

## **Real-time prediction of noise signals for active control based on Bayesian forecasting and time series analysis**

**Siu-Kai, LAI<sup>1</sup>**

**Department of Civil and Environmental Engineering, The Hong Kong Polytechnic University**

**Hung Hom, Kowloon, Hong Kong, P.R. China**

**Yi-ting, ZHANG<sup>2</sup>**

**Department of Civil and Environmental Engineering, The Hong Kong Polytechnic University**

**Hung Hom, Kowloon, Hong Kong, P.R. China**

### **ABSTRACT**

Rapid urbanization has intensified environmental noise pollution due to hectic construction activity. Generally, conventional noise barriers/enclosures can provide passive noise control, but do little to mitigate low-frequency noise, as the acoustic wavelength is significantly longer than the dimension of barrier-type structures, leading to inefficient coupling. In this study, we consider a smart barrier/enclosure featuring a combination of both passive and active noise control functions for dealing with construction noise, in which the passive approach is regarded as a low-pass filter to remove the high-frequency components of noise signals. The filtered signals are mainly focused on a relatively narrow bandwidth that can be treated by active noise control. An active control technique based on the superposition principle is a promising alternative approach, in which anti-noise speakers emit antiphase acoustic waves to reduce undesirable noise. The effectiveness of active control systems is mainly governed by adaptive algorithms. However, a time delay issue is still inevitable for most adaptive systems. To reduce the time delay and enhance the accuracy of adaptive systems, real-time estimation of noise signals based on a Bayesian inference-based dynamic linear model is proposed in this work. Bayesian forecasting is a learning approach that enables the computation of conditional probabilities, and a dynamic linear model is a mathematical tool for times series analysis. On-site measured data were used as observation values for time series analysis with the dynamic linear model. When the Bayesian forecasting approach is combined with times series analysis, prior information and measurement data are naturally integrated for better condition identification and assessment. Based on the Bayesian statistical framework, a “*forecast-observation-analysis*” cycle can be formulated to mitigate time-delay issues in adaptive noise control.

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<sup>1</sup> [sk.lai@polyu.edu.hk](mailto:sk.lai@polyu.edu.hk)

<sup>2</sup> [yi-ting.zhang@connect.polyu.hk](mailto:yi-ting.zhang@connect.polyu.hk)

## 1. Introduction

Due to the advancement of construction technologies, there has been a rapid increase in the use of powered mechanical equipment for construction work. The noise generated by operating machines (e.g., hammering or drilling) can cause annoyance and psychological discomfort to human beings in the vicinity. The simple way to reduce these adverse effects is to use passive noise control to obstruct the propagation of noise, e.g., through deploying noise barriers or enclosures. However, such an approach is not useful for the isolation of low-frequency noise, especially below 1,000 Hz, although some previous studies have tackled this issue using acoustic metamaterials [1, 2]. An alternative approach is the use of active noise control (ANC), based on the principle of superposition, for attenuating low-frequency noise. The use of ANC strategies in noise barriers/enclosures to deal with environmental noise problems was proposed in the 1990s [3, 4]. In conventional active control systems, it is desirable for anti-noise devices to be governed digitally; in this way, noise signals from electro-acoustic or electro-mechanical transducers can be sampled and processed in real time using a digital signal processing system.

The development of digital signal processing chips has enabled low-cost implementation of powerful adaptive algorithms. A suitable algorithm is the kernel of an adaptive ANC system. In the past, standard Least Mean Square (LMS) algorithms [5] have been used, in which a secondary-path transfer function is introduced as a convergence controller. However, this leads to an inevitable time delay effect because the error signal is not correctly aligned in time with the reference signal. To cope with this problem, the Filtered-x Least Mean Square (FxLMS) algorithm [6] was proposed to place an identical filter for controlling the reference signal by a weighted optimization.

