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Sensorless Control of Permanent Magnet Synchronous Motor Based on the fifth-order Cubature Kalman Filter

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ABSTRACT

The rotor position signal is the key parameter to control the permanent magnet synchronous motor (PMSM). In order to solve the problems caused by using the position sensor of the PMSM traditional control, such as the high cost, large volume, low reliability and susceptibility to environment disturbance and so on, a estimation method of motor speed and rotor position based on fifth-order Cubature Kalman filter (fifth-order CKF) is proposed for sensorless control of PMSM. Firstly, the discrete mathematical model of permanent magnet synchronous motor in α-β coordinate system is established, the fifth order cubature criterion is introduced, and the fifth-order CKF algorithm is derived. Then the model of sensorless vector control system with double closed loop of speed and current is built by using the fifth-order CKF algorithm. The Matlab/simulink simulation is carried out. The simulation results show that the fifth-order CKF algorithm has high estimation accuracy, steady running and good robustness, which can meet the needs of actual motor control performance.

Keywords: The fifth-order Cubature Kalman filter, PMSM sensorless control, PMSM simulation
I-INCE Classification of Subject Number:76

1. INTRODUCTION

In recent years, with the rapid development of power electronics technology, new motor control theory andrare earth permanent magnet materials, the permanent magnet synchronous motor (PMSM) has been widely used. The PMSM has the advantages of small size, low loss and high efficiency. Nowadays, more and more attention has been paid to energy saving and environmental protection, so it is necessary for PMSM to be studied. Accurate rotor position or speed signal is the key to implementing PMSM control. However, the application and development of PMSM are greatly limited by the large volume and the poor anti-harsh environment ability of the speed sensor installed at the motor shaft end. Therefore, in order to avoid various defects caused by mechanical sensors to speed regulation system, scholars have proposed the PMSM sensorless control method, which estimates the rotor position or speed through the motor terminal voltage and current, and achieves sensorless closed-loop control. Many methods are used to realize the PMSM sensorless control, including Extended Kalman filter (EKF)[1~2], Unscented Kalman filter (UKF)[3~4], high frequency injection method[5~6] and Cubature Kalman filter (CKF)[7~8]. The EKF sensorless control for PMSM has good control effect, but PMSM is a typical nonlinear system. the model needs to be linearized by taking the first term obtained by the Taylor series expansion of the non-linear function, which leads to the high-order truncation error and inadequate estimation accuracy. At the same time, it is difficult to calculate the complex Jacobian matrix. The UKF algorithm can achieve the high estimation accuracy. The mean and covariance are calculated by using the vector and matrix operations, so Jacobian matrix can't need to be calculated , but the UKF algorithm has the disadvantages of the long calculating time and easy filtering divergence. The high frequency injection method has better performance when motor running at low speed, but it has high frequency noise problem, which needs the higher requirement of the motor hardware. The CKF algorithm calculates the weights and volume points, and generates new points after the transformation of the non-linear equation to get the state estimation at the next moment. There is no need to linearize the system, but the accuracy of CKF estimation is still limited.

To overcome the limitation of the PMSM speed estimation accuracy, a sensorless control method for PMSM based on the fifth-order Cubature Kalman filter(fifth-order CKF) is proposed. Based on the PMSM mathematical model, the fifth-order spherical-radial cubature rule is deduced, and the motor rotor position and speed signals are obtained by the fifth-order CKF algorithm to construct the motor vector system and realize sensorless control. Finally, the Matlab/simulink simulation is used to verify the proposed algorithm. The results show that the proposed algorithm has higher estimation accuracy and stability than the CKF algorithm.

The rest of this paper is organized as follows: the PMSM discrete mathematical model is first established in the section 2. Next, the fifth-order CKF algorithm is introduced in the section 3. Furthermore, the simulation results are given in section 4. Finally, the conclusion section ends this paper in section 5.

