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Sound transmission loss of metamaterial plates with periodically attached local resonators

Yong Xiao[1], Jiajia Guo, Jihong Wen

Laboratory of Science and Technology on Integrated Logistics Support, College of Intelligence Science, National University of Defense Technology, Changsha 410073, P. R. China

ABSTRACT

This work is concerned with the study of sound transmission loss (STL) of metamaterial plates comprising periodic arrays of local resonators attached to a host plate. The influences of the periodic lattice constant, the resonator mass, and the damping of the resonators and the host plate, on the STL are analysed systematically by using a plane wave expansion (PWE) method. It is shown that the lattice constant can affect the STL in a dramatic way, which provides new guidelines for designs aiming at lower frequency sound insulation.

Keywords: Acoustic Metamaterial, Sound Insulation, Local Resonator
I-INCE Classification of Subject Number: 33

1. INTRODUCTION

The development of acoustic metamaterials[1, 2] provides many new ideas for the manipulation of wave propagation in acoustic/elastic media. For instance, the concept of acoustic metamaterial has been introduced to design metastructures by attaching periodic arrays of local resonators to structural waveguides such as longitudinal rods[3], flexural beams [4, 5] and plates [6, 7]. Such metastructures can realize low frequency elastic wave band gaps that can find applications in the field of vibration and noise control. To insulate low frequency sound, lightweight membrane-type acoustic metamaterials made of periodic arrays of small membrane-mass units attached to a host frame structure have been proposed and demonstrated [8]. However, such membrane-type acoustic metamaterials many have limitations in many practiccal enginering due to the low reliability of thin and soft membranes used in the construction. In contrast, plate-type metamaterials (named “metamaterial plates” in this work) made of conventional plate strcutures as a host plate attached with periodic local resonators were proposed as an alternative kind of acoustic metamaterial for low frequency sound insulation[9]. However, previous investigations[9, 10] on sound insulation problem of such metamaterial plates are only focused to the case of sub-wavelength design, by which the spacing between adjacent local resonators, or in orther words, the lattice constant, is much smaller than the

[1] xiaoy@vip.sina.com

operating flexural wavelength in the host plate. Thus it is still an open question whether and how a larger lattice constant influence the sound insulation performance. In conjunction with answering this question, a systematic analysis of the effects of resonator and plate properties on the sound insulation properties is also required to provide more guidance for the design of such structures.

This paper is organized as four sections. Following this introduction, the second section presents the plane wave expansion (PWE) method [9] to deal with sound transmission problems of metamaterial plates. The third section is devoted to the examination of STL behavior of metamaterial plates with different choice of lattice constant and resonator parameters. Finally, the fourth section concludes this paper.

2. METHOD

A general metamaterial plate is considered to be a plate system consisting of 2D periodic multiple arrays of spring-mass resonators attached to a thin, uniform plate, as shown in Figure 2. The purpose of this section is to briefly present the PWE method [9] for the calculation of STL of such a general metamaterial plate. In Figure 2, the plate is surrounded by air, and an oblique plane sound wave (p_{inc}) varying harmonically in time is incident upon the plate from the lower domain (z<0) with elevation angle θ and azimuth angle ϕ. The associated incident sound wavenumber vector is denoted by $\mathbf{k}_0$. In addition, the reflected and transmitted sound waves are denoted by p_{ref} and p_{tr}, respectively.

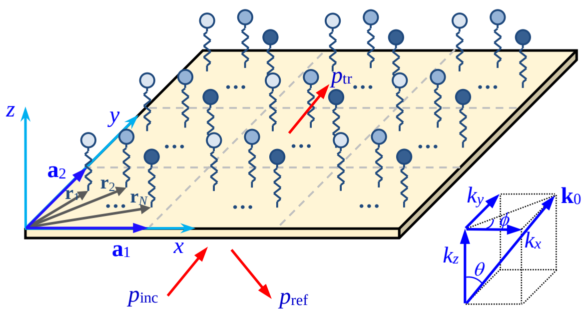

Fig. 1. Schematic diagram of a general metamaterial plate consisting of 2D periodic multiple arrays of resonators attached to a uniform plate.

