MADRID
inter.noise 2019
June 16 - 19
NOISE CONTROL FOR A BETTER ENVIRONMENT

Three-dimensional vibration analysis of functionally graded material plates in thermal environment

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ABSTRACT

The three-dimensional vibrations of functionally graded material plates exposed to thermal environment are considered based on a novel high-order layerwise theory. The formulation is based on the three-dimensional theory of elasticity so that no other assumptions on deformations and stresses along the thickness direction are introduced. The governing equations of the plates is discretized by modified Chebyshev polynomials of first kind in the transfer domain and the remaining domains are approximated by spectral method in which each of the fundamental unknowns is invariantly expanded as basic functions and the problems are stated in a variational form by the aid of penalty parameters which provides complete flexibility to describe any specified boundary conditions. FGM plates with temperature-dependent material properties subjected to uniform temperature rise, linear temperature rise and nonlinear temperature rise are considered. The vibration characteristics of the FGM plates at different temperature are brought out through parametric study.

Keywords: Vibration, FGM Plate, Thermal environment
I-INCE Classification of Subject Number: 76

1. INTRODUCTION

With the development of new industries and modern processes, functionally graded materials (FGMs) have been increasingly utilized to build various structural components in many fields of modern engineering practices to satisfy special functional requirements due to their outstanding material properties. As is well-known, the practical engineering structures may fail and collapse because of material fatigue resulting from vibrations. Therefore, it is of particular importance to understand the structural vibration and reduce it through proper design. The vibration modal information of FGM structural components, such as their natural frequencies and mode shapes, play a vital role in the safety evaluation, dynamic analysis and reliable design of the whole engineering

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structures. Hence, an accurate vibration characteristic determination of the FGM plates is essential.

Heretofore, the vibration analysis of FGM structures has aroused much attention in the open literature [1-4]. A variety of theoretical models such as the classical plate theory (CPT), the shear deformation plate theories and the three-dimensional plate theory had been proposed. The CPT were depended on the well-known Kirchhoff–Love's hypothesis: (i) thin plates; (ii) linear small deformation; (iii) transverse normal stresses can be neglected; (iv) straight lines normal to the referential surface do not change during deformation. In the CPT, a FGM plate is normally treated as a single-layered thin plate referring to its middle surface. It is commonly accepted that they can yield results with enough accuracy when the thickness ratio is far less than one. However, the transverse stress components are ignored in the CPT. This omission makes them generally incompetent for the dynamic modelling of plates which are composed of inhomogeneous materials. With the ever-increasing applications of FGM plates, a variety of first-order deformation theories (FSDTs) and higher-order deformation theories (HSDTs) have been proposed. The FSDTs were developed based on the displacement assumptions of Reissner and Mindlin that the in-plane displacements vary linearly across the transverse direction while the transversal components remain constant. The FSDTs are more accurate than the CPT but they are adequate only for moderately thick FGM plates. To overcome these drawbacks, a variety of HSDTs based on nonlinear displacement field assumptions through the thickness coordinate have been proposed. The HSDTs are better qualified to thin and moderately thick plates than the CPT and FSDTs. However, they are still inadequate for thick FGM plates because the transverse normal stain and stress are generally been ignored.

Essentially, the CPT, FSDTs and HSDTs are two-dimensional (2-D) theories on the basis of certain kinematic assumptions and equivalent single layer approach. They can only predict the flexural modes and are not able to determine the thickness twist and shear modes. The presence of material inhomogeneity of FGM plates will result in a 3-D stress field in nature. As a consequence, even the 2-D theories are simpler, less memory capacity and high velocity, the usage of 3-D elasticity theory is an essential requirement in the accurate assessment of their vibration characteristics.

The rise in temperature reduces both strength and stiffness of FGM. A thorough understanding of the effects of temperature in FGM plates is critical in ensuring safety design. There seems to be no such solutions available. This paper is therefore devoted to the three-dimensional vibration analysis of FGM plates in thermal environment. FGM plates with temperature-dependent material properties subjected to uniform temperature rise, linear temperature rise and nonlinear temperature rise are considered. The vibration characteristics of the FGM plates at different temperature are brought out through parametric study.