2.THE PMSM DISCRETE MATHEMATICAL MODEL

According to the relationship among the voltage, flux, electromagnetic torque and mechanical motion of the surface-mounted PMSM, the PMSM mathematical model can be obtained, which can be expressed under the synchronous rotating coordinate system as follows:

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$$\begin{cases} \dfrac{di_\alpha}{dt} = -\dfrac{R_s}{L} I_\alpha + \dfrac{\Psi_m}{L} \varpi_r \sin\theta_r + \dfrac{V_\alpha}{L} \\ \dfrac{di_\beta}{dt} = -\dfrac{R_s}{L} I_\beta + \dfrac{\Psi_m}{L} \varpi_r \cos\theta_r + \dfrac{V_\beta}{L} \\ \dfrac{d\varpi_r}{dt} = \dfrac{1}{J}\left(T_e - T - D\varpi_r\right) \\ \dfrac{d\theta_r}{dt} = \varpi_r \end{cases} \tag{1}$$

Where L is the stator inductance, I_α is the current of the α axle, I_β is the current of the β axle, R_s is the stator resistance, Ψ_m is the rotor flux, ω_r is the mechanical angular acceleration of the rotor, θ_r is the rotor electromagnetic angle position, J is the rotational inertia of the rotor.

The state space equation of the PMSM nonlinear mathematical model can be established from the equation (1), as shown in equation (2).

$$\begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{f}\left(\boldsymbol{x}(t)\right) + \boldsymbol{Bu}(t) + \boldsymbol{v}(t) \\ \boldsymbol{y}(t) = \boldsymbol{h}(\boldsymbol{x}(t)) + \boldsymbol{w}(t) \end{cases} \tag{2}$$

Where $\boldsymbol{x}(t)$ is the state variable, which can be defined as $\boldsymbol{x} = \left[I_\alpha \ I_\beta \ \varpi_r \ \theta_r\right]^T$, $\boldsymbol{u}(t)$ is the input, which can be defined as $\boldsymbol{u}(t) = \left[V_\alpha \quad V_\beta\right]^T$, $\boldsymbol{y}(t)$ is defined as the system current output. $\boldsymbol{v}(t)$ is the system noise, $\boldsymbol{w}(t)$ is the measurement noise, both of which are the zero mean Gaussin noise, and the covariance matrices are Q and R, respectively.

Combining with the above analysis, the $\boldsymbol{f}\left(\boldsymbol{x}(t)\right)$, $\boldsymbol{h}(\boldsymbol{x}(t))$ and $\boldsymbol{Bu}(t)$ in the state space equation can be deduced respectively as follows.

$$\boldsymbol{f}(\boldsymbol{x}(t)) = \begin{bmatrix} \dfrac{-R_s i_\alpha + \varpi_r \psi_m \sin\theta_r}{L} \\ -\dfrac{R_s i_\beta + \varpi_r \psi_m \cos\theta_r}{L} \\ -\dfrac{B}{J} \\ \varpi_r \end{bmatrix} \tag{3}$$

$$\boldsymbol{Bu}(t) = \begin{bmatrix} \dfrac{V_\alpha}{L} \\ \dfrac{V_\beta}{L} \\ 0 \\ 0 \end{bmatrix} \tag{4}$$

$$\boldsymbol{h}(\boldsymbol{x}(t)) = \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \tag{5}$$

By taking the sampling period as T and making the equation (2) discretize, the discretized state space equation of the PMSM can be got as follows.

$$\begin{cases} \boldsymbol{x}_k = \boldsymbol{f}(x_{k-1}) + \boldsymbol{Bu}(t)T + \boldsymbol{v}_{k-1} \\ \boldsymbol{y}_k = \boldsymbol{h}(x_{k-1}) + \boldsymbol{w}_{k-1} \end{cases} \tag{6}$$

Where

$$\boldsymbol{h}(\boldsymbol{x}_{k-1}) = \begin{bmatrix} i_{\alpha,k} \\ i_{\beta,k} \end{bmatrix} \tag{7}$$

$$\boldsymbol{f}(\boldsymbol{x}_{k-1}) = \begin{bmatrix} \left(1 - \frac{TR_s}{L}\right) i_{\alpha,k-1} + \frac{Ty_m \varpi_{r,k-1} \sin\theta_{r,k-1}}{L} \\ \left(1 - \frac{TR_s}{L}\right) i_{\beta,k-1} - \frac{Ty_m \varpi_{r,k-1} \cos\theta_{r,k-1}}{L} \\ \left(1 - \frac{TB}{J}\right) \varpi_{r,k-1} \\ q_{r,k-1} + T\varpi_{r,k-1} \end{bmatrix} \tag{8}$$

The equation (6) is the PMSM discrete mathematical model.