The periodic lattice of the plate system shown in Figure 2 is characterized by two basis vectors: $\mathbf{a}_1=(a_{1x}, a_{1y})$ and $\mathbf{a}_2=(a_{2x}, a_{2y})$. Thus the position of each unit cell can be described by a direct lattice vector

$$\mathbf{R} = \bar{m}\mathbf{a}_1 + \bar{n}\mathbf{a}_2, \tag{1}$$

where $\bar{m}$ and $\bar{n}$ are integers.

Note that, in general, there are N resonators attached in each unit cell, as shown in Figure 2. The spring constant and mass of the jth resonator in each unit cell are $k_{\mathrm{r},j}$ and $m_{\mathrm{r},j}$. Resonator damping can be introduced by means of a complex spring constant, i.e., $k_{\mathrm{r},j} \rightarrow k_{\mathrm{r},j}(1+\mathrm{i}\eta_{\mathrm{r},j})$, where $\eta_{\mathrm{r},j}$ is the loss factor. The location of the attachment point of each resonator can be described by the following vector

$$\mathbf{r}_j + \mathbf{R} = \mathbf{r}_j + (\bar{m}\mathbf{a}_1 + \bar{n}\mathbf{a}_2),\ j = 1, 2, \ldots, N, \tag{2}$$

where $\mathbf{r}_j=(x_j, y_j)$ represents the location of the jth resonator, as illustrated in Figure 2.

Assume the amplitude of the incident sound wave is P_0, and then the incident sound wave can be expressed by

$$p_{\mathrm{inc}}(x,y,z,t)=p_{\mathrm{inc}}(x,y,z)\mathrm{e}^{\mathrm{i}\omega t}=P_0\mathrm{e}^{-\mathrm{i}(k_x x+k_y y+k_z z)}\mathrm{e}^{\mathrm{i}\omega t}, \tag{3}$$

where

$$k_x=k_0\sin\theta\cos\phi,\quad k_y=k_0\sin\theta\sin\phi,\quad k_z=k_0\cos\theta, \tag{4}$$

where $k_0=\omega/c_0$ denotes the sound wavenumber in air, and c_0 is the sound speed in air.

Let $\mathbf{r}=(x, y)$ and $\mathbf{k}=(k_x, k_y)$, hence the incident sound pressure can be written as

$$p_{\mathrm{inc}}(\mathbf{r},z,t)=P_0\mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{r}}\mathrm{e}^{-\mathrm{i}k_z z}\mathrm{e}^{\mathrm{i}\omega t}. \tag{5}$$

The governing equation for the vibration of the thin plate can be expressed by

$$\begin{aligned}D\nabla^4 w(\mathbf{r})-\rho h\omega^2 w(\mathbf{r})&=p_{\mathrm{inc}}(\mathbf{r},z)_{z=0}+p_{\mathrm{ref}}(\mathbf{r},z)_{z=0}-p_{\mathrm{tr}}(\mathbf{r},z)_{z=0}\\&\quad+\sum_{j=1}^{N}\sum_{\mathbf{R}}f_j(\mathbf{r}_j+\mathbf{R})\delta[\mathbf{r}-(\mathbf{r}_j+\mathbf{R})],\end{aligned} \tag{6}$$

where $\nabla^4=(\partial^2/\partial x^2+\partial^2/\partial y^2)^2$, $w(\mathbf{r})$ represents the transverse displacement of the plate and the time dependence $\exp(\mathrm{i}\omega t)$ is suppressed. In addition, $f_j(\mathbf{r}_j+\mathbf{R})$ refers to the force applied to the plate by the resonator locating at $\mathbf{r}_j+\mathbf{R}$, and $\delta[\mathbf{r}-(\mathbf{r}_j+\mathbf{R})]$ is a two dimensional delta function defined by

$$\delta[\mathbf{r}-(\mathbf{r}_j+\mathbf{R})]=\delta[x-(\mathbf{r}_j+\mathbf{R})_x]\delta[y-(\mathbf{r}_j+\mathbf{R})_y]. \tag{7}$$