In general, ANC systems can be classified into three categories according to their structural design: feedforward, feedback, and hybrid feedforward/feedback [5, 7]. In terms of computational algorithms, a modified version of the LMS algorithm called the Filtered-x LMS is widely used due to its simple and effective computations. Because of this main advantage, the time-domain FxLMS algorithm was developed, which offered easier implementation and higher robustness; however, it has distinct limitations, because convergence results are greatly affected by a slight disturbance [8]. To deal with the convergence issue, the frequency-domain FxLMS algorithm was then proposed, and high-order adaptive filters were applied to verify computer simulations [9, 10]. In addition, a hybrid active noise control system has also been proposed [11]. This system implements active control for the uncorrelated disturbance of reference signals by making use of structural design and time-domain signal processing. Based on these adaptive algorithms, the integration of ANC and time-frequency analysis can be a useful tool for condition monitoring and fault diagnosis in structural dynamics [12]. For real-time experiments, a multi-channel ANC system has also been designed to perform the attenuation of nonstationary and intermittent noises [13].

In the literature, many research studies have focused on the development of complicated computational algorithms, which may lead to high computational cost and long convergence time. In real-life engineering environments, construction noise as a major noise pollution source is a critical issue in city development. Therefore, we explore the use of ANC with a simpler computational algorithm for the mitigation of construction noise. In what follows, a digital signal preprocess before ANC is presented, which uses a Bayesian forecasting approach and a dynamic linear model. If a high accuracy prediction based on the reference signal can be provided before the controlling process, the efficiency of such an adaptive algorithm can be greatly optimized by processing the reference signal to generate a new acoustic wave. Therefore, the Bayesian inference-

based dynamic linear model can be used to carry out “one-step” (short-term) signal prediction to mitigate the impact of time delay.

Bayesian forecasting is one of the best known approaches in statistical analysis. It has been widely used in multiple disciplines, such as econometrics, structural dynamics, and aerospace engineering [14]. Nevertheless, there are few examples of its being used for signal prediction in acoustic engineering. This approach possesses great potential, especially in acoustic engineering, because there are many types of modeling for noise signals and many parametric uncertainties. The basic element of a Bayesian updating model includes prior information, a likelihood function, and posterior information. The Bayesian strategy has been widely adopted for decision rules used to classify data patterns. It provides a way to infer unknown model parameters from known measurements. Building upon the Bayesian statistical framework, the developed model can be verified and updated with reduced uncertainty when new monitoring data become available.

Dynamic linear models (DLMs) are a mathematical tool for time series analysis, which can be used to describe a routine way of viewing context that changes with time. West and Harrison [15] summarized various structures and theories of dynamic models that incorporate Bayesian forecasting. The derivation details about how to analyze time series data were also presented [16]. Due to its versatility and applicability, there are many potential applications in mechanical and structural engineering. For instance, Bayesian forecasting has been combined with DLM for gas turbine performance detection [17]. In addition, Zhang et al. [18] recently conducted an online condition assessment of high-speed trains based on a combination of time series analysis and Bayesian inference.

In this work, a digital signal preprocess, based on Bayesian forecasting and time series analysis, is explored for analyzing noise data measured on-site. A real-time updating process for the adaptive fitness of digital signals is demonstrated. An illustrative case is presented and discussed. For signal processing of construction noise, both computational effort and convergence rate are crucial. Hence, a DLM with state space priors will be developed in the Bayesian context to simplify the adaptive algorithm of ANC in real-life engineering environments. Based on the present study, a longer time-length prediction study will be undertaken, and verified based on the next segment of observation data.

## 2. Methodology

### 2.1 Dynamic Linear Model

Dynamic linear models are a time series analysis method that uses mathematical and statistical models, in which “dynamic” refers to changes over time [16]. To describe a time series using such models, observation signals are generally broken into several elements, e.g., value, gradient, and curvature. Indeed, it is simple to use DLMs to form a series of analysis models. The modeling process with a second-order DLM needs only two elements, i.e., value and gradient, as shown in Figure 1 [17].