3. THE FIFTH ORDER CKF ALGORITHM

The accuracy of the traditional CKF estimation can currently reach the third order. For a discrete system, if the Bayesian estimation theoryis combined with the high-order cubature rule, the high-order Cubature Kalman filter method can be deduced[9] . If the high-order cubature rule adopts fifth-order spherical-radial cubature rule, the fifth-order cubature Kalman filter algorithm can be derived, whose accuracy can reach the fifth-order.

The typical fifth-order cubature formula is defined as follows.

$$\begin{aligned} \boldsymbol{I}_1(f) = \int_{R^n} \boldsymbol{f}(\boldsymbol{x}) N(\boldsymbol{x}; 0, \boldsymbol{I}) d\boldsymbol{x} = \hat{W}_0 \, f(0) + \hat{W}_1 \sum_{i=1}^{2n} \boldsymbol{f}([v]_i) \\ + \hat{W}_{1,1} \sum_{i=1}^{2n(n-1)} \boldsymbol{f}([v,v]_i), n \geq 2 \end{aligned} \tag{9}$$

According to reference [10], the coefficients in equation (9) need to satisfy the following equations.

$$\begin{cases} \hat{W}_0 + 2n\hat{W}_1 + 2n(n-1)\hat{W}_{1,1} = I_0 \\ 2v^2 \hat{W}_1 + 4(n-1)v^2 \hat{W}_{1,1} = I_2 \\ 2v^2 \hat{W}_1 + 4(n-1)v^4 \hat{W}_{1,1} = I_4 \\ 4v^4 \hat{W}_{1,1} = I_{2,2} \end{cases}$$

$$I_0 = \int_{R^n} exp(\text{-}\boldsymbol{x}^T \boldsymbol{x}) d\boldsymbol{x} = \sqrt{\pi^n} \, , \quad I_2 = \int_{R^n} x^2{}_i exp(\text{-}\boldsymbol{x}^T \boldsymbol{x}) d\boldsymbol{x} = \sqrt{\pi^n} / 2 \, .$$

$$I_4 = \int_{R^n} \boldsymbol{x}^4{}_i exp(\text{-}\boldsymbol{x}^T \boldsymbol{x}) d\boldsymbol{x} = 3\sqrt{\pi^n}/4, \quad I_{2,2} = \int_{R^n} \boldsymbol{x}_i^2 \boldsymbol{x}_j^2 exp(\text{-}\boldsymbol{x}^T \boldsymbol{x}) d\boldsymbol{x} = \sqrt{\pi^n}/4.$$

Thus the unique solution of the coefficients in equation (9) is

$$v = \sqrt{3/2}, \quad \hat{\mathrm{W}}_0 = \sqrt{\pi^n}\left[1-(7-n)n/18\right].$$

$$\hat{\mathrm{W}}_1 = \sqrt{\pi^{\mathrm{n}}}\left(4-n\right)/18, \quad \hat{\mathrm{W}}_{1,1} = \sqrt{\pi^{\mathrm{n}}}/36.$$

The cubature points set and weights can be obtained as follows.

$$\boldsymbol{\xi}_i = \begin{cases} [0]_i, & i=1 \\ [\sqrt{3}]_i, & i=2,\cdots 2n+1 \\ [\sqrt{3},\sqrt{3}]_i, & i=2n+2,\cdots 2n^2+1 \end{cases}$$

$$\varpi_i = \begin{cases} 1-\frac{n(7-n)}{18}, & i=1 \\ (4-n)/18, & i=2,\cdots,2n+1 \\ 1/36, & i=2n+2,\cdots,2n^2+1 \end{cases}$$

The fifth-order CKF algorithm steps containing the time update and the measurement update are as follows.

(1) Time update

1) Calculate the cubature points $\boldsymbol{X}_{k,i}\left(i=0,1,\cdots,2n^2+1\right)$:

$$\boldsymbol{X}_{k,i} = \hat{\boldsymbol{X}}_k + \boldsymbol{S}_k \boldsymbol{\xi}_i \tag{10}$$

Where the $\hat{\boldsymbol{X}}_k$ is the optimal estimation of the state at time *k*, the $\boldsymbol{S}_k$ is the Cholesky decomposition of the state covariance matrix $\boldsymbol{P}_k$ at time *k*.

