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The Cremer Impedance for Double-lined Rectangular Ducts

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ABSTRACT

The Cremer impedance refers to the locally reacting boundary condition that maximizes the propagation damping of a certain acoustic mode in a uniform waveguide with infinite length. Previously, the Cremer impedance for rectangular ducts with only one lined wall is obtained by setting the first order derivative of the waveguide eigenvalue equation to zero, which leads to the merging of the fundamental and the first non-plane mode. By symmetry, this solution (referred to as the 'double root' hereafter) can be used on two opposite walls, which is equivalent to a rectangular duct with twice the height. However, as suggested by Zorumski and Mason, it is alternatively possible to create conditions for two different impedances on opposite walls by requiring both the first and second order derivative to be zero. By this means that the second higher order mode will also merge with the two lower modes. In this paper, two such solutions ('triple root') are proposed and compared with the double root. Some improvement in damping is found in both the low and high frequency range for the new triple roots. Alternative ways to create double roots compared to the Cremer symmetric case based on symmetry-breaking are also discussed.

Keywords: double-lined, maximum damping, Cremer impedance, mode merging, triple eigenvalue
I-INCE Classification of Subject Number: 37

1. Introduction

The Cremer impedance, first proposed by Cremer [1] in 1953, is the theoretically optimum locally reacting boundary condition that can lead to the maximum axial damping of a certain acoustic mode in an infinitely long duct. The original solution of the Cremer

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impedance was dedicated for the fundamental mode (the 'plane wave') in a single-lined rectangular duct in the absence of a mean flow. Tester [2, 3] improved the concept in 1973 by giving the solution for an arbitrary mode in the presence of a 'plug' flow, that is, a uniform mean flow that 'slips' on the duct wall. According to Cremer and Tester, two acoustic modes will merge, i.e., become identical under the optimum condition, hence making the optimum transverse wavenumber a branch point of the waveguide eigenvalue equation. Further improvement on the Cremer impedance was conducted by Kabral et al. [4] and Zhang et al., [5, 6], who removed the assumption Tester used when solving the branch point equation [3] and obtained the so-called 'exact' solution that can be applied also in the low frequency range.

The concept of the Cremer impedance is not necessarily confined to single-lined structures. Cremer and Tester implied that their result for a single-lined rectangular duct can be used for a symmetric duct of twice the height provided the source is symmetric [2]. On the other hand, a different configuration can be realized by keeping the optimum condition (the first order derivative of the eigenvalue equation equals zero) but breaking the symmetry, i.e., applying different impedances on the two opposite walls. In both scenarios, it is still two modes that merge. Solutions of this kind are referred to as the 'double root' hereafter.

Alternatively, the so-called 'triple root' solutions can also be obtained by setting not only the first but also the second order derivative of the eigenvalue equation to zero. This was first demonstrated in references [7, 8] for annular ducts. Following this work, two triple roots for double-lined rectangular ducts (without flow) are studied here and compared with the double root given by Cremer and Tester. One of the triple roots has a purely real eigenvalue and gives a damping that vanishes above the eigenfrequency, which is barely observed in the literature. In addition, an active wall, i.e., a negative resistance is required for this root. But unlike all the previous negative resistances [4-6] that are found in the presence of a mean flow, this case is without flow, demonstrating that an optimum impedance with a negative real part is not necessarily a consequence of the mean flow.

2. Double- and Triple Root Solutions for Double-Lined Rectangular Ducts

The considered structure is schematically illustrated in Fig. 1. For such a structure, the eigenvalue equation can be expressed as

$$H = (\bar{\beta}_1 + \bar{\beta}_2) k_y h \cdot kh \cos(k_y h) + \mathrm{i}\left[(k_y h)^2 + \bar{\beta}_1 \bar{\beta}_2 (kh)^2\right] \sin(k_y h) = 0. \quad (1)$$

where $\bar{\beta}_1$ and $\bar{\beta}_2$ are the normalized wall admittance on the two lined walls, k is the wavenumber and h the duct height.

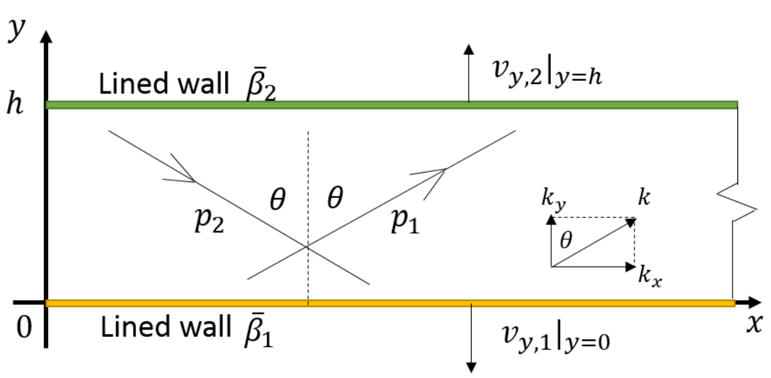

Figure 1. Sketch of an infinitely-long double-lined 2-D rectangular duct.