Due to the periodicity of the considered plate system, the plate displacement can be expanded as a summation of plane waves

$$w(\mathbf{r})=\sum_{\mathbf{G}}W_{\mathbf{G}}\mathrm{e}^{-\mathrm{i}(\mathbf{k}+\mathbf{G})\cdot\mathbf{r}}. \tag{8}$$

where $\mathbf{G}$ denotes the reciprocal-lattice vector, given by

$$\mathbf{G}=m\mathbf{b}_1+n\mathbf{b}_2, \tag{9}$$

where m and n are integers, $\mathbf{b}_1=(b_{1x}, b_{1y})$ and $\mathbf{b}_2=(b_{2x}, b_{2y})$ are basis vectors of the reciprocal lattice. Similar to Eq. (8), the reflected and transmitted sound pressures can be expressed by

$$p_{\mathrm{ref}}(\mathbf{r},z)=\sum_{\mathbf{G}}P_{\mathrm{ref},\mathbf{G}}\mathrm{e}^{-\mathrm{i}(\mathbf{k}+\mathbf{G})\cdot\mathbf{r}}\mathrm{e}^{+\mathrm{i}k_{z,\mathbf{G}}z}, \tag{10}$$

$$p_{\mathrm{tr}}(\mathbf{r},z)=\sum_{\mathbf{G}}P_{\mathrm{tr},\mathbf{G}}\mathrm{e}^{-\mathrm{i}(\mathbf{k}+\mathbf{G})\cdot\mathbf{r}}\mathrm{e}^{-\mathrm{i}k_{z,\mathbf{G}}z}, \tag{11}$$

where

$$k_{z,\mathbf{G}}=\begin{cases}\sqrt{k_0^2-|\mathbf{k}+\mathbf{G}|^2};\ k_0^2\geq|\mathbf{k}+\mathbf{G}|^2\\-\mathrm{i}\sqrt{|\mathbf{k}+\mathbf{G}|^2-k_0^2};\ k_0^2<|\mathbf{k}+\mathbf{G}|^2\end{cases}. \tag{12}$$

By modeling time-harmonic vibration of the resonators, one can find that the force $f_j(\mathbf{r}_j)$ is related to the plate displacement $w(\mathbf{r}_j)$ through[6, 9]

$$f_j(\mathbf{r}_j)=-D_{\mathrm{r},j}w(\mathbf{r}_j), \tag{13}$$

where

$$D_{\mathrm{r},j}=\frac{-\omega^2 m_{\mathrm{r},j}}{1-\omega^2/[\omega_{\mathrm{r},j}^2(1+\mathrm{i}\eta_{\mathrm{r},j})]} \tag{14}$$

represents the dynamic stiffness of the jth resonator, and $\omega_{\mathrm{r},j}=(k_{\mathrm{r},j}/m_{\mathrm{r},j})^{1/2}=2\pi f_{\mathrm{r},j}$ refers to the resonance frequency of the jth resonator. At the air-plate interfaces, the continuity conditions of normal velocity imply that

$$\left.\frac{\partial[p_{\mathrm{inc}}(\mathbf{r},z)+p_{\mathrm{ref}}(\mathbf{r},z)]}{\partial z}\right|_{z=0}=\rho_0\omega^2 w(\mathbf{r}), \tag{15}$$

$$\left.\frac{\partial[p_{\mathrm{tr}}(\mathbf{r},z)]}{\partial z}\right|_{z=0} = \rho_0\omega^2 w(\mathbf{r}). \tag{16}$$

Substituting Eqs.(5), (8), (10) and (11) into Eqs. (15) and (16) and considering the sums to be true for all choices of **r**, one can find the following relations between the sound pressure coefficients and plate displacement coefficients

$$P_{\mathrm{ref},\mathbf{G}} = P_0\delta_{\mathbf{0-G}} - \frac{\mathrm{i}\rho_0\omega^2}{k_{z,\mathbf{G}}}W_{\mathbf{G}}, \tag{17}$$

$$P_{\mathrm{tr},\mathbf{G}} = \frac{\mathrm{i}\rho_0\omega^2}{k_{z,\mathbf{G}}}W_{\mathbf{G}}, \tag{18}$$