2) Calculate the cubature points transferred through the state equation $\boldsymbol{X}_{k+1/k,i}$:

$$\boldsymbol{X}_{k+1/k,i} = \boldsymbol{f}\left(\boldsymbol{X}_{k,i}\right) \tag{11}$$

3) Calculate the predictive values at time *k*+1, which can be shown as follows:

$$\hat{\boldsymbol{X}}_{k+1/k} = \sum_{i=1}^{2n^2+1} \varpi_i \boldsymbol{X}_{k+1/k,i} \tag{12}$$

4) Estimate the state error covariance matrix $\boldsymbol{P}_{k+1/k}$ at time *k*+1, and it can be shown as follows：

$$\boldsymbol{P}_{k+1/k} = \sum_{i=1}^{2n^2+1} \varpi_i \left(\boldsymbol{X}_{k+1/k,i} - \hat{\boldsymbol{X}}_{k+1/k}\right)\left(\boldsymbol{X}_{k+1/k,i} - \hat{\boldsymbol{X}}_{k+1/k}\right)^T + \boldsymbol{Q}_k \tag{13}$$

(2) Measurement update

1) Calculate the updated state cubature points $\boldsymbol{X}_{k+1/k,i}$:

$$\boldsymbol{X}_{k+1/k,i} = \hat{\boldsymbol{X}}_{k+1/k} + \boldsymbol{S}_{k+1/k} \xi_i \; i=1,2,\cdots,2n^2+1 \tag{14}$$

Where the $\boldsymbol{S}_{k+1/k}$ is the matrix obtained by the Cholesky decomposition of the matrix $\boldsymbol{P}_{k+1/k}$.

2) Calculate the cubature points $\boldsymbol{Y}_{k+1,i}$ transferred through the measurement equation, and can be expressed as follows:

$$\boldsymbol{Y}_{k+1,i} = \boldsymbol{h}\left(\boldsymbol{X}_{k+1/k,i}\right) \tag{15}$$

3) Calculate the measured predictive values $\hat{\boldsymbol{Y}}_{k+1}$ at time k+1, which can be shown as follows:

$$\hat{\boldsymbol{Y}}_{k+1} = \sum_{i=1}^{2n^2+1} \varpi_i \boldsymbol{Y}_{k+1,i} \tag{16}$$

4) Estimate the measured estimation error covariance matrix $\boldsymbol{P}_{k+1}^{yy}$ and the cross-correlation covariance matrix $\boldsymbol{P}_{k+1/k}^{xy}$ at time k+1:

$$\begin{cases} \boldsymbol{P}_{k+1}^{yy} = \sum_{i=1}^{2n^2+1} \varpi_i \left(\boldsymbol{Y}_{k+1,i} - \hat{\boldsymbol{Y}}_{k+1} \right) \left(\boldsymbol{Y}_{k+1,i} - \hat{\boldsymbol{Y}}_{k+1} \right)^T + \boldsymbol{R}_k \\ \boldsymbol{P}_{k+1/k}^{xy} = \sum_{i=1}^{2n^2+1} \varpi_i \left(\boldsymbol{X}_{k+1/k,i} - \hat{\boldsymbol{X}}_{k+1/k} \right) \left(\boldsymbol{Y}_{k+1,i} - \hat{\boldsymbol{Y}}_{k+1} \right)^T \end{cases} \tag{17}$$

5) Calculate the filter gain matrix $\boldsymbol{K}_{k+1}$ at time k+1:

$$\boldsymbol{K}_{k+1} = \boldsymbol{P}_{k+1/k}^{xy} \left(\boldsymbol{P}_{k+1}^{yy} \right)^{-1} \tag{18}$$

6) Calculate the state estimation values $\hat{\boldsymbol{X}}_{k+1}$ at time k+1:

$$\hat{\boldsymbol{X}}_{k+1} = \hat{\boldsymbol{X}}_{k+1/k} + \boldsymbol{K}_{k+1} \left(\boldsymbol{Y}_{k+1} - \hat{\boldsymbol{Y}}_{k+1} \right) \tag{19}$$

7) Calculate the state error covariance matrix $\boldsymbol{P}_{\mathrm{k}+1}$:

$$\boldsymbol{P}_{\mathrm{k}+1} = \boldsymbol{P}_{k+1/k} - \boldsymbol{K}_{k+1} \boldsymbol{P}_{k+1}^{yy} \boldsymbol{K}_{k+1}^T \tag{20}$$

Given the initial conditions, the fifth-order CKF can be implemented through the above process. The flow chart of the algorithm is shown in Figure 1.