2. THEORETIC FORMULATION

2.1 Model description

The geometry of the FGM plates studied in this work is illustrated in Fig. 1. The length, width and thickness of the plate are assumed to be a, b and h, respectively. The bottom surface of the plate where an orthogonal coordinate system x, y and z is fixed is taken as the reference surface. The x, y and z axes are taken in the length, width and thickness directions, respectively. The displacements of the plate in the x, y and z directions are denoted by u, v and w, respectively. Consider the FGM plates made from a mixture of two material phases, for example, a metal and a ceramic as shown in Fig. 1(b).

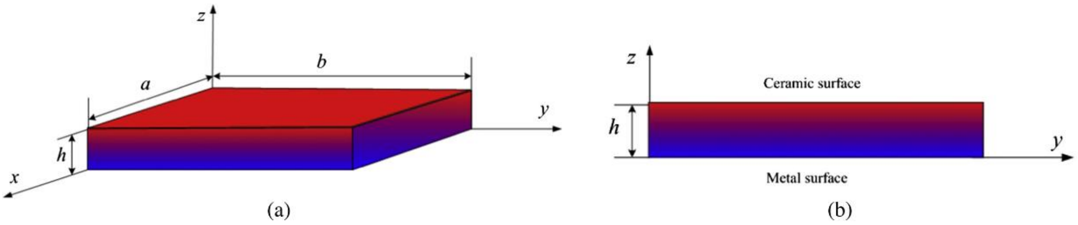

Fig. 1. Schematic diagram of the rectangular FGM plates: (a) the geometry and coordinates; (b) FGM plate of two material phases.

Herein, the top surface of the plate is ceramics-rich whereas the bottom surface is metal-rich. Young's modulus and density per unit volume are assumed to vary continuously through the plate thickness according to a power-law distribution as

$$P(z,T) = \left(P_u - P_l\right)(z/h)^p + P_l \tag{1}$$

where P denotes a generic material property, such as elastic modulus E, the Poisson ratio ν, mass density ρ and thermal expansion coefficient α. The subscripts u and l represent the corresponding values at the upper and lower constituents. p is the volume fraction index that is the positive real value. A typical material property P can be expressed as a function of temperature, see Ref. [5]

$$P = P_0\left(P_{-1}T^{-1} + 1 + P_1T + P_2T^2 + P_3T^3\right) \tag{2}$$

in which T=T_0+$\Delta T(z)$ and T_0 =300K (room temperature). P_0, P_{-1}, P_1, P_2 and P_3 are the coefficients of temperature $T(K)$ and are unique to the constituent materials. $\Delta T(z)$ is temperature rise only through the thickness direction. The thermal conductivity is assumed to be temperature independent.

2.2. Kinematic relations and constitutive relation

According to the 3-D theory of elasticity, the linear strain–displacement relations at an arbitrary point in the space of the FGM plate are found to be

$$\begin{aligned}
\varepsilon_x &= \frac{\partial u}{\partial x}, \quad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \\
\varepsilon_y &= \frac{\partial v}{\partial y}, \quad \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \\
\varepsilon_z &= \frac{\partial w}{\partial z}, \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}
\end{aligned} \tag{3}$$

where ε_x, ε_y and ε_z are the normal strains. γ_{yz}, γ_{xz} and γ_{xy} stand for the shear strain components. Stress-strain relations at the point can be derived as

$$\boldsymbol{\sigma} = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{bmatrix}, \quad \boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} C_{11}(z) & C_{12}(z) & C_{12}(z) & 0 & 0 & 0 \\ C_{12}(z) & C_{11}(z) & C_{12}(z) & 0 & 0 & 0 \\ C_{12}(z) & C_{12}(z) & C_{11}(z) & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44}(z) & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44}(z) & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44}(z) \end{bmatrix} \tag{4}$$

$$\boldsymbol{\sigma} = \mathbf{C}\boldsymbol{\varepsilon}$$

in which σ_x, σ_y, σ_z and τ_{yz}, τ_{xz}, τ_{xy} denote the normal and shear stresses. **C** is the material stiffness matrix in the material principal coordinates:

$$C_{11}(z)=\frac{E(1-\nu)}{(1+\nu)(1-2\nu)},\quad C_{12}(z)=\frac{\nu E}{(1+\nu)(1-2\nu)},\quad C_{44}(z)=\frac{E}{2(1+\nu)} \tag{5}$$