By inserting expaned plane wave formulations of plate displacement (8), and pressures (10)-(11), the governing equation (6) can be written in a matrix form [9]

$$(\mathbf{K}_{\mathrm{p}} + \mathrm{i}\omega\mathbf{C}_{\mathrm{f}} - \omega^2\mathbf{M}_{\mathrm{p}} + \mathbf{D}_{\mathrm{r}})\mathbf{W}_{\mathbf{G}} = 2P_0 S\cdot\boldsymbol{\delta}_{\mathbf{0-G}}, \tag{19}$$

where, $\mathbf{K}_{\mathrm{p}}$ and $\mathbf{M}_{\mathrm{p}}$ represent the plate stiffness and mass matrices, $\mathbf{C}_{\mathrm{f}}$ is regarded as the fluid loading matrix, and $\mathbf{D}_{\mathrm{r}}$ refers to the dynamic stiffness matrix of the resonators. Eq. (19) can be adopted for the prediction of plate displacement coefficient $W_{\mathbf{G}}$. The sound pressure coefficients can be obtained by using Eqs. (17) and (18). It should be noted out that, although Eq. (11) suggests the transmitted pressure comprises a number of components, only those having a real wavenumber in the z-direction could transmit energy. Therefore, the oblique sound power transmission coefficient is [9]

$$\tau(\theta,\phi) = \frac{\sum_{\mathbf{G}}\left|P_{\mathrm{tr},\mathbf{G}}\right|^2 \mathrm{Re}(k_{z,\mathbf{G}})}{\left|P_0\right|^2 k_z}. \tag{20}$$

The diffuse field power transmission coefficient can be obtained by averaging the oblique transmission coefficient over the elevation angle θ and azimuth angle ϕ

$$\tau_{\mathrm{diff}} = \frac{\int_0^{2\pi}\int_0^{\pi/2}\tau_{\mathrm{p}}(\theta,\phi)\sin\theta\cos\theta\,\mathrm{d}\theta\,\mathrm{d}\phi}{\int_0^{2\pi}\int_0^{\pi/2}\sin\theta\cos\theta\,\mathrm{d}\theta\,\mathrm{d}\phi} = \frac{1}{\pi}\int_0^{\pi}\int_0^{\pi/2}\tau_{\mathrm{p}}(\theta,\phi)\sin 2\theta\,\mathrm{d}\theta\,\mathrm{d}\phi. \tag{21}$$

The oblique or diffuse field sound transmission loss (STL) is defined by

$$\mathrm{STL} = 10\log_{10}(\frac{1}{\tau}). \tag{22}$$

3. RESULTS

3.1 Effects of lattice constant

To examine the effects of lattice constant, we first calculate the normal incidence STL of various metamaterial plates made of the same host plate attached with an array of resonators of different lattice constant a. The results are shown in Fig. 2. The host plate is a homogenous aluminum plate of thickness h=2mm, and the material properties are: Young's modulus E=70GPa, and poisson ratio σ=0.33. The ratio of resonator mass to the plate mass is defined by $r_{\mathrm{m}}=m_{\mathrm{r}}/m_{\mathrm{p}}$. Here, m_{r} is the mass of resonator in one unit cell, and m_{p} is the mass of the plate within one unit cell. For all the cases considered in Fig. 2, the resonator mass ratio is chosen as r_{m}=0.25, and the resonance frequency of local resonators is f_{r} =300Hz, at which the half flexural wavelength of the host plate is $\lambda/2$=0.128m. The lattice constant a is increased from a subwavelength scale (a=0.03m) to a half wavelength scale (a=0.18m). The results in Fig. 2 indicate that the choice of a relatively larger lattice constant can affect the STL in a dramatic way. Specifically, the frequency locations of the STL peak and dip both shift to a lower frequency range even though the resonance frequency f_r is fixed to 300 Hz. This provides a new idea to lower the STL peak frequency without the need to increase the resonator mass or reduce the resonator stiffness.