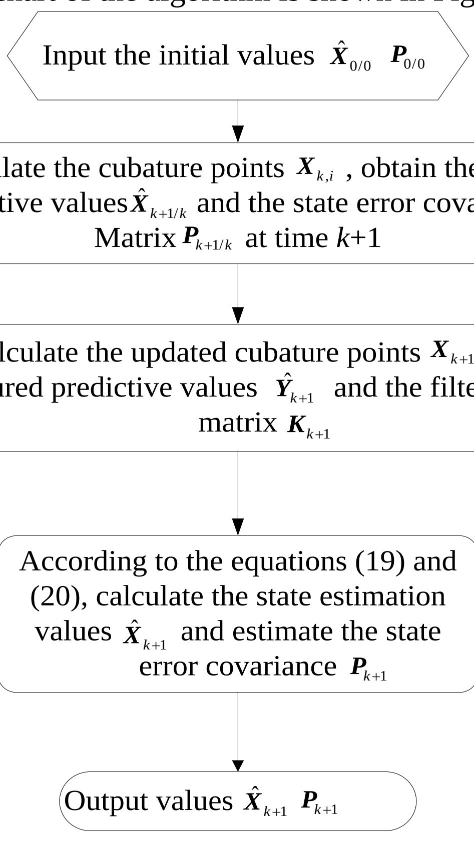


Figure 1. The flow chart of the fifth order CKF algorithm

4. SIMULATION ANALYSIS

On the basis of the above analysis, the PMSM sensorless control system models based on CKF and fifth-order CKF are established in Matlab/simulink, respectively. The PMSM sensorless control system of the fifth-order CKF is shown in Figure 2. The i_d=0 control strategy is used to realize the PMSM space vector control. The speed loop control is implemented by using the PI control method, and the rotor speed and position are estimated by the fifth-order CKF algorithm, so as to realize the PMSM sensorless control.

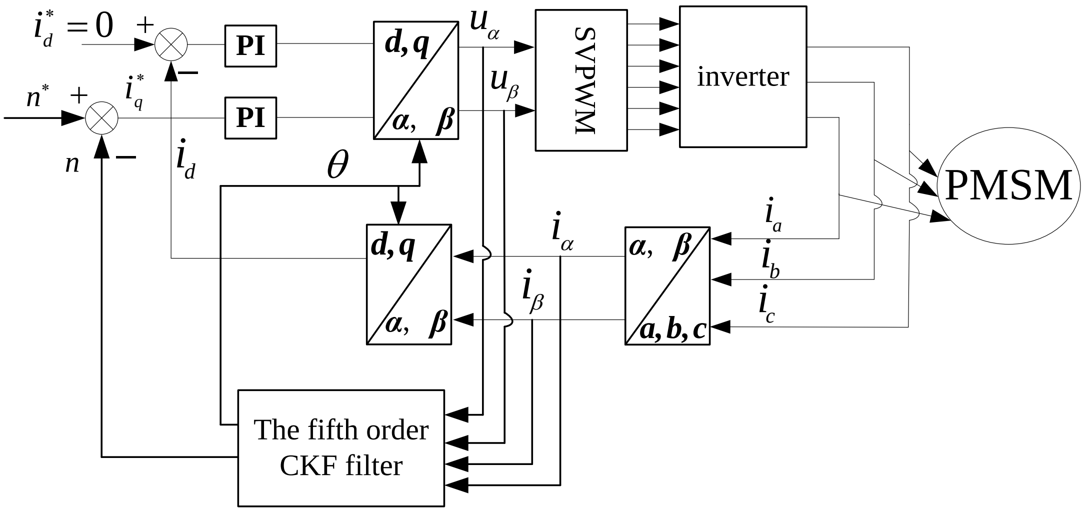
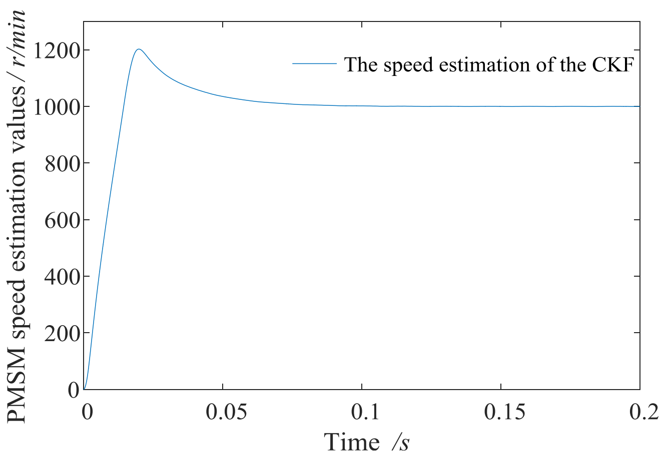