From the Modified Chebyshev polynomial based higher-order layerwise theory, the displacement field for the considered plate may be represented by

$$\begin{aligned} u(x,y,z,t)&=u^b(x,y,t)\psi^b(z)+\sum_{j=1}^{J}u^j(x,y,t)\psi^j(z)+u^t(x,y,t)\psi^t(z)\\ v(x,y,z,t)&=v^b(x,y,t)\psi^b(z)+\sum_{j=1}^{J}v^j(x,y,t)\psi^j(z)+v^t(x,y,t)\psi^t(z)\\ w(x,y,z,t)&=w^b(x,y,t)\psi^b(z)+\sum_{j=1}^{J}w^j(x,y,t)\psi^j(z)+w^t(x,y,t)\psi^t(z) \end{aligned} \tag{6}$$

where u^b, u^j, u^t, v^b, v^j, v^t, w^b, w^j and w^t are the in-plane displacement variables for the plate while

$$\begin{aligned} \psi^b(z)&=1-\frac{z}{h},\\ \psi^t(z)&=\frac{z}{h}-1,\\ \psi^j(z)&=T_{j+1}\left(\frac{2z-h}{h}\right)-T_{j-1}\left(\frac{2z-h}{h}\right) \end{aligned} \tag{7}$$

in which $T_{j\text{-}1}$ is the j-1 term of the Chebyshev polynomials of first kind:

$$T_{j-1}(x)=\cos[(j-1)\arccos x],\quad j=1,2,\cdots \tag{8}$$

Substituting Eq. (6) into Eq. (3) Therefore, the strain–displacement relations can be rewritten as

$$\boldsymbol{\varepsilon}=\begin{bmatrix}\mathbf{I} & \mathbf{I} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\boldsymbol{\varepsilon}^b\\ \boldsymbol{\varepsilon}^i\\ \boldsymbol{\varepsilon}^t\end{bmatrix},\quad \boldsymbol{\varepsilon}^\lambda=\begin{bmatrix}\varepsilon_x^\lambda & \varepsilon_y^\lambda & \varepsilon_z^\lambda & \gamma_{yz}^\lambda & \gamma_{xz}^\lambda & \gamma_{xy}^\lambda\end{bmatrix}^T,\quad \lambda=b,i,t \tag{9}$$

Similarly, the stress-strain relations for the plate can be redefined as

$$\boldsymbol{\sigma}_k=\begin{bmatrix}\mathbf{C} & \mathbf{C} & \mathbf{C}\end{bmatrix}\begin{bmatrix}\boldsymbol{\varepsilon}^b\\ \boldsymbol{\varepsilon}^i\\ \boldsymbol{\varepsilon}^t\end{bmatrix} \tag{10}$$

Therefore, the strain energy U_s and kinetic energy T_k can be written in terms of strain and stress components as:

$$\begin{aligned} U_s&=\frac{1}{2}\int_x\int_y\int_0^h\begin{Bmatrix}(\boldsymbol{\varepsilon}^b)^T\mathbf{C}\boldsymbol{\varepsilon}^b+(\boldsymbol{\varepsilon}^i)^T\mathbf{C}\boldsymbol{\varepsilon}^b+(\boldsymbol{\varepsilon}^t)^T\mathbf{C}\boldsymbol{\varepsilon}^b+\\ (\boldsymbol{\varepsilon}^b)^T\mathbf{C}\boldsymbol{\varepsilon}^i+(\boldsymbol{\varepsilon}^i)^T\mathbf{C}\boldsymbol{\varepsilon}^i+(\boldsymbol{\varepsilon}^t)^T\mathbf{C}\boldsymbol{\varepsilon}^i+\\ (\boldsymbol{\varepsilon}^b)^T\mathbf{C}\boldsymbol{\varepsilon}^t+(\boldsymbol{\varepsilon}^i)^T\overline{\mathbf{C}}_k\boldsymbol{\varepsilon}^t+(\boldsymbol{\varepsilon}^t)^T\mathbf{C}\boldsymbol{\varepsilon}^t\end{Bmatrix}dxdydz\\ T_k&=\frac{1}{2}\int_x\int_y\int_0^h\rho\left(\dot{u}^2+\dot{v}^2+\dot{w}^2\right)dxdydz \end{aligned} \tag{11}$$