The DLM can be formulated using a set of state-space equations. Based on the information available at the last time cycle  $t-1$ , the guess values for the parameters (value and gradient) can be calculated. It can be regarded as a special case of the state-space model, and the integration of the state-space equation is as follows [16-18]:

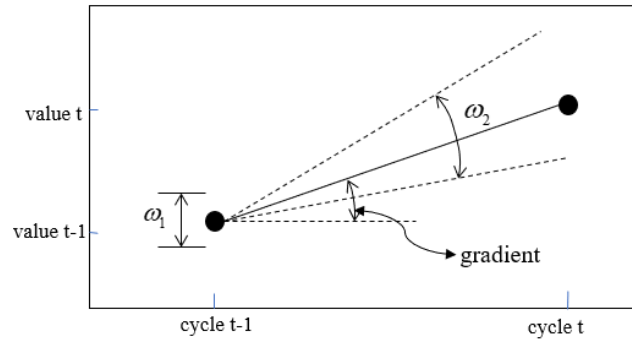
$$\text{Observation equation: } Y_t = F_t^T \cdot \Theta_t + v_t, v_t \sim N[0, V_t] \quad (1)$$

$$\text{Evaluation equation: } \Theta_t = G_t^T \cdot \Theta_{t-1} + w_t, w_t \sim N[0, W_t] \quad (2)$$

where  $F_t$  is a  $p \times 1$  vector of known parameters,  $G_t$  is a  $p \times p$  matrix of known constants, and  $\Theta_t$  is the state vector that contains two elements, i.e.,  $\mu_t$  and  $\beta_t$ . In the form of a state-space model, they can be written as

$$\begin{aligned} \Theta_t &= \begin{pmatrix} \mu_t \\ \beta_t \end{pmatrix}, \quad F_t = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad G_t = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \\ W_t &= \begin{pmatrix} \sigma_{level}^2 & 0 \\ 0 & \sigma_{trend}^2 \end{pmatrix}, \quad V_t = \sigma_{observation}^2 \end{aligned} \quad (3)$$

where  $\sigma_{level}^2$  and  $\sigma_{trend}^2$  are the variance of the parameter values and the change rate of the parameters, respectively. The measurement variance is equivalent to the square of its standard deviation  $\sigma_{observation}^2$ .



**Figure 1:** Second-order DLM process modeling

## 2.2 Bayesian Forecasting

Based on Bayes' rule, prior density provides concise and coherent transferred information to the posterior probability of a model parameter. With a time-varying observed process, it can be formed using a probability density function. The notation is given by [16-18]

$$(\Theta_t | D_t) \sim N[m_t, C_t] \text{ with } D_t = Y_1, Y_2, \dots, Y_t \quad (4)$$

where  $\{D_t = Y_1, Y_2, \dots, Y_t\}$  stand for the observations.

In terms of noise signal inference, the Bayesian updating approach can be utilized based on observation data to provide real-time estimation for undesired noise with a short-term time length. Therefore, the perturbation of random events can be avoided, thereby reducing the computational effort of adaptive control systems. It is worth noting that the noise signals being processed are limited to relatively stable and regular construction noise patterns, such as automobile engines or turbine motors. The combination of Bayesian inference and a dynamic linear model can provide a “*forecast-observation-analysis*” cycle, which can promote high-accuracy inference before adaptive processing. In another word, it dramatically mitigates the inevitable time delay effect by ensuring accuracy. The logic of Bayesian inference equations is presented next; the derivations can be found in Ref. [15]. Given the state at time  $t-1$ :

$$(\Theta_{t-1} | D_{t-1}) \sim N[m_{t-1}, C_{t-1}] \quad (5)$$

Predictions are performed by the following one-step forecast equations at time  $t$ :

$$(\Theta_t | D_{t-1}) \sim N[a_t, R_t] \quad (6)$$

$$(Y_t | D_{t-1}) \sim N[f_t, Q_t] \quad (7)$$

where the means and variances are given by

$$a_t = G_t \cdot m_{t-1}, \quad f_t = F_t^T \cdot a_t, \quad R_t = G_t C_{t-1} G_t^T + W_t, \quad Q_t = F_t^T R_t F_t + V_t \quad (8)$$

Taking into account the measurement at time  $t$ , corrections are given by updating the equation

$$(\Theta_t | D_t) \sim N[m_t, C_t] \quad (9)$$

where the means and variances become

$$m_t = a_t + A_t e_t \quad (10)$$

with  $A_t = R_t F_t / Q_t$ ,  $e_t = Y_t - f_t$  and  $C_t = R_t - A_t A_t^T Q_t$ .