Figure 2. the PMSM control system of the fifth order CKF

The PMSM parameters used in the simulation are as follows: the stator resistance $R_S = 0.958\Omega$, the stator inductance $L_s = 8.5mH$, the flux $\psi_f = 0.1827Wb$, the moment of inertia $J = 0.003kg \cdot m^2$, the damping coefficient $B = 0.008N \cdot m \cdot s$, the pole of the rotor $p_n = 4$.The system noise covariance matrix $Q = [0.01 \quad 0.01 \quad 0.21 \quad 0.001]^T$, the measurement noise covariance matrix $R = [0.02 \quad 0.02]^T$, the initial error covariance matrix $P = [0.5 \quad 0.5 \quad 0.5 \quad 0.5]^T$. When starting in no-load operation condition, the given speed of the PMSM is 1000r/min. The CKF and fifth order CKF algorithms are used to estimate the PMSM speed respectively, and the results are shown in figures as follows.



Figrue3. the speed estimation curve of the CKF

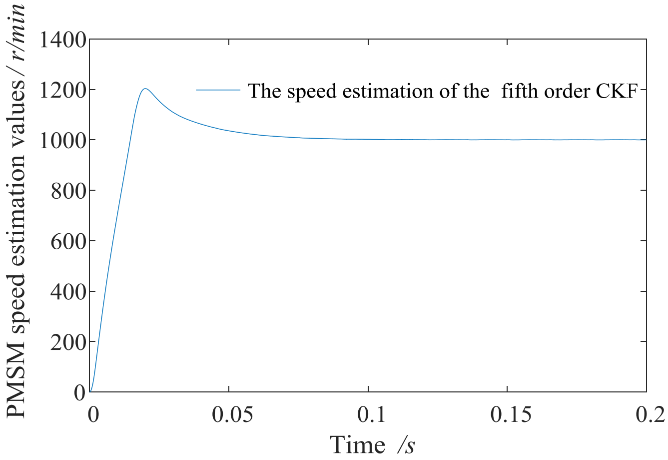


Figure 4. the speed estimation curve of the fifth order CKF

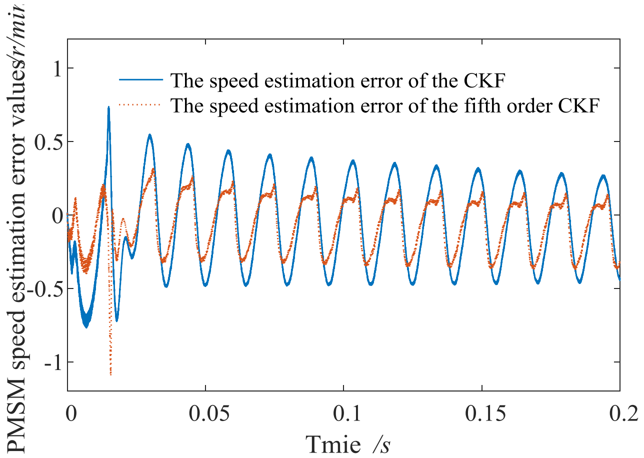


Figure 5. the speed estimation error curves of the CKF and fifth order CKF

From the speed estimation cures in figure 2 and figure 4, it can be seen that both the CKF and the fifth order CKF algorithm can track the given speed of the PMSM faster. Although the fifth-order CKF algorithm is more complex than CKF, it does not affect the response speed of rotor speed tracking. As we can see from figure 5, the speed estimation error of the CKF is larger than the fifth order CKF, the root mean square value of the CKF estimation error is 0.102, and the root mean square value of the CKF is 0.038. When the fifth order CKF speed estimation is adopted, although a step error may occur at the beginning, after the peak value, the error of the speed estimation value gradually decreases and tends to be stable. Therefore, the simulation results show that the fifth-order CKF algorithm can accurately and quickly track the motor speed.