The thermal stresses for the plate can be written as follows using Eq. (4)

$$\boldsymbol{\alpha}=\Delta T(z)\begin{bmatrix}\alpha_1 & \alpha_2 & \alpha_3 & \alpha_{23} & \alpha_{13} & \alpha_{12}\end{bmatrix}^{\mathrm{T}} \tag{12}$$

Therefore, the thermal energy functions of the plate can be expressed as [24]:

$$U_{\text{thermal}} = -\iint \int_0^h \left(\boldsymbol{\varepsilon}^{nl}\right)^{\mathrm{T}} \mathbf{C}\boldsymbol{\alpha} dxdydz \tag{13}$$

where $\boldsymbol{\varepsilon}^{nl}$ indicate the nonlinear strain components:

$$\boldsymbol{\varepsilon}^{nl} = \begin{bmatrix} \frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \\ \frac{1}{2}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 \\ \frac{1}{2}\left(\frac{\partial u}{\partial z}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial z}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial z}\right)^2 \\ \frac{\partial u}{\partial y}\frac{\partial u}{\partial z} + \frac{\partial v}{\partial y}\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\frac{\partial w}{\partial z} \\ \frac{\partial u}{\partial x}\frac{\partial u}{\partial z} + \frac{\partial v}{\partial x}\frac{\partial v}{\partial z} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial z} \\ \frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\frac{\partial v}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \end{bmatrix} \tag{14}$$

The Lagrangian energy function (L) of the FGM plate is expressed in terms of the aforementioned energy functions as:

$$L = T_k - U_s - U_{thermal} \tag{15}$$

The in-plane displacement variables of the plate can be expressed as follows:

$$\begin{aligned} u^{\lambda} &= \sum_{n=1}^{N}\sum_{m=1}^{M} u^{\lambda mn} P_m(x) P_m(y) \\ v^{\lambda} &= \sum_{n=1}^{N}\sum_{m=1}^{M} v^{\lambda mn} P_m(x) P_m(y) \qquad \lambda = b,i,t \\ w^{\lambda} &= \sum_{n=1}^{N}\sum_{m=1}^{M} w^{\lambda mn} P_m(x) P_m(y) \end{aligned} \tag{16}$$

in which $u^{\lambda mn}$, $v^{\lambda mn}$ and $w^{\lambda mn}$ are the expansion coefficients. $P_m(x)$ and $P_n(y)$ represent the mth and nth Chebyshev polynomials. M and N stand for the total number of polynomials considered in the spectral expansion.

The boundary equations a FGM plate can be defined as:

$$\begin{aligned} x = l_x &\begin{cases} \sigma_x(y,z) = \bar{\sigma}_x^{l_x}(y,z) \text{ or } u(y,z) = \bar{u}^{l_x}(y,z) \\ \tau_{xy}(y,z) = \bar{\tau}_{xy}^{l_x}(y,z) \text{ or } v(y,z) = \bar{v}^{l_x}(y,z), \quad l_x = 0, a \\ \tau_{xz}(y,z) = \bar{\tau}_{xz}^{l_x}(y,z) \text{ or } w(y,z) = \bar{w}^{l_x}(y,z) \end{cases} \\ y = l_y &\begin{cases} \tau_{xy}(x,z) = \bar{\tau}_{xy}^{l_y}(x,z) \text{ or } u(x,z) = \bar{u}^{l_y}(x,z) \\ \sigma_y(x,z) = \bar{\sigma}_y^{l_y}(x,z) \text{ or } v(x,z) = \bar{v}^{l_y}(x,z), \quad l_y = 0, b \\ \tau_{yz}(x,z) = \bar{\tau}_{yz}^{l_y}(x,z) \text{ or } w(x,z) = \bar{w}^{l_y}(x,z) \end{cases} \end{aligned} \tag{17}$$

where $\bar{\sigma}_x^{l_x}, \bar{\tau}_{xy}^{l_x}, \bar{\tau}_{xz}^{l_x}, \bar{\tau}_{xy}^{l_y}, \bar{\sigma}_y^{l_y}, \bar{\tau}_{yz}^{l_y}, \bar{u}^{l_x}, \bar{v}^{l_x}, \bar{w}^{l_x}, \bar{u}^{l_y}, \bar{v}^{l_y}, \bar{w}^{l_y}$ are the boundary conditions.

