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Bayesian estimation of sound power level

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ABSTRACT

The possibility of using the Bayesian inference to determining sound power level for precision, engineering and survey methods was presented in this paper. Minimum size of the original random sample necessary to estimate the expected value with required accuracy has been determined for estimation of sound power level for each methods. The simulation experiment has been performed for that purpose. The inference has been carried out based on results of non-parametric statistical tests at significance level $\alpha = 0.05$. The statistical analysis has shown that the minimum size of original random sample n used to estimate the values of sound power level should be 4 elements for precision and engineering methods, and 2 elements for survey method. The study on usefulness and effectiveness of the Bayesian approach to determination of sound power level in real-life situation was carried out with the use of data representing actual results. The data used to illustrate the proposed solutions and carry out the analysis were results of sound power levels of reference sound power source B&K 4205 were used.

Keywords: sound power level, Bayesian inference, uncertainty
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1. INTRODUCTION

Sound power level is one of the main parameters that describe noise source. This parameter is commonly used in acoustics, among other things, to model distribution of equivalent A-weighted sound pressure level in the environment [1, 2] and to noise hazard

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assessment in the working environment as well as for comparison itself between machines and devices of a certain type [3,4]. Therefore, the exact value of sound power level is very important. The exact value of this parameter is determined based on precision method for anechoic or hemi-anechoic rooms according to ISO 3745:2012 [5]. In the in-situ conditions use of the precision method to determination of sound power level it is not possible. Therefore in the industrial conditions two methods are used to determine this parameter, i.e. engineering and survey methods according to ISO 3744:2010 [6] and ISO 3746:2010 [7], respectively.

For the reasons mentioned above, it seems to be necessary to implement solutions of non-classical statistic to increase the accuracy of determining sound power level of industrial devices in the in-situ conditions. These techniques are based on non-parametric statistical methods, allowing to determine the distribution of a random variable without any information on belonging or not to any specific class of distributions and with a limited sample size.

For these reasons mentioned above, particular attention was paid to the possibility of using the Bayesian inference to determining sound power level of noise sources. This approach is used with success in point [8, 9] and interval estimation [10] of the noise indicators expected value and uncertainty.

Discussion of the algorithm, together with an example illustrating its functioning, will be presented further in this paper. The study on usefulness and effectiveness of the Bayesian approach to determination of sound power level in real-life situation was carried out with the use of data representing actual results.

2. BAYES INFERENCE

2.1. Bayes' theorem

Consider an estimate the random parameter θ from data $\mathbf{x}$. Then the associated conditional density $p\,(\theta|\mathbf{x})$ is called the posterior density because the estimate is conditioned "after the measurements" have been acquired. Estimators based on this posterior density are usually called Bayesian because they are constructed from Bayes' theorem, since $p\,(\theta|\mathbf{x})$ is difficult to obtain directly. That is, Bayes' rule is defined as follows [11]

$$p\,(\theta|\mathbf{x}) = \frac{f\,(\mathbf{x})}{p\,(\mathbf{x})} = \frac{p\,(\mathbf{x}|\theta)\,p\,(\theta)}{\int_{\Omega} p\,(\mathbf{x}|\theta)\,p\,(\theta)\ \mathrm{d}\theta}, \tag{1}$$

where $p\,(\theta)$ is called the prior density (before measurement), $p\,(\mathbf{x}|\theta)$ is called the sampling density or likelihood (more likely to be true), $p\,(\mathbf{x})$ is called the marginal data density or evidence (normalizes the posterior to assure its integral is unity), Ω is called the parameters space. Bayesian methods view the sought-after parameter as random possessing a "known" prior density [12].

This approach requires knowledge of two distributions. The first is the sampling density $p\,(\mathbf{x}|\theta)$. It was determined on the basis of simple random sample $\mathbf{x}$ of size n was sampled from the investigated population and determined make use of kernel density estimator [13]. The second is the prior density $p\,(\theta)$. In this experiment, a uniform distribution on the interval from 50 to 70 dB(A) was used as the prior distribution for each methods.

The Bayesian inference requires the use of numerical methods. In the hereby paper, the random walk Metropolis-Hastings sampling method (random walk M–H) [11] was used to generate samples θ_i^{BAY} from a posterior distribution. This sampling method is one of the main technique provides a means for drawing random samples from a posterior probability density function. The random walk Metropolis-Hastings method is a special case of the Metropolis-Hastings algorithm [14, 15].