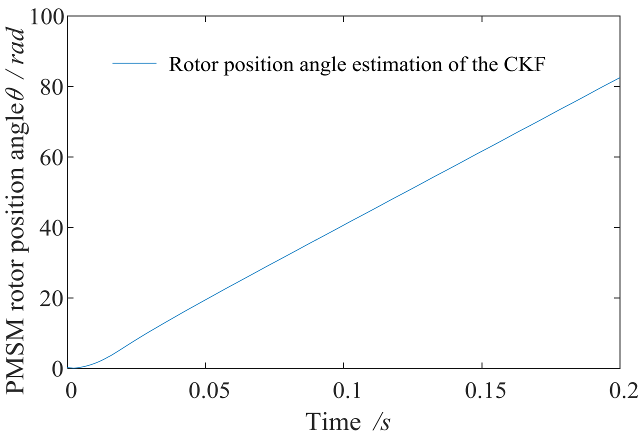


Figure 6. the rotor position estimation curve of the CKF

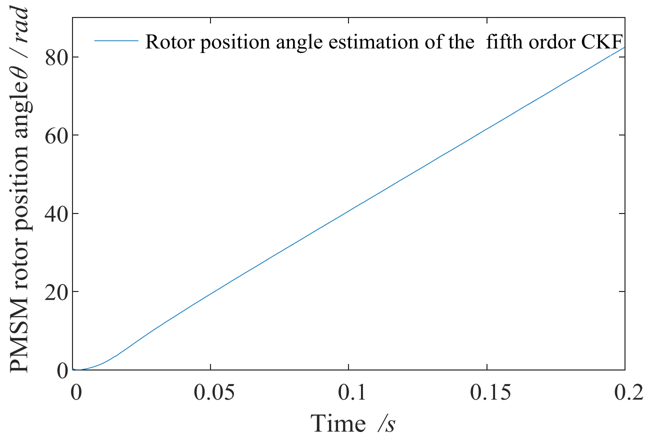


Figure 7. the rotor position estimation curve of the fifth order CKF

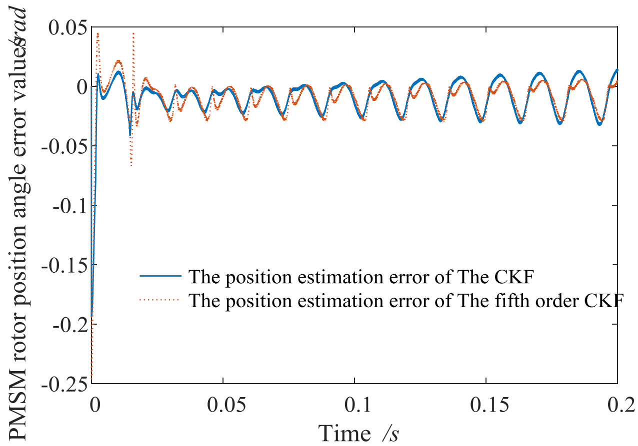


Figure 1. the rotor position estimation curves of the CKF and the fifth order CKF

From the results of the rotor position simulation in the figure 6 and the figure 7, the fifth order CKF can track the rotor position faster. As we can see from the figure 8, the root mean square value of the CKF position estimation error is 3.9354e-04, and the root mean square value of the CKF is 4.4198e-04. when starting at the beginning, the accuracy of the fifth order CKF position estimation is less than the CKF, but when the PMSM runs steadily, the fluctuation of the fifth order CKF position estimation values is less than the CKF, and the error decreases gradually, so the fifth order CKF algorithm can estimate the rotor position with much accuracy.

5. CONCLUSIONS

The fifth-order CKF algorithm is applied in this paper to the PMSM sensorless control. The fifth-order CKF algorithm can effectively improve the PMSMspeed estimation accuracy, and requires less motor hardware parameters. Firstly, the PMSM discrete mathematical model is established, and the fifth-order spherical-radial cubature rule is deduced. Then, In order to verify the effectiveness of the fifth-order CKF algorithm, the PMSM sensorless control model based on the fifth-order CKF is built and simulated in MATLAB/Simulink. The simulation results are compared with the traditional CKF. The results show that the fifth-order CKF algorithm can estimate the motor speed and rotor position in real time, and the error is smaller than the third-order CKF, which can be effectively used in the PMSM sensorless motor control.

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