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Analysis of vibration characteristics of pipeline system with passive vibration absorber

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ABSTRACT

The purpose of this paper is to investigate the influence of passive vibration absorber in pipeline system. Passive vibration absorber is installed on the piping system. It analyzes the natural vibration characteristics and forced vibration characteristics of the piping system. The theoretical innovation of this paper combines the transfer matrix method with the Lagrangian equation in order to analyze the vibration characteristic of piping system with passive vibration absorber. Based on the vibration mechanics, the vibration differential equation of a single pipe is established. Using the continuum transfer matrix method, the transfer matrix of space piping system sets up. Considering the kinetic energy and potential energy of the pipeline system, the Lagrange equation of the second kind was used to establish the vibration differential equations of the pipeline system with the passive vibration absorber. Then , the mode superposition method was used to solve the forced vibration response. This paper analyzed a L-shaped beam with passive vibration absorber. The correctness of the method was proved by comparing the theoretical calculation results with the finite element results. The continuum transfer matrix method with Lagrange equation of the second kind has high accuracy in calculating the vibration characteristics of piping system.

Keywords: Pipeline system, Passive vibration absorber, Continuum transfer matrix
I-INCE Classification of Subject Number: 45

1. INTRODUCTION

Pipeline systems have been widely applied to areas such as designing heat exchange tubes in steam pipes, chemical plants, pump discharge lines, oil pipelines

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marine risers. As the piping systems work, undesirable noise and vibrations are produced. The vibration would lead to a piping system surge, deteriorate the work environment or even paralyze the pipe system and machines. Therefore, it is of great significance to control the vibration of pipeline systems and an extensive effort has been made in the analysis of the piping system vibrations[1-2].

This paper simplifies the pipeline system to Euler-Bernoulli beam model, taking an L-shaped beam structure with the consideration of passive vibration absorber as an example for analysis.Warminski[3] formulated the third order partial differential nonlinear equations for an L-shaped beam structure with different flexibility in the two orthogonal directions, without taking into account rotary inertia effects. Ozonato[4] studied post-buckled chaotic vibrations of an L-shaped beam structure considering only in-plane bending nonlinear motions. Using Lagrange formulation, Georgiades[5] derived the linear equations of motion for an L-shaped beam structure when considering the rotary inertia terms. That study demonstrated well separated in-plane and out-of-plane motions, which has also been shown with Abaqus Finite Element simulations. In the analysis of vibration system, the passive vibration absorber is simplified to mass-spring system. Mass-spring system has been widely studied by researches. Related research to alter the natural frequencies and mode shape of beam, plate and shell structures using tuned mass damper and penalty function methods for various applications including optimal vibration control can be found in Refs[6-11].

Different from the traditional discrete transfer matrix method, the continuum transfer matrix method is used in this paper. And Lagrange equation of the second kind is introduced to consider the impact of passive vibration absorber, the vibration of pipeline system is divided into in-plane vibration and out-of-plane vibration, their free vibration characteristics and forced vibration characteristics are separately analyzed. This method can calculate pipeline system with arbitrary angles and arbitrary constraint conditions, which has the advantages of easy programming and high calculation accuracy. The method can provide theoretical support for the application of passive vibration absorber on pipeline system and theoretical reference for parameter determination of vibration absorber.

2. PIPELINE DYNAMICS EQUATION

2.1 Vibration Transfer Matrix of Pipeline

Transverse vibration transfer matrix of straight pipe:

$$\begin{bmatrix} U_y(l) \\ \theta_z(l) \\ M_z(l) \\ Q_y(l) \end{bmatrix} = \left[T_{x-y}\right] \begin{bmatrix} U_y(0) \\ \theta_z(0) \\ M_z(0) \\ Q_y(0) \end{bmatrix} = \begin{bmatrix} V_1(kl) & \frac{V_2(kl)}{k} & \frac{V_3(kl)}{EIk^2} & \frac{V_4(kl)}{EIk^3} \\ kV_4(kl) & V_1(kl) & \frac{V_2(kl)}{EIk} & \frac{V_3(kl)}{EIk^2} \\ k^2 EIV_3(kl) & kEIV_4(kl) & V_1(kl) & \frac{V_2(kl)}{k} \\ k^3 EIV_2(kl) & k^2 EIV_3(kl) & kV_4(kl) & V_1(kl) \end{bmatrix} \begin{bmatrix} U_y(0) \\ \theta_z(0) \\ M_z(0) \\ Q_y(0) \end{bmatrix} \tag{1}$$