2.2. Point estimation of distribution parameters by Bayes' rule

The random walk M–H method is able to generate correlated sample $\mathbf{x}_{\mathbf{BAY}}$ from a continuous posterior distribution. In the Bayesian inference the form of parameter θ depends not only on the prior and posterior distributions, but also on the loss function L. If the loss function is a quadratic function of form [16]

$$L(\theta, d) = C\,(d - \theta)^2\,, \quad C > 0, \tag{2}$$

where d-decision. The Bayesian estimate of θ parameter is expected value of posterior distribution, which was determined as the mean from the Markov chain after removed cycles burned [12]

$$\hat{\bar{\theta}}^{\mathrm{BAY}} = \hat{E}\left[p\,(\theta|\mathbf{x})\right] = \frac{1}{N-S}\sum_{i=S+1}^{N}\theta_i^{\mathrm{BAY}}, \tag{3}$$

where θ_i^{BAY} - elements of random sample $\mathbf{x}_{\mathbf{BAY}}$ from posterior distribution.

However, the Bayesian estimate of the standard deviation of the $\hat{\bar{\theta}}^{\mathrm{BAY}}$ parameter as follows [12]

$$\hat{s}_{\mathrm{BAY}} = \sqrt{\frac{1}{k}\sum_{i=1}^{1}\left(\theta_i^{\mathrm{BAY}} - \hat{\bar{\theta}}^{\mathrm{BAY}}\right)^2}. \tag{4}$$

3. RESEARCH MATERIAL

The study on usefulness and effectiveness of the Bayesian inference to determination of sound power level in real-life situation was carried out with the use of data representing actual results. The data used to illustrate the proposed solutions and carry out the analysis were results of sound power levels of reference sound power source B&K 4205 were used. The sound power level of this source has been determined using the precision, engineering and survey methods based on measurements of A-weighted sound levels (L_{Aeq}). Measurements of L_{Aeq} were made with a SVAN 959 (SVANTEK, Poland) equipped with SV type preamps and $\frac{1}{2}$ inch free-field 40AN microphone from G.R.A.S. The results of the background noise corrected A-weighted sound levels recorded at each measurement point which has been used to determine the sound power level of source using each method are presented in Figure 1. These data constituted the examined populations with sizes $K = 20$ for precision and engineering methods, and $K = 8$ for survey method.

3.1. Precision method

The precision method for determining the sound power level was based on ISO 3745:2012 [5]. The measurements were carried out in the anechoic room located in the AGH University of Science and Technology in Krakow, Poland. The $K = 20$ measurement points were located on a hemispherical measurement surface with a radius

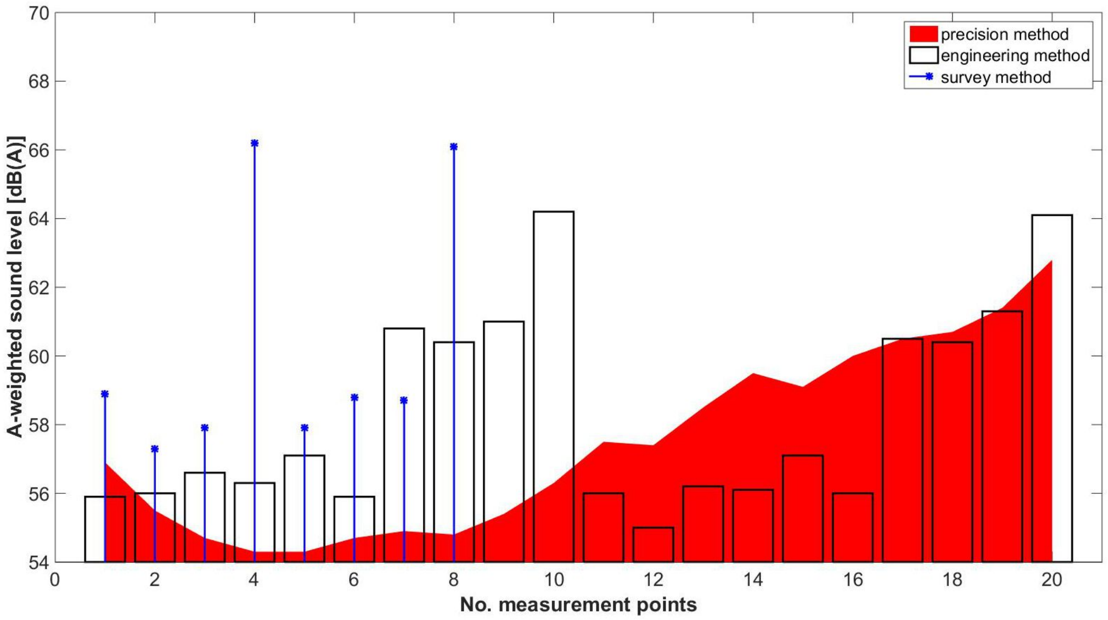

Figure 1: The background noise corrected A-weighted sound levels for each method.