In this paper, the plate vibration problems are characterized in a modified variational form. The variational equation for the plate in the case of free vibration is written as

$$L = T_k - U_s - U_{thermal} - \Pi_{bc} \tag{18}$$

where

$$\Pi_{bc} = \iint_0^h \sum_{l_x} \left\{ k_u^{l_x} \left(u|_{x=l_x} - \bar{u}^{l_x} \right) + k_v^{l_x} \left(v|_{x=l_x} - \bar{v}^{l_x} \right) + k_w^{l_x} \left(w|_{x=l_x} - \bar{w}^{l_x} \right) \right\} dydz$$
$$+ \iint_0^h \sum_{l_y} \left\{ k_u^{l_y} \left(u|_{y=l_y} - \bar{u}^{l_y} \right) + k_v^{l_y} \left(v|_{y=l_y} - \bar{v}^{l_y} \right) + k_w^{l_y} \left(w|_{y=l_y} - \bar{w}^{l_y} \right) \right\} dxdz \quad (19)$$

notations $k_u^{l_x}, k_v^{l_x}, k_w^{l_x}, k_u^{l_y}, k_v^{l_y}, k_w^{l_y}$ stand for the penalty parameters. Physically, the penalty parameters can be referred to as elastic stiffness along the boundaries, and the penalty function (Eq. (19)) is the strain energy associated with the straining of this elastic restraint.

Finally, minimizing Eq. (18) with respect to the coefficients and, the characteristic equations for a FGM plate can be finally derived and rearranged in matrix form as

$$\left\{ \mathbf{K} - \omega^2 \mathbf{M} \right\} \mathbf{G} = \mathbf{0} \quad (20)$$

where **K** and **M** are the stiffness and mass matrices. **G** is the generalized freedom. Therefore, the vibration characteristics of the plates can be obtained directly by solving Equation (20).

3. NUMERICAL RESULTS

In this section, the theoretical formulations presented above have been implemented in a MATLAB code to compute the natural frequencies and mode shapes of FGM plates with different thicknesses, a variety of boundary conditions to validate and demonstrate the flexibility of the proposed high-order layerwise theory.

Generally, there are three possible combinations of classical boundary conditions at each boundary of a plate, i.e., the completely free (F), simply-supported (S), and completely clamped (C) restraints. For simplification, a symbolism, e.g., FCSF is used to indicate that the plate under consideration have F, C, S, and F boundary conditions referred to the boundaries x=0, x=a, y=0 and y=b, respectively.

Table 1 shows the first eight frequency parameters $\Omega = \omega(b^2/h)\sqrt{\rho_c/E_c}$ for square Al/Al2O3 FGM plates with SSSS boundary consitons. The material properties are E_{Al2O3}=380 GPa, ν_{Al2O3}=0.3, ρ_{Al2O3}=3800 kg/m, E_{Al}=70 GPa, ν_{Al}=0.3, ρ_{Al}=2707 kg/m^3. The volume fraction index is considered to be 0, 1, 5. The plate is assumed to be moderately thick, i.e., h/a=0.1. The plates are exposed to room temperature. The 3-D solutions Published by Huang et al. [6] are also shown in Table 1. Huang et al. [6] employed a Ritz method. From the table ,we can see that the presnt theory convege quickly and mach well with solutions published by Huang et al. [6].

Table 1. Frequency parameters $\Omega = \omega(b^2/h)\sqrt{\rho_c/E_c}$ for moderately thick, Al/Al2O3 FGM square plateswith SSSS boundary congdiotns. .

p	J	Mode							
		1	2	3	4	5	6	7	8
0	0	6.381	15.21	15.21	19.48	19.48	23.33	27.55	28.42
	1	5.794	13.90	13.90	19.48	19.48	21.43	26.18	26.18
	2	5.776	13.80	13.80	19.48	19.48	21.21	25.87	25.87
	3	5.776	13.80	13.80	19.48	19.48	21.21	25.87	25.87
	4	5.776	13.80	13.80	19.48	19.48	21.21	25.87	25.87
	3-D [6]	5.777	13.81	13.81	19.48	19.48			
1	0	4.889	11.71	11.71	16.20	16.20	18.02	22.00	22.00
	1	4.434	10.67	10.67	16.20	16.20	16.50	20.19	20.19