where

$$V_1(kx) = \frac{1}{2}\left(\cos h(kx) + \cos(kx)\right) \tag{2}$$

$$V_2(kx) = \frac{1}{2}\left(\sin h(kx) + \sin(kx)\right) \tag{3}$$

$$V_3(kx) = \frac{1}{2}(\cos h(kx) - \cos(kx)) \tag{4}$$

$$V_4(kx) = \frac{1}{2}(\sin h(kx) - \sin(kx)) \tag{5}$$

$$k^4 = \frac{\rho A}{EI}\omega^2 \tag{6}$$

E is elastic modulus; *I* is moment of inertia; *ρ* is density; *A* is cross-sectional area; *ω* is the natural frequency.

Longitudinal transfer matrix of straight pipe：

$$\begin{bmatrix} U(l) \\ F(l) \end{bmatrix} = [T_X]\begin{bmatrix} U(0) \\ F(0) \end{bmatrix} = \begin{bmatrix} \cos(\beta l) & \frac{1}{-\beta EA}\sin(\beta l) \\ \beta EA\sin(\beta l) & \cos(\beta l) \end{bmatrix}\begin{bmatrix} U(0) \\ F(0) \end{bmatrix} \tag{7}$$

where

$$\beta = \omega\sqrt{\frac{\rho}{E}} \tag{8}$$

Torsion transfer matrix of straight pipe：

$$\begin{bmatrix} \theta(l) \\ M_n(l) \end{bmatrix} = [T_{tor}]\begin{bmatrix} \theta(0) \\ M_n(0) \end{bmatrix} = \begin{bmatrix} \cos(\sigma l) & \frac{1}{-\sigma GI_\rho}\sin(\sigma l) \\ \sigma GI_\rho\sin(\sigma l) & \cos(\sigma l) \end{bmatrix}\begin{bmatrix} \theta(0) \\ M_n(0) \end{bmatrix} \tag{9}$$

where

$$\sigma = \omega\sqrt{\frac{\rho}{G}} \tag{10}$$

G is shear elastic modulus，I_ρ is polar moment of inertia.

When considering the axial movement, lateral movement in x-z plane, lateral movement in x-y plane and torsional movement of straight pipe simultaneously, the state variables of the straight pipe are as follows:

$$[S] = [U_x\ F_x\ U_z\ \theta_y\ M_y\ F_z\ U_y\ \theta_z\ M_z\ F_y\ \theta_x\ M_x]^T \tag{11}$$

The overall pipeline transfer matrix [H] is obtained:

$$[H] = \begin{bmatrix} TX & & & \\ & TZ & & \\ & & TY & \\ & & & TOR \end{bmatrix} \tag{12}$$

The structure composed of the two beams is connected by the displacement coordination condition and the force balance condition, thereby obtaining the transfer matrix of the system. By substituting a specific boundary, the natural frequency and mode shape function of the system can be obtained.

2.2 Vibration Analysis of L-shaped Pipeline with Passive Vibration Absorber

Placing the passive vibration absorber in the direction of out-of-plane vibration of L-shaped pipeline. The analysis model is shown as follows:

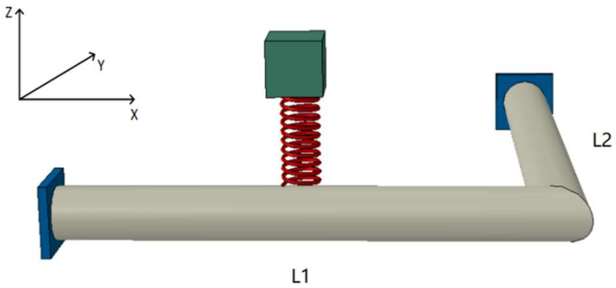

Fig. 1 Out-of-plane vibration analysis model of L-shaped pipe

Assume that the out-of-plane transverse vibration mode function of pipe L_1 is $\overline{\phi}(x)$, the torsional vibration mode function is $\gamma(x)$. The out-of-plane transverse vibration mode function of pipe L_2 is $\overline{\phi}(y)$, the torsional vibration mode function is $\gamma(y)$, the kinetic energy and potential energy of the system are obtained:

$$\begin{aligned} T = &\frac{1}{2}m\dot{\zeta}^2 + \frac{1}{2}\sum_{i=1}^{\infty}\int_0^{L_1}\rho I_\rho \gamma_i^2(x)dx\dot{q}_i^2(t) + \frac{1}{2}\sum_{i=1}^{\infty}\int_0^{L_1}\rho A\overline{\phi}_i^2(x)dx\dot{q}_i^2(t) \\ &+\frac{1}{2}\sum_{i=1}^{\infty}\int_0^{L_2}\rho I_\rho \gamma_i^2(y)dy\dot{q}_i^2(t) + \frac{1}{2}\sum_{i=1}^{\infty}\int_0^{L_2}\rho A\overline{\phi}_i^2(y)dy\dot{q}_i^2(t) \end{aligned} \tag{13}$$

$$\begin{aligned} V = &\frac{1}{2}k(\zeta - \sum_{i=1}^{\infty}\overline{\phi}_i(x_0)q_i(t))^2 + \frac{1}{2}\sum_{i=1}^{\infty}\int_0^{L_1}GI_\rho \gamma_i^{'2}(x)dxq_i^2(t) \\ &+\frac{1}{2}\sum_{i=1}^{\infty}\int_0^{L_1}EI\overline{\phi}_i^{"2}(x)dxq_i^2(t) + \frac{1}{2}\sum_{i=1}^{\infty}\int_0^{L_2}GI_\rho \gamma_i^{'2}(y)dyq_i^2(t) \\ &+\frac{1}{2}\sum_{i=1}^{\infty}\int_0^{L_2}EI\overline{\phi}_i^{"2}(y)dyq_i^2(t) \end{aligned} \tag{14}$$

Substituting Equation(13) and Equation(14) into the Lagrange equation of the second kind, the out-of-plane free vibration equations of the pipeline system are obtained:

$$m\ddot{\zeta} + k(\zeta - \sum_{i=1}^{\infty}\overline{\phi}_i(t)q_i(t)) = 0 \tag{15}$$

$$\begin{aligned} &\int_0^{L_1}\rho I_\rho \gamma_i^2(x)dx\ddot{q}_i(t) + \int_0^{L_1}\rho A\overline{\phi}_i^2(x)dx\ddot{q}_i(t) + \int_0^{L_2}\rho I_\rho \gamma_i^2(y)dy\ddot{q}_i(t) \\ &+\int_0^{L_2}\rho A\overline{\phi}_i^2(y)dy\ddot{q}_i(t) - k\overline{\phi}_i(x_0)(\zeta - \sum_{i=1}^{\infty}\overline{\phi}_i(x_0)q_i(t)) + \int_0^{L_1}GI_\rho \gamma_i^{'2}(x)dxq_i(t) \\ &+\int_0^{L_1}EI\overline{\phi}_i^{"2}(x)dxq_i(t) + \int_0^{L_2}GI_\rho \gamma_i^{'2}(y)dyq_i(t) + \int_0^{L_2}EI\overline{\phi}_i^{"2}(y)dyq_i(t) = 0 \end{aligned} \tag{16}$$

Which can expressed in matrix form:

$$\begin{bmatrix} k-mw^2 & -k\overline{\phi}_1(x_0) & -k\overline{\phi}_2(x_0) & \cdots & -k\overline{\phi}_i(x_0) \\ -k\overline{\phi}_1(x_0) & diagB_1 & k\overline{\phi}_1(x_0)\overline{\phi}_2(x_0) & \cdots & k\overline{\phi}_1(x_0)\overline{\phi}_i(x_0) \\ -k\overline{\phi}_2(x_0) & k\overline{\phi}_1(x_0)\overline{\phi}_2(x_0) & diagB_2 & \cdots & k\overline{\phi}_2(x_0)\overline{\phi}_i(x_0) \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ -k\overline{\phi}_i(x_0) & k\overline{\phi}_1(x_0)\overline{\phi}_i(x_0) & k\overline{\phi}_2(x_0)\overline{\phi}_i(x_0) & \cdots & diagB_i \end{bmatrix} \begin{bmatrix} \zeta \\ q_1(t) \\ q_2(t) \\ \cdots \\ q_i(t) \end{bmatrix} = \begin{bmatrix} BB \end{bmatrix} \begin{bmatrix} \zeta \\ q_1(t) \\ q_2(t) \\ \cdots \\ q_i(t) \end{bmatrix} = 0 \tag{17}$$

$$\begin{aligned} diagB_i = &\int_0^{L_1}GI_\rho \gamma_i^{'2}(x)dx + \int_0^{L_1}EI\overline{\phi}_i^{"2}(x)dx + \int_0^{L_2}GI_\rho \gamma_i^{'2}(y)dy \\ &+\int_0^{L_2}EI\overline{\phi}_i^{"2}(y)dy + k\overline{\phi}_i^2(x_0) - w^2\int_0^{L_1}pI_\rho \gamma_i^2(x)dx - w^2\int_0^{L_1}pA\overline{\phi}_i^2(x)dx \\ &-w^2\int_0^{L_2}pI_\rho \gamma_i^2(y)dy - w^2\int_0^{L_2}pA\overline{\phi}_i^2(y)dy \end{aligned} \tag{18}$$

Solving the determinant of [BB], the natural frequencies of out-of-plane vibration of pipeline system can be obtained.