of r = 2 m in a hemi-free field according to Table E.1 in Annex E of ISO 3745:2012. The measurement results recorded on February 14th, 2018 in the following meteorological conditions:

- relative humidity: $RH = 24$ %,
- temperature: $t = 19.7$ °C = 292.85 K,
- static pressure: $p_s = 101.6$ kPa.

On the grounds of the recorded A-weighted sound levels, value of sound power level for precision method L_{Wpr} is defined by [5] as

$$L_{\mathrm{Wpr}} = \overline{L_{\mathrm{pc}}} + 10\log\left(\frac{S}{S_0}\right) + C_1 + C_2 \quad [\mathrm{dB(A)}], \tag{5}$$

where $\overline{L_{\mathrm{pc}}}$ is the background noise corrected surface sound pressure level in dB(A), $S = 2\pi r^2$ is the area of the hemispherical measurement surface in m^2, $S_0 = 1$ m^2, C_1 is the reference quantity correction in dB(A), C_2 is the acoustics radiation impedance correction in dB(A). The correction factors C_1 and C_2 are defined in Equation 14 of ISO 3745:2012.

The obtained value of sound power level in accordance with Equation 5 is $L_{\mathrm{Wpr}} = 72.0$ dB(A).

3.2. Engineering method

The engineering method for determining the sound power level was based on ISO 3744:2010 [7]. The measurements were made on the asphalt playing field. The $K = 20$ measurement points were located on a hemispherical measurement surface with a radius of $r = 2$ m in an essentially free field over a reflecting plane according to Table B.1 in Annex B of ISO 3744:2010. At a distance of 20 m from the measuring surface there were no sound reflecting surfaces. The measurement results recorded on February 17th, 2018 from 11:05 a.m. to 01:15 p.m. in the following meteorological conditions:

- relative humidity: $RH = 75$ %,
- temperature: $t = 2.8$ °C = 275.95 K,
- static pressure: $p_s = 102.4$ kPa,
- wind speed: $v = 1 - 2.6$ m/s,
- wind direction: S – SW.

Based on the measurement results of the A-weighted sound levels, value of sound power level for engineering method L_{Wen} is defined by [6] as

$$L_{\mathrm{Wen}} = \overline{L_{\mathrm{p}}} - K_{1\mathrm{A}} - K_{2\mathrm{A}} + 10\log\left(\frac{S}{S_0}\right) + C_1 + C_2 \quad [\mathrm{dB(A)}], \tag{6}$$

where $\overline{L_{\mathrm{p}}}$ is the surface sound pressure level in dB(A), $K_{1\mathrm{A}}$ is the background noise correction in dB(A) is defined in Equation 16 of ISO 3744:2010, $K_{2\mathrm{A}}$ is the environmental correction factor in dB(A) according to Annex A or subsection 4.3.1 of ISO 3744:2010, $S = 2\pi r^2$ is the area of the hemispherical measurement surface in m^2, $S_0 = 1$ m^2, C_1 is the reference quantity correction in dB(A), C_2 is the acoustics radiation impedance correction in dB(A). The correction factors C_1 and C_2 are calculated according to Annex G of ISO 3744:2010.

The obtained value of sound power level in accordance with Equation 6 is $L_{\mathrm{Wen}} = 72.6$ dB(A).

3.3. Survey method

The survey method for determining the sound power level was based on ISO 3746:2010 [7]. The measurements were made on the on the paved parking. The $K = 8$ measurement points were located on a hemispherical measurement surface with a radius of $r = 2$ m over a reflecting plane according to Table B.1 in Annex B of ISO 3746:2010. At a distance of 25 m from the measuring surface there were no sound reflecting surfaces. The measurement results recorded on February 17th, 2018 from 08:00 p.m. to 08:40 p.m. in the following meteorological conditions:

- relative humidity: $RH = 81$ %,
- temperature: $t = 1.0$ °C = 274.15 K,
- static pressure: $p_s = 102.2$ kPa,
- wind speed: $v = 0.5 - 1$ m/s,
- wind direction: N.