	2	4.426	10.63	10.63	16.20	16.20	16.40	20.04	20.04
	3	4.426	10.63	10.63	16.20	16.20	16.39	20.04	20.04
	4	4.426	10.63	10.63	16.20	16.20	16.39	20.04	20.04
	3-D [6]	4.426	10.63	10.63	16.20	16.20			
5	0	4.190	9.960	9.960	12.64	12.64	15.24	17.86	18.54
	1	3.795	9.050	9.050	12.64	12.64	13.89	16.92	16.92
	2	3.773	8.933	8.933	12.64	12.64	13.63	16.56	16.56
	3	3.772	8.929	8.929	12.64	12.64	13.62	16.55	16.55
	4	3.771	8.927	8.927	12.64	12.64	13.62	16.54	16.54
	3-D [6]	3.772	8.93	8.93	12.64	12.64			

Table 2 shows the first seven frequency parameters $\Omega=\omega b^2\sqrt{\rho h/D}$ for square isotropic plates with different boundary conditions. The thickness ratio is selected as h/a=0.1 and the material properties are E=210 GPa, ν=0.3, ρ=7800 kg/m. The plates are exposed to room temperature. The results are compared with other published solutions by using the 3-D Ritz method with modified Fourier series polynomials [7]. It is seen that the present results are in good agreement with Refernce [7]. In general, the present solutions agree very well with those available in the literature. From Tables 1-2, it can be conculde that the poposed high-order layerwise theory is sufficient for the 3-D vibration analysis of FGM plates with different boundary conditions.

Table 2. Frequency parameters $\Omega=\omega b^2\sqrt{\rho h/D}$ for moderately thick isotropic square plateswith FFFF and SSSS boundary congdiotns.

B.C.	$M\times N$	Mode						
		1	2	3	4	5	6	7
FFFF	9×9	12.738	18.955	23.346	31.975	31.975	55.498	55.498
	10×10	12.725	18.955	23.345	31.965	31.965	55.492	55.492
	11×11	12.725	18.954	23.345	31.957	31.957	55.490	55.490
	12×12	12.723	18.955	23.345	31.956	31.956	55.490	55.490
	13×13	12.723	18.954	23.345	31.955	31.955	55.490	55.490
	14×14	12.723	18.954	23.345	31.955	31.955	55.489	55.489
	15×15	12.723	18.954	23.345	31.954	31.954	55.489	55.489
	3-D [7]	12.728	18.956	23.346	31.965	31.965	55.493	55.493
SSSS	9×9	19.088	45.616	45.616	64.369	64.369	70.097	85.497
	10×10	19.088	45.616	45.616	64.369	64.369	70.096	85.488
	11×11	19.088	45.616	45.616	64.369	64.369	70.096	85.483
	12×12	19.088	45.616	45.616	64.369	64.369	70.096	85.483
	13×13	19.088	45.616	45.616	64.369	64.369	70.096	85.483
	14×14	19.088	45.616	45.616	64.369	64.369	70.096	85.483
	15×15	19.088	45.616	45.616	64.369	64.369	70.096	85.483
	3-D [7]	19.098	45.636	45.636	64.384	64.384	70.149	85.500

Table 3 shows the numerical results of frequency parameters of the first six modes for Si3N4/SUS304 FGM square plates subjected to CCCC boundary conditions uniform temperature rise with the results published by other researchers [5, 7, 8]. The material properties are given in Table 4. The plate is assumed to be moderately thick with thickness ratio of h/b =0.1.

Table 3. Comparisons of first six natural frequency parameters for Si3N4/SUS304 FGM square plates subjected to CCCC boundary conditions and uniform temperature rise (a=0.2m, h/b =0.1, k=2.0, T_0=300 K).

ΔT	Solutions	Mode					
		1	2	3	4	5	6
0	Present	4.1116	7.8497	7.8497	11.009	12.976	13.100
	HSDT[8]	4.1062	7.8902	7.8902	11.183	12.588	13.187
	HSDT[5]	4.1165	7.9696	7.9696	11.220	13.106	13.209
	3-D[9]	4.1658	7.9389	7.9389	11.121	13.097	13.223
300	Present	3.6708	7.2202	7.2202	10.234	12.117	12.245
	HSDT[8]	3.6636	7.2544	7.2544	10.3924	11.705	12.318
	HSDT[5]	3.6593	7.3098	7.3098	10.4021	12.198	12.305
	3-D[9]	3.7202	7.3010	7.3010	10.3348	12.226	12.356
500	Present	3.2431	6.5966	6.5966	9.4504	11.237	11.370
	HSDT[8]	3.2357	6.6281	6.6281	9.5900	10.829	11.435
	HSDT[5]	3.2147	6.6561	6.6561	9.5761	11.271	11.381
	3-D[9]	3.2741	6.6509	6.6509	9.5192	11.313	11.447

Table 4. Temperature-dependent coefficients of elastic modulus E (GPa), Poisson's ratio ν, mass density ρ (kg/m3), and thermal expansion coefficient α (1/K) for ceramics and metals (from Ref. [10]).