3. ANALYSIS OF EXAMPLE

The left end of pipe L_1 is the origin of coordinates. The free vibration characteristics and forced vibration characteristics of the model are analyzed. Parameters involved in the illustration are listed in Table 1.

Table 1 Geometrical and material properties of T-shaped pipe

The length of pipe	L_1=10m L_2=8m
Inner diameter	d=0.035m
Outer diameter	D=0.040m
Elastic modulus	E=2.1*10^11 Pa
Mass density	ρ=7850 kg m^{-3}
Poisson's ratio	μ=0.3

3.1 Free Vibration Characteristics of In-plane Vibration

The model is shown in Figure.1. Change the mass, stiffness and setting position of the passive vibration absorber, solve the natural frequencies of pipeline system and compare the theoretical calculation results with finite element calculation results. Take the first three natural frequencies for analysis, as shown in the following table:

Table 2 Calculation of out-plane vibration natural frequency

The situation of passive vibration absorber	Finite Element Method (Hz)	Proposed method (Hz)	Relative error (%) (Based on FEM)
No passive vibration absorber	0.5672 2.0483 3.4278	0.5676 2.0499 3.4308	0.0688 0.0781 0.0875
m=1.5kg k=3·10^8N·m^{-1} Acting position(3,0)	0.5661 2.0005 3.3892	0.5666 2.0029 3.3944	0.0866 0.1220 0.1520
m=2.5kg k=3·10^8N·m^{-1} Acting position(3,0)	0.5654 1.9694 3.3650	0.5660 1.9731 3.3736	0.1044 0.1899 0.2544
m=1.5kg k=3·10^7N·m^{-1} Acting position(3,0)	0.5661 2.0010 3.3892	0.5665 2.0030 3.3943	0.0636 0.1230 0.1505
m=1.5kg k=3·10^8N·m^{-1} Acting position(5,0)	0.5611 1.9437 3.3951	0.5616 1.9297 3.3984	0.0820 0.7208 0.0957

From the calculation results in Table 2, it can be seen that the calculation error of this method is very small. Changing the mass and position of the passive vibration absorber can change the natural frequency of the system. Changing the stiffness of the passive vibration absorber from $3 \cdot 10^{\wedge} 8N\ m^{-1}$ to $3 \cdot 10^{\wedge} 7N\ m^{-1}$ has almost no effect on the natural frequency of the piping system. The out-of-plane vibration characteristics is similar to in-plane vibration and the natural frequency of out-of-plane vibration is much smaller than in-plane vibration.

3.2 Forced Vibration Characteristics of Out-of-plane Vibration

To solve the out-of-plane forced vibration response of the L-shaped pipeline system, an out-of-plane exciting force is applied, the amplitude of the exciting force is 200N and the frequency of the exciting force is 15Hz. The position of the force and response position are changed. According to the parameters of this example, using the modal truncation method, the first ten modes are intercepted and the results are compared with the finite element calculation results. The calculation results are shown in the following table:

Table 3 Calculation of out-of-plane forced vibration response

Parameters and acting position of passive vibration absorber	The acting position of force (m)	The response position (m)	Proposed method (mm)	FEM (mm)	Relative error (%) (Based on FEM)
m=1.5kg k=3·10^8N·m^{-1} Acting position (3,0)	(5,0)	(2,0)	4.2447	4.2494	0.1117
		(5,0)	-7.3390	-7.0510	4.0844
		(10,4)	1.7292	1.7578	1.6234
	(8,0)	(2,0)	-5.3032	-5.1412	3.1510
		(5,0)	5.0190	4.9937	0.5052
		(10,4)	-1.2332	-1.2353	0.1643
	(10,4)	(2,0)	-1.7921	-1.7746	0.9890
		(5,0)	1.7292	1.7578	1.6236
		(10,6)	-5.5270	-5.5868	1.0704

It can be seen from Table 3, the results calculated using the continuous transfer matrix method are very close to those calculated by ANSYS, the relative errors are within 5%. The out-of-plane vibration calculation results prove the accuracy of this method.