Then, the logic of inference can be demonstrated as follows:

**Derivative steps:**

- (1) Posterior for  $\Theta_{t-1}$ :  $(\Theta_{t-1} | D_{t-1}) \sim N[m_{t-1}, C_{t-1}]$ .
- (2) Prior for  $\Theta_t$ :  $(\Theta_t | D_{t-1}) \sim N[m_{t-1}, R_t]$  with  $R_t = C_{t-1} + W_t$ .
- (3) One-step forecasting:  $(Y_t | D_{t-1}) \sim N[f_t, Q_t]$  with  $f_t = m_{t-1}$  and  $Q_t = R_t + V_t$ .
- (4) Posterior for  $\Theta_t$ :  $(\Theta_t | D_t) \sim N[m_t, C_t]$  with  $m_t = m_{t-1} + A_t e_t$  and  $C_t = A_t V_t$

where  $A_t = \frac{R_t}{Q_t}$  and  $e_t = Y_t - f_t$ .

Finally, if a forecast of the next  $k$  time steps is of interest, the  $k$ -step forecasting equations are given by [17]

$$(\Theta_{t+k} | D_t) \sim N[a_t(k), R_t(k)] \quad (11)$$

$$(Y_{t+k} | D_t) \sim N[f_t(k), Q_t(k)] \quad (12)$$

with the following parameters:

$$\begin{aligned} a_t(k) &= G_{t+k} \cdot a_t(k-1), \quad f_t(k) = F_{t+k}^T \cdot a_t(k), \\ R_t(k) &= G_{t+k} R_t(k-1) G_{t+k}^T + W_{t+k}, \quad Q_t = F_{t+k}^T R_t(k) F_{t+k} + V_{t+k} \end{aligned} \quad (13)$$

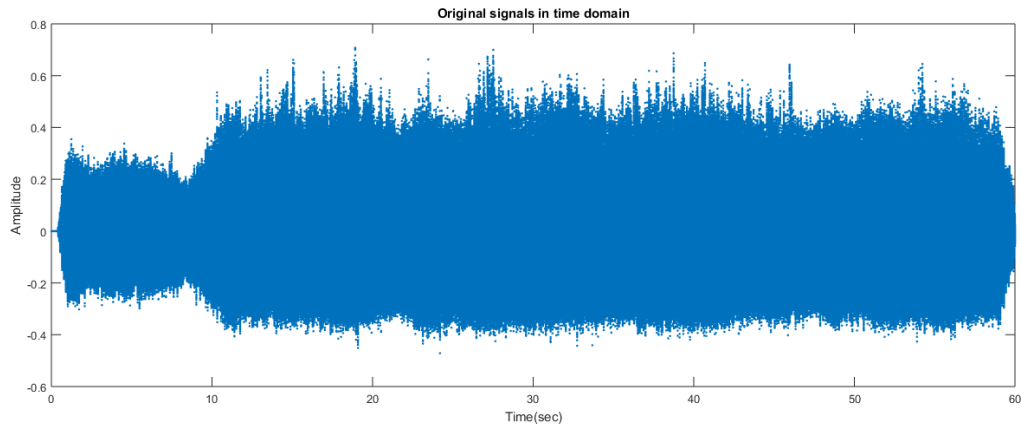
### 3. Data analysis and discussion

#### 3.1 Main frequency position detection and inference

A set of engine machine noise sequences is plotted in Figure 2. In this study, the noise signals from construction machines were measured on-site, see Figure 3. Using a time-domain analysis, the noise sequences are separated into many equivalent-time frames (i.e., 0.02 s), with windows added to reduce the loss of these signals. Given the disturbance of multiple sources and the application of fast Fourier transform in later signal processing, every equivalent-time frame signal is treated as a stationary signal. Before framing, a finite impulse response low-pass filter is adopted to simulate the filtering of high-frequency components in the original signals, the reason being that ANC is mainly focused on low-frequency components.