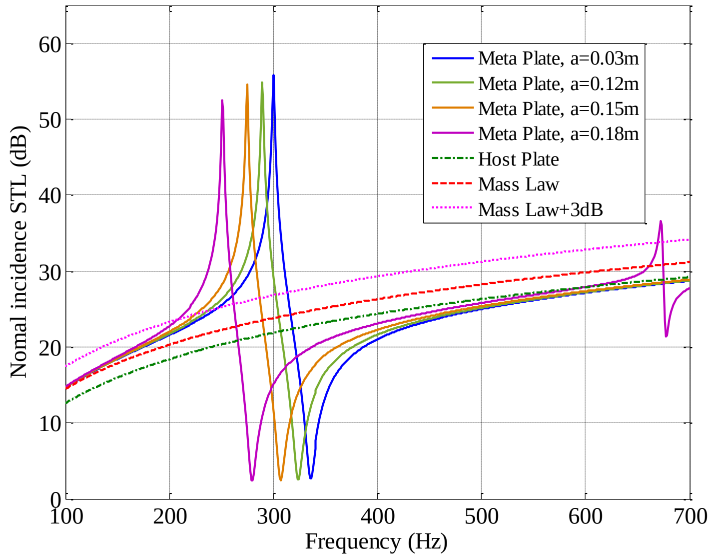

Fig. 2. Comparison of normal incidence STL of metamaterial plates with diffrent lattice constant.

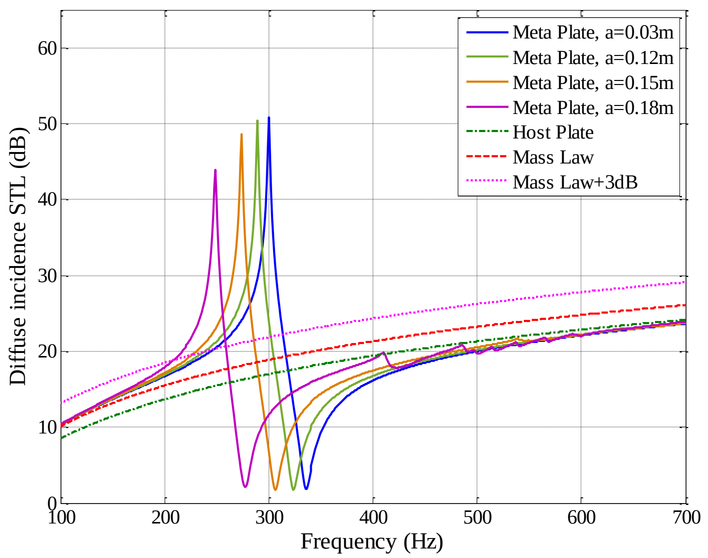

Fig. 3. Comparison of diffuse incidence STL of metamaterial plates with diffrent lattice constant.

The diffuse incidence STL of each metamaterial plate is also calculated and depicted in Fig. 3. One can observe that the effects of the lattice constant are similar to those shown in Fig. 2.

3.2 Effects of resonator mass

To examine the effects of resonator mass, the case of metamaterial plates with a subwavelength lattice constant (a=0.03m) is considered first. Fig. 4 shows the normal incidence STL of three metamaterial plates with different mass ratio of local resonators, which are r_m=0.25, r_m=0.5 and r_m=1, respectively. The resonance frequency of local resonators are fixed to f_r =300 Hz, and other parameters of these metamaterial plates are all the same as previous examples. Results in Fig. 4 indicates that as the mass ratio increases, the STL dip shifts to a higher frequency range, while the frequency location of the STL peak remains the same. But at the same time, the STL below the dip frequency is generally increased, leading to a broadened bandwidth of improved STL.

