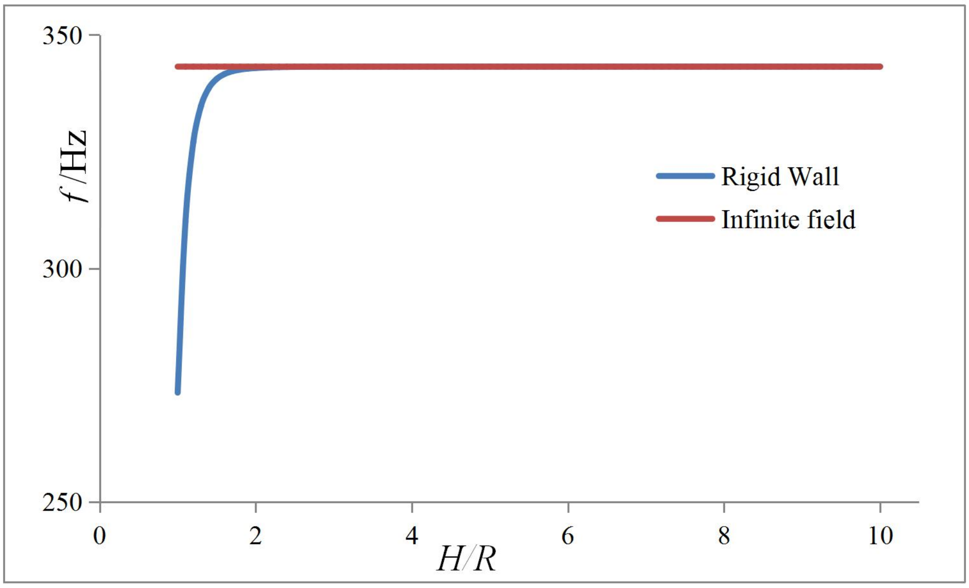

Fig.4 Rigid wall effect on coupled modal frequency of mode (1,5)

From Fig.4, one can note that coupled modal frequency in the case of 'Rigid wall' is much smaller than that from the case of 'Infinite field' when the shell is near the rigid wall. But, with the increase of distance H, coupled modal frequency in case of 'Rigid wall' will increase and approach that in the case of 'Infinite field', which means the reduction of rigid wall effect. So the rigid wall effect could be neglected if the shell is far enough away.

4. CONCLUSIONS

(1) An effective approach is proposed to analyze the natural vibration of cylindrical shell with finite length submerged near a rigid wall.

(2) Rigid wall will lead to obvious reduction of coupled modal frequency.

(3) With the increase of distance between shell axis and rigid wall, effect of rigid wall on coupled modal frequency could finally be neglected.

5. ACKNOWLEDGEMENTS

The authors wish to express their gratitude to the National Natural Science Foundation of China (Contract No.11802213, 11502176, 51579109) that has supported this work.

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