To reduce computational effort, a frequency-domain analysis is adopted. The convergence results are presented in Figure 4 using the short-time fast Fourier transform and power spectrum density. The short-time main frequency point can be determined as the maximum power point in the frequency spectrum within a sub-frame. The main frequency peak is prominent in the frequency spectrum, which can be used to determine the spectrum bandwidth. It is efficient to investigate the internal regularity of engine noise data. In Figure 5, it is obvious that the main frequency component is centered at roughly

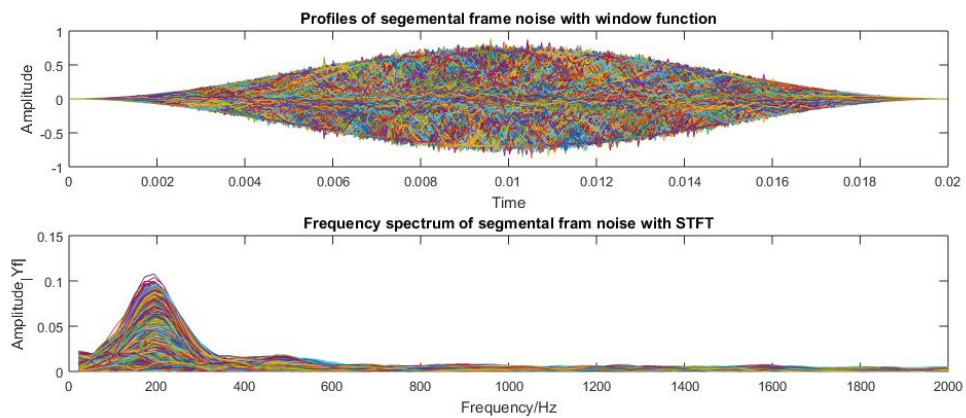
193 Hz. In other words, the principal frequency does not change dramatically as time goes on. [Figure 6](#) traces the peak value of the power spectrum in every frame signal over time.



**Figure 2:** On-site measured sound signals



**Figure 3:** On-site measurement equipment setup

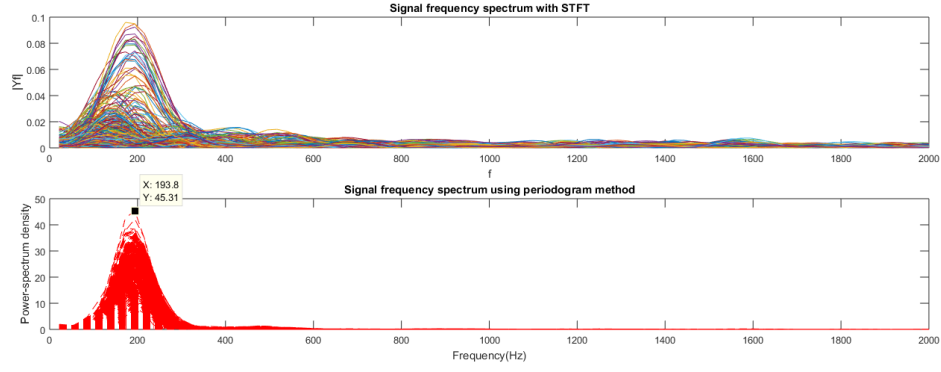


**Figure 4:**

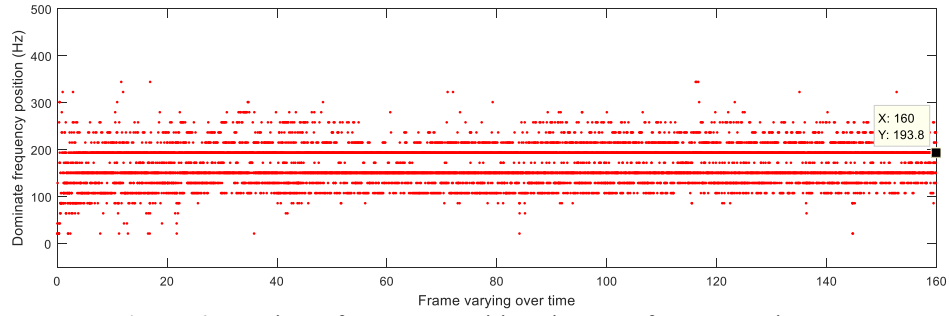
(Top) Profiles of segmental frame noise with equivalent-time cycle of 0.02 s