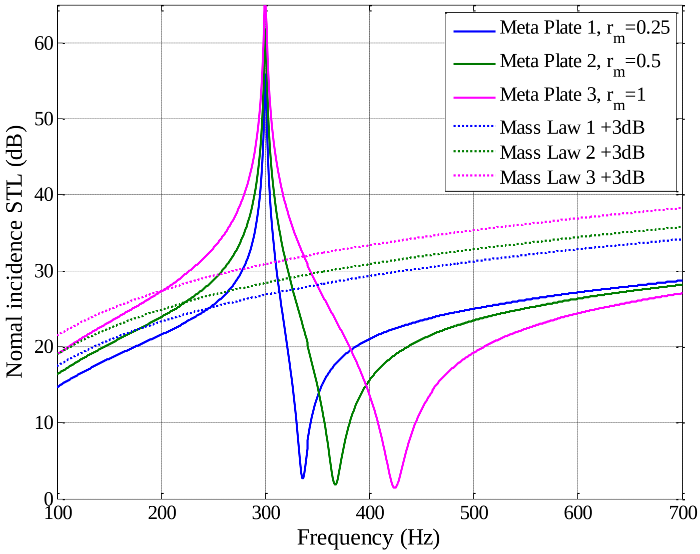

Fig. 4. Comparison of normal incidence STL of metamaterial plates with diffrent mass ratio of resonators (a=0.03m , f_r=300Hz).

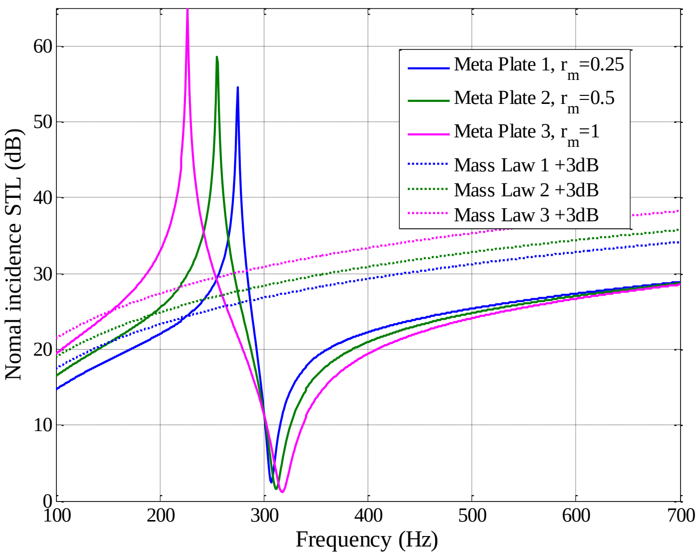

Fig. 5. Comparison of normal incidence STL of metamaterial plates with diffrent mass ratio of resonators (a=0.15m , f_r=300Hz).

We further consider the case of metamaterial plates with a larger lattice constant (a=0.15m) comparable to the half flexural wavelength ($\lambda/2$=0.128m). The normal incidence STL of metamaterial plates with various resonator mass ratio are calculated and presented in Fig. 5. It is seen the effects of the mass ratio indicated by Fig. 5 is very different from those observed in Fig. 4 for the case of subwavelength lattice constant

(a=0.03m). In Fig. 5, the frequency location of the STL dip is almost unchanged as the mass ratio increases. However, the STL peak frequency deceases with the increase in mass ratio, although the resonance frequency is fixed to f_r =300 Hz.

3.3 Effects of resonator damping and plate damping

The effects of resonator damping are examined first for the case of metamaterial plates with a subwavelength lattice constant (a=0.03m). The normal incidence STL is calculated according to different choices of resonator damping loss factor η_r , as shown in Fig. 6. It is seen the resonator damping has a smoothing effect on the STL peak and dip. As the resonator damping increases, the STL around the peak frequency decreases, while the STL near the dip frequency increases.