Following the optimum condition given by Cremer and Tester [1-3], i.e., $\frac{\partial H}{\partial(k_y h)} = 0$, and keeping $\bar{\beta}_1 = \bar{\beta}_2$ (a symmetric configuration), a double root can be obtained, as listed in Table 1 with the denotation 'Double root 1'. Since the configuration is symmetric, it is believed that only the symmetric modes, i.e., the plane wave mode and the second higher order mode will merge.

Alternatively, an almost anti-symmetric configuration can be obtained by setting the two admittances as $\bar{\beta}_1 = a_1 + ib$ and $\bar{\beta}_2 = a_2 - ib$. The solution that satisfies this form is referred to as double root 2. Unlike double root 1, it becomes the plane wave mode and the first higher order mode that merge for this case.

Taking one step forward, two triple roots are obtained by setting the second order derivative of the eigenvalue equation also to zero, i.e., $\frac{\partial^2 H}{\partial(k_y h)^2} = 0$. These two roots are listed in Table 1 with the denotation 'Triple root 1' and 'Triple root 2', respectively. Both of these two solutions can merge three modes, the fundamental, the first and the second higher order mode, although differences in the details can be expected given the configuration or "symmetry" of the two solutions.

Table 1. The double- and triple-root solutions for double-lined rectangular ducts. Note that double root 1 corresponds to the classical Cremer solution [1].

Solution	$k_y h$	$\bar{Z}/\left(\frac{kh}{\pi}\right)$	Configuration	Merging modes	Dominant modes
Double root 1	$4.21 + 2.25i$	$0.464 - 0.372i$	Symmetric	0:th and 2:nd	0:th and 2:nd
Double root 2	$3.17 + 1.71i$	$0.681 - 0.475i$ $0.542 + 0.243i$	"Anti-symmetric"	0:th and 1:st	0:th and 1:st
Triple root 1	$4.20 + 2.61i$	$0.502 - 0.428i$ $0.460 - 0.312i$	"Symmetric"	0:th, 1:st and 2:nd	0:th and 2:nd
Triple root 2	4.60	$0.651 - 0.143i$ $-0.651 - 0.143i$	"Anti-symmetric"	0:th, 1:st and 2:nd	0:th and 1:st

3. Damping

The motivation of applying the Cremer impedance on both instead of only one of the two walls in a 2-D rectangular duct is to achieve more damping. In this chapter, double root 1 given by Cremer and Tester [1-3] is used as the benchmark to evaluate the other three solutions in terms of the axial damping, and the different behaviors of the four solutions are interpreted in light of the specific mode merging scenario shown in Table 1.

The damping within a finite distance (the duct height h in this case) along the axial direction of a duct can be calculated via

$$\mathrm{TL}_{dis} = -20\log_{10}\left[\exp\left(\mathrm{Im}(k_x h)\right)\right]. \tag{2}$$

A comparison of the axial damping using Eq. (2) between double root 1 and the two triple roots is presented in Fig. 2. As shown in the figure, triple root 1 gives a similar damping as that of double root 1, especially in the low frequency range since the symmetric modes are dominant in merging. However, with the first anti-symmetric mode mathematically 'forced' to merge, a slightly larger damping can be found in the mid-to-high frequency range. For triple root 2, on the contrary, the fundamental and the first higher mode dominate and therefore the low frequency damping is larger than that of double root 1. In this sense, triple root 1 and triple root 2 should be applied above and below approximately the first cut-on frequency, respectively to achieve a better damping compared with double root 1. An intriguing point regarding triple root 2 is that it gives no damping at all above the eigenfrequency ($kh = 4.6$), which can be explained as that the acoustic power is dissipated at one wall while amplified at the other, thus cancelling out the damping. For double root 2, although it is the plane wave mode and the first higher order mode that merge, which is supposed to lead to a larger damping in the low frequency range, it actually gives a much poorer damping behavior compared with the other three solutions.

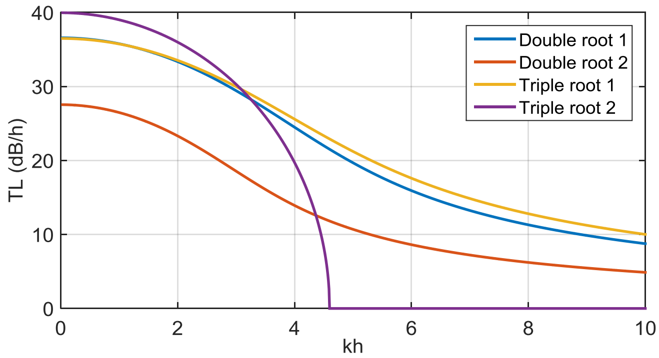

Figure 2. A comparison of the axial damping of the four solutions.

4. CONCLUSIONS

In this paper, the concept of the Cremer impedance is extended to 2-D rectangular ducts with opposite lined walls, and four solutions, two double roots and two triple roots are obtained and compared (Table 1). It is found that the two triple roots can provide a larger damping in different frequency range compared with Cremer's solution, and for one of the triple roots (No. 2) which has a purely real eigenvalue and a negative resistance, the damping behavior is peculiar and similar to a hard wall case (Fig. 2).

To achieve more damping in the low frequency range, an anti-symmetric configuration which can merge the plane wave and the first higher order mode may be a suitable candidate, and an impedance form other than the one used for double root 2 is worth trying in the future work.

5. ACKNOWLEDGEMENTS

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