(Bottom) Frequency spectrum of segmental frame noise using short-time Fourier transform





**Figure 5:** Frequency spectrum of segmental frame noise with STFT and PSD

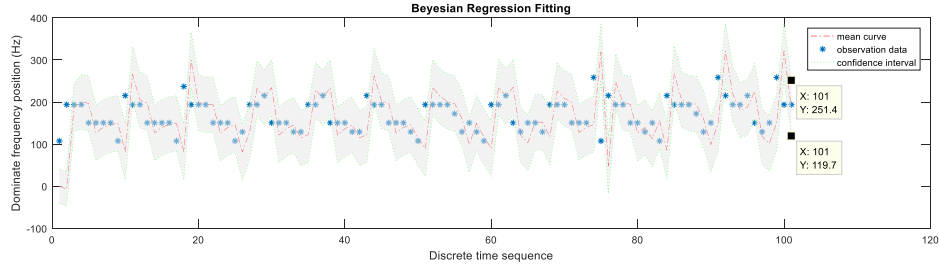


**Figure 6:** Dominant frequency positions in every frame over time

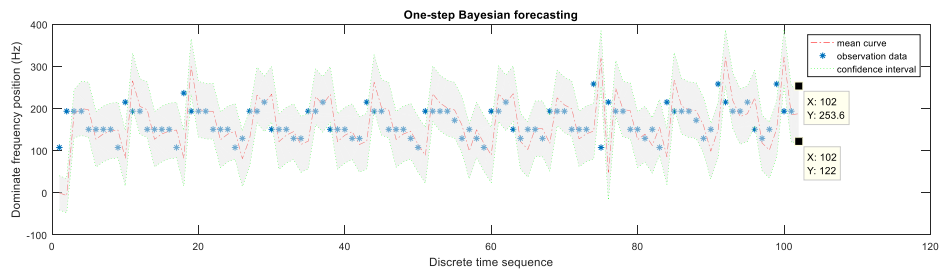
As mentioned earlier, the main frequency can be determined as the peak value in the short-time power spectrum, which is 193 Hz in this case. However, using an ANC technique, the main frequency component process is not enough to slow down the real sound pressure level, unless a specific spectrum bandwidth can be treated. In this case, the main frequency position varying over time can be treated as a set of time sequences, and the Bayesian inference-based dynamic linear method can be used to provide a short-time frequency bandwidth for real-time inference. Direct comparison with the time domain analysis shows that frequency domain analysis can reduce the computational effort by avoiding the disturbance of scaled and shifted impulses that can make the system cumbersome. With increased training data, the accuracy of inference can be enhanced, and one-step forecasting becomes possible. The predicted length is approximately 10 frames, with high accuracy. The predicted length enlargement to chase the time delay effect is a crucial research topic. Figures 7-9 show the real-time inference of the main frequency position.

In Figure 9, at the beginning of inference, there is a fluctuating segment that cannot cover the observation data. With the presence of more observation data, the adaptive updating process can converge and continuously provide a “*forecast-observation-analysis*” cycle. The confidence bandwidth is 95%. To enlarge the inference length, a  $k$ -step forecasting process is applied, and the results are calculated using  $k=10$ . By further increasing the  $k$  value, the covered bandwidth can be enlarged, but the accuracy is lower. The triangle symbols stand for the observation data in the following time sequence, which is the same as the blue asterisks in the top of Figure 9. To verify the accuracy of  $k$ -length inference, the triangle symbols are plotted. In other words, when  $k=10$ , inference accuracy can still be maintained, as the data (triangle symbols) remain in the confidence bandwidth, as presented in the bottom of Figure 9. Because the  $k$  value is not too large, short-term prediction is still workable and accurate using the Bayesian inference-based dynamic