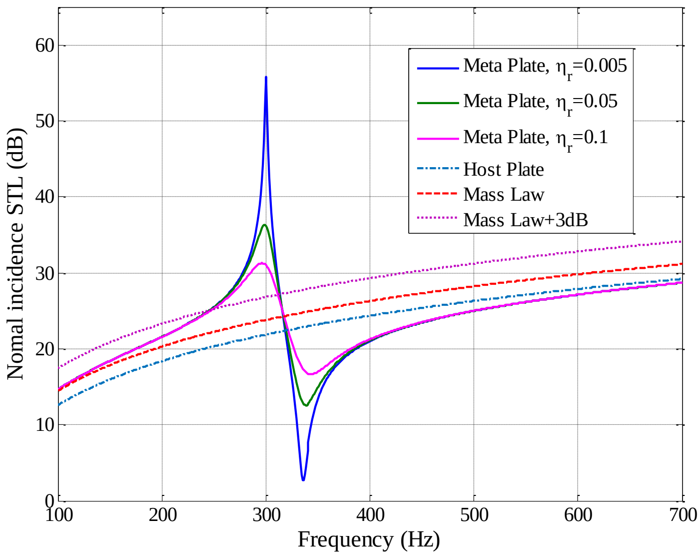

Fig. 6. Comparison of normal incidence STL of metamaterial plates with diffrent resonator damping(a=0.03m , f_r=300Hz, η_p=0.005).

Numerical results associated with increased plate damping are not depicted here, since we find that the plate damping has little influence within the considered low frequency range for such case of metamaterial plates with a subwavelength lattice constant.

Now we consider the case of metamaterial plates with a larger lattice constant (a=0.15m) comparable to the half flexural wavelength (λ/2=0.128m). Comparison of normal incidence STL of metamaterial plates with different resonator damping and plate damping are shown in Fig. 7. It is seen that the effects of the resonator damping are similar to those observed in Fig. 6. However, it is noted that the plate damping also has influence. Similar to the effects of resonator damping, plate damping can suppress the STL around the peak and increase the STL around the dip. But such effects are weaker compared with the resonator damping.

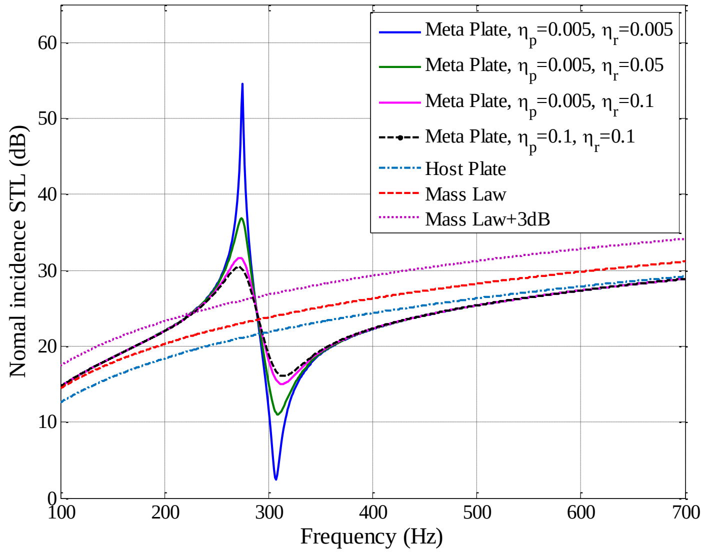

Fig. 7. Comparison of normal incidence STL of metamaterial plates with different resonator damping and plate damping (a=0.15m , f_r=300Hz).

4. CONCLUSIONS

This work presents a systematic analysis of STL of metamaterial plates with a periodic array of local resonators. The STL is calculated by using a plane wave expansion (PWE) method. The influence of the periodic lattice constant, the resonator mass, and the damping of resonators and host plate on the STL is examined. It is shown that a choice of a lattice constant that is comparable to or larger than the half flexural wavelength results in very different STL characteristics in comparison with the subwavelength case, which has been usually addressed in existing publications. In particular, for metamaterial plates with a lattice constant large enough, the frequency location of the STL peak always decrease with increasing lattice constant, and it also decease with increasing resonator mass ratio, even though the resonance frequency of local resonators is fixed. It is also shown that both the resonator and plate damping have a smoothing effect on the STL peak and dip if the lattice constant is large enough. This work provides new knowledge of the STL characteristics of metamaterial plates with local resonators, and gives new ideas to the design of metamaterial plates for low frequency sound insulation.

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