	Material	P_0	P_{-1}	P_1	P_2	P_3
E	SUS304	0	201.04	3.08E-04	-6.53E-07	0
	Si3N4	0	348.43	-3.07E-04	2.16E-07	-8.95E-11
ν	SUS304	0	0.3262	-2.00E-04	3.79E-07	0
	Si3N4	0	0.24	0.00E+00	0	0
ρ	SUS304	0	8166	0	0	0
	Si3N4	0	2370	0	0	0
α	SUS304	0	1.23E-05	8.09E-06	0	0
	Si3N4	0	5.87E-06	9.10E-06	0	0

4. CONCLUSIONS

In this paper, a novel high-order layerwise theory has been developed for the vibration analysis of FGM plates with general boundary conditions and exposed to thermal environment. The formulation is based on the three-dimensional theory of elasticity so that no other assumptions on deformations and stresses along the thickness direction are introduced. The governing equations of the plates is discretized by modified Chebyshev polynomials of first kind in the transfer domain and the remaining domains are approximated by spectral method. The penalty technique is introduced to release the requirement of boundary conditions such that it is able to provide complete ability to satisfy any specified boundary conditions. The convergence, accuracy and reliability of the theory are validated. It is shown that our results meet well with data reported by other researchers.

5. ACKNOWLEDGEMENTS

The authors gratefully acknowledge the financial support from the National Natural Science Foundation of China (Nos. 51709066 and 51775125), the Fundamental Research Funds for the Central Universities of China (No. HEUCFM180303 and HEUCF180305) and China Postdoctoral Science Foundation funded project (2018T110277 and 2017M621252).

6. REFERENCES

1. Jin G, Ye T, Wang X, et al. "*A unified solution for the vibration analysis of FGM doubly-curved shells of revolution with arbitrary boundary conditions*". Composites Part B Engineering, 2016, 89: 230-252.
2. Ye T, Jin G, Su Z. "*Three-dimensional vibration analysis of laminated functionally graded spherical shells with general boundary conditions*". Composite Structures, 2014, 116:571-588.
3. Ye T, Jin G, Su Z. "*Three-dimensional vibration analysis of functionally graded sandwich deep open spherical and cylindrical shells with general restraints*". Journal of Vibration and Control, 2016, 22(15), 3326–3354.
4. Reddy JN, Cheng ZQ. "*Frequency correspondence between membranes and functionally graded spherical shallow shells of polygonal planform*". International Journal of Mechanical Sciences, 2002; 44: 967–985.
5. Kim YW. "*Temperature dependent vibration analysis of functionally graded rectangular plates*". Journal of Sound and Vibration, 2005, 284(3): 531-49.
6. Huang CS, McGee III OG, Wang KP. "*Three-dimensional vibrations of cracked rectangular parallelepipeds of functionally graded material*". International Journal of Mechanical Sciences, 2013, 70: 1-25.
7. Jin G, Su Z, Shi S, Ye T, Gao S. "*Three-dimensional exact solution for the free vibration of arbitrarily thick functionally graded rectangular plates with general boundary conditions*". Composite Structures, 2014, 108: 565–577.
8. Yang J, and Shen HS. "*Vibration characteristics and transient response of shear-deformable functionally graded plates in thermal environments*." Journal of Sound and Vibration, 2002, 255: 579-602.
9. Li Q, Iu VP, and Kou KP. "*Three-dimensional vibration analysis of functionally graded material sandwich plates*." Journal of Sound and Vibration 2009, 324: 733-750.
10. Ootao Y Kawamura R, Tanigawa Y, Imamura R. "*Optimization of material composition of nonhomogeneous hollow sphere for thermal stress relaxation making use of neural network*". Computer Methods in Applied Mechanics and Engineering, 1999, 180: 185–201.