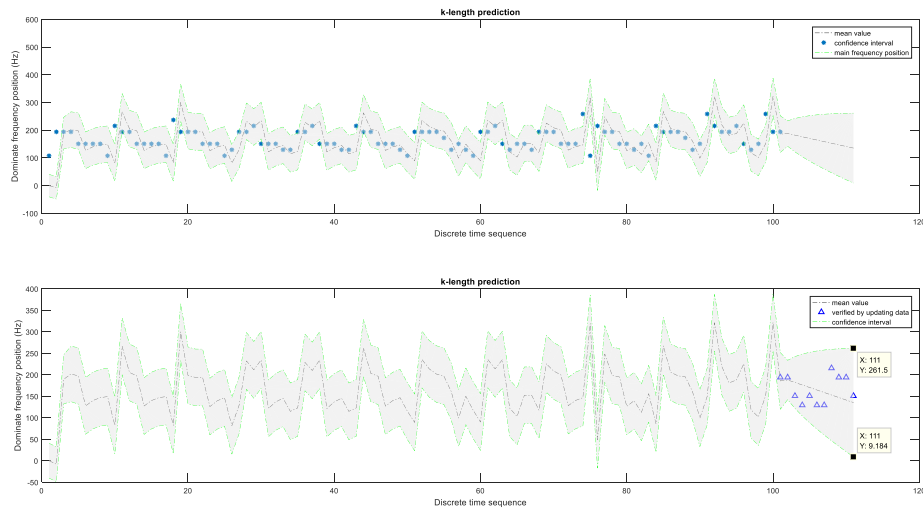
linear method. In short, the Bayesian approach can act as a filter when it is applied to a time series analysis due to its basic form. By choosing the variance  $\omega$ , the behavior of the filter can be adjusted to be highly sensitive or offer a high degree of smoothing.



**Figure 7:** Regression fitting of the main frequency position varying over time, using Bayesian inference



**Figure 8:** One-step inference of the main frequency position, using Bayesian inference and DLM

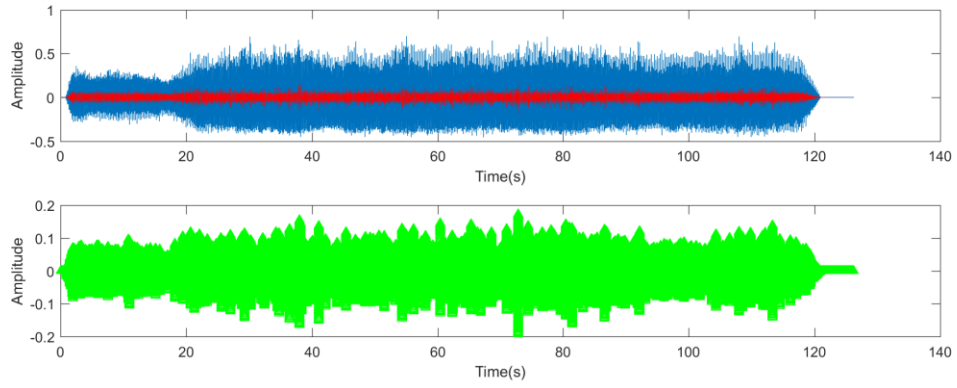


**Figure 9:** A  $k$ -step inference (when  $k=10$ ) of the main frequency position based on acquired data

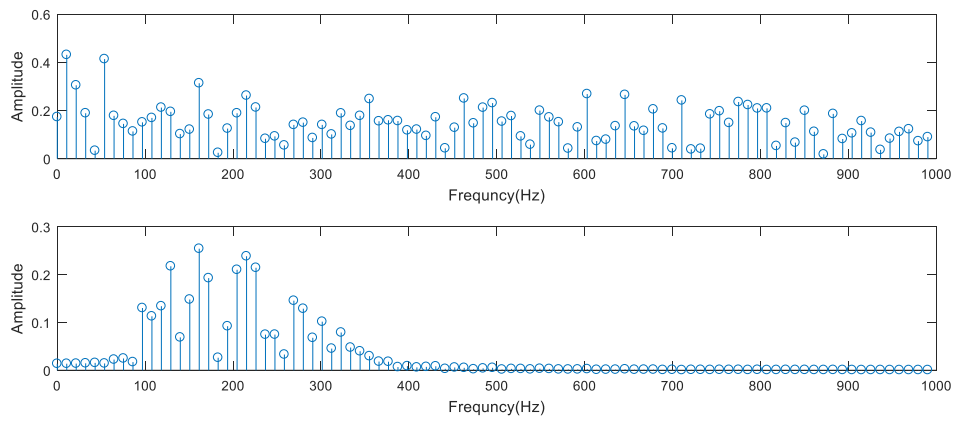
### 3.2 Value of main frequency component inference