The value of sound power level for survey method L_{Wsu} was also determined based on Equation 6, where $\overline{L_{\mathrm{p}}}$ is the surface sound pressure level in dB(A), $K_{1\mathrm{A}}$ is the background noise correction in dB(A) is defined in Equation 15 of ISO 3746:2010, $K_{2\mathrm{A}}$ is the environmental correction factor in dB(A) according to Annex A or section

4 of ISO 3746:2010, $S = 2\pi r^2$ is the area of the hemispherical measurement surface in m^2, $S_0 = 1$ m^2, C_1 is the reference quantity correction in dB(A), C_2 is the acoustics radiation impedance correction in dB(A). The correction factors C_1 and C_2 are calculated according to Annex G of ISO 3744:2010.

The obtained value of sound power level in accordance with Equation 6 is $L_{\mathrm{Wsu}} = 75.0$ dB(A).

4. SIMULATION EXPERIMENTS, RESULTS AND DISCUSSION

The experiment has been conducted in order to specify the minimum size of the original sample size n in order to determine the expected values of the sound power level with required accuracy.

This experiment served to determine the impact of original random sample size n on the estimation accuracy of sound power level. For that reason, 1000 random samples with sizes $n = 2, 3, \ldots, K$ were drawn from the examined population. The original random sample size n simulates the number of measurement points based on which the sound power level is determined.

The reconstruction of probability distributions was performed based on the same number of Markov chain elements N for each sample with size n. The distributions were determined based on $N = 110000$ elements of which the first 10000 were treated as burned cycles.

Thus receiving 1000 posterior distributions with 100000 elements for each original sample size n. Each distribution was used to determine the Bayesian estimate of the expected value of sound power level from Equation 3. The result was 1000-element probability distributions of sound power level which were subjected to further statistical analysis.

First, the Kruskal-Wallis non-parametric test has been performed at the significance level $\alpha = 0.01$ in order to check if there are statistically significant differences in estimated sound power level for various original sample sizes. The test gave the probability values of $p = 2.60 \times 10^{-4}$ and $p = 1.07 \times 10^{-23}$ for data from precision and engineering methods, respectively. These values are much less than the assumed level of significance which proves the existence of statistically significant differences in values of estimated parameter.

For precision and engineering methods, the Tukey-Kramer multiple comparison test at the level of significance $\alpha = 0.05$ was conducted in order to find out between which groups there are difference. The results of the Tukey-Kramer test a indicate the original random sample size n based on which the estimated expected values of sound power level are statistically different at the assumed level of significance.

However, for survey method the Kruskal-Wallis test gave the probability values of $p = 0.13$. This value indicates no statistically significant differences in estimated sound power level for various original sample sizes.

The statistical analysis has shown that the minimum size of original sample n used to estimate sound power level should be $n = 4$ for precision and engineering methods, while $n = 2$ for survey method.

The dispersion of obtained results was also analysed by determining the 95% confidence intervals using the percentiles of the Bayesian distribution for each probability distribution obtained using the original random sample of size n. The 95%

interval width ($IW_{95\%}$) was defined as

$$IW_{95\%} = p_{97.5} - p_{2.5} \quad [\mathrm{dB(A)}], \tag{7}$$

where $p_{2.5}$ and $p_{97.5}$ are the 2.5th and 97.5th empirical percentiles of the posterior distribution of sound power level.

Interval widths obtained for precision method fall into ranges from 0.01 dB(A) to 7.65 dB(A) for the original random sample of size $n = 20$ and $n = 2$, respectively. However, for engineering method fall into ranges from 0.12 dB(A) to 7.78 dB(A) for the original random sample of size $n = 20$ and $n = 2$, respectively. On the other hands the $\mathrm{IW}_{95\%}$ obtained for survey method fall into ranges from 0.36 dB(A) to 8.52 dB(A) for the original random sample of size $n = 8$ and $n = 2$, respectively. The results clearly show that the dispersion decreases when the size of original random sample increases.

5. CONCLUSIONS

The paper determines the minimum size of the original random sample necessary to estimate the expected value with required accuracy for different methods of determining sound power level using Bayesian inference.

The statistical analysis was carried out on the basis of Kruskal-Wallis test. Next, multiple comparison procedures were used for pairwise comparisons between the means using non-parametric Tukey-Kramer test at significance level $\alpha = 0.05$.

The statistical analysis has shown that the minimum size of original random sample n used to estimate the values of sound power level should be 4 elements for precision and engineering methods, and 2 elements for survey method.

The results of this experiment clearly indicate that this approach can be successfully used to estimate not only sound power level, but also other acoustic parameters.

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