When the position of exciting force is (5,0) and the position of passive vibration absorber is (3,0), the out-of-plane vibration response of each point in pipeline system are solved and compare the results with the finite element calculation results, as shown in Figure 2 and Figure 3:

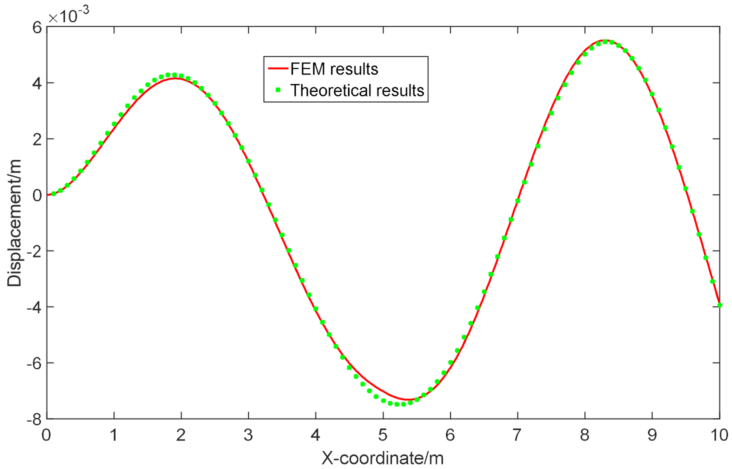

Figure 2 Comparison of vibration response results of out-of-plane vibration of pipeline L_1

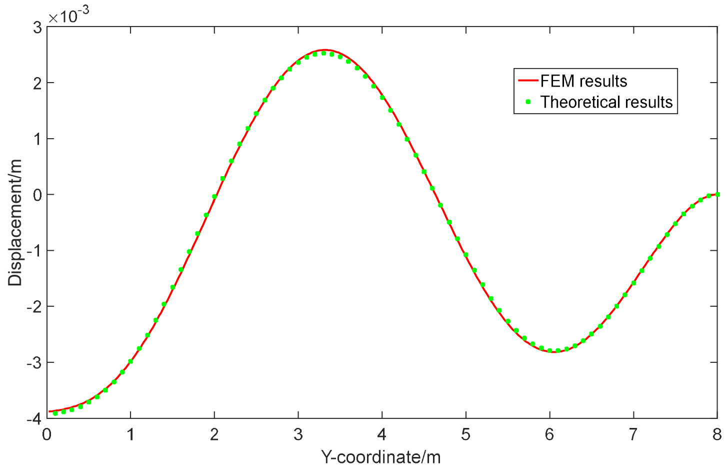

Figure 3 Comparison of vibration response results of out-of-plane vibration of pipeline L_2

It can be seen that the results of the continuous transfer matrix method and the finite element method are very close when calculating each position in the pipeline system, the theoretical method has a high accuracy when calculating out-of-plane vibration.

In out-of-plane vibration, the effects of torsional vibration cannot be ignored. When the torsional vibration is ignored in analysis, the calculation results will have large errors, which is consistent with the conclusion of Ref [5] .

4. CONCLUSIONS

In this paper, taking the L-shaped pipe with passive vibration absorber as an example, the continuous matrix transfer matrix method with Lagrange equation of the second kind is introduced to analyze the free vibration characteristics and forced vibration characteristics of pipeline system. The passive vibration absorber is simplified as a mass-spring unit. The natural frequency and vibration response under harmonic exciting force are obtained. The conclusions are obtained:

①. Using the continuous transfer matrix method with Lagrange equation of the second kind for free vibration analysis of L-shaped pipe with passive vibration absorber, the error between the theoretical method and FEM is very small, which proves the method has high accuracy when calculate the free vibration characteristics of pipeline system.

②. When the forced vibration analysis is performed, the position of the exciting force and response point are changed, multiple sets of calculations are performed. The comparison with FEM results shows that the results are in good agreement. The position of the force and the parameters of the passive vibration absorber remain unchanged, the forced vibration response of the entire pipeline is solved. Compared with the results of FEM, it is proved that the continuum transfer matrix method with the Lagrange equation of the second kind has high accuracy.

③. The continuum transfer matrix method with Lagrange equation of the second kind can calculate arbitrary boundary conditions and arbitrary angles of the pipeline system, it has the advantages of accurate calculation results, convenient programming and fast calculation speed, which can provide theoretical support for the application of passive vibration absorber. The method can provide theoretical reference for the parameter design of the passive vibration absorber and the design of the distribution position of the passive vibration absorber.

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