Through detection of the main frequency component, the observation data in the time domain can be narrowed down using band-pass filtering between 100 Hz and 300 Hz. The filtered component is presented in Figures 10 and 11. As mentioned earlier, the Bayesian inference-based dynamic linear method can be used to predict the trend of data development over time. The regression fitting is provided in Figure 12. Based on this, a  $k$ -length prediction ( $k=10$ ) is also presented in Figure 13.

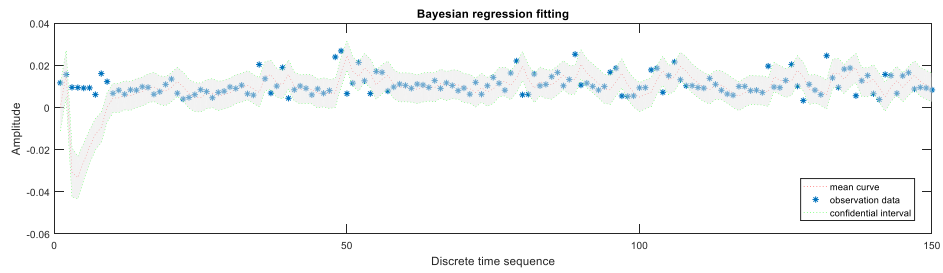




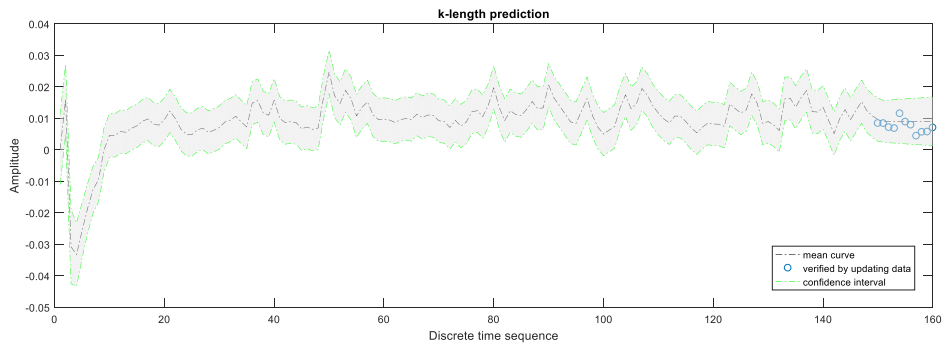
**Figure 10:** Observation data in the time domain before and after band-pass filtering



**Figure 11:** Observation data in the frequency domain before and after band-pass filtering



**Figure 12:** Bayesian fitting of observation data after band-pass filtering (100–300 Hz)



**Figure 13:** A  $k$ -step (when  $k=10$ ) prediction of observation data after band-pass filtering (100–300 Hz)

#### 4. Conclusions

Detection logics for ANC can be developed using many learning methods, but the Bayesian inference method based on DLMs has several advantages over other approaches. The most important is its statistical nature. Updating the processing of posterior information based on prior information enables the formation of an adaptive “*forecasting-observation-updating*” cycle. For ANC systems, a time-domain algorithm is generally limited by cumbersome disturbances, while a frequency-domain algorithm based on complicated transforms (e.g., fast Fourier transform or wavelet transform) is limited by convergence time. In this study, we attempted to deal with both problems by using an effective signal pre-process approach. From the perspective of practical engineering, computational effort and convergence time are two key issues. The combination of Bayesian inference and DLM can provide conjectural characteristics of noise signals to the real-time ANC system. This is a simple method for predicting the varying trend of the main frequency bandwidth and reducing the computational effort of the ANC algorithm. Once a complete system is implemented, a comparison of ANC algorithms with or without Bayesian inference and DLM can be made to identify the advantages of this method. A detailed analysis of smart noise barriers/enclosures is currently in progress, and will be subsequently presented.

#### 5. Acknowledgements

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