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## **Study on proper arrangement of damping material with SEA parameter as objective function**

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### **ABSTRACT**

**A structural optimization method of subsystems to realize desired SEA parameters was proposed by the authors in the past studies. This method is based on a combination of SEA and FEM calculation, calculating repeatedly until satisfying the value of objective functions under arbitrary constraints. As a result of applying the proposed method to a simple structure consisting of two flat plates connected in an L shaped configuration, the design variable is taken as the thickness of the FEM element, a subsystem structure with the desired value of the CLF or power flow between subsystems for the one frequency band or multi frequency bands were constructed. However, it is difficult to apply the optimal results to real machine structure because of setting the thickness of the FEM element as the design variable. In this paper, the method is also validated through numerical analyses, using a finite element method, of a flat plate and an L shaped plate, the subsystem is grouped into a plural elements, and the each grouped element is set as a design variable, which should take a discrete value, the total mass is taken as a constraint function in order to minimize the subsystem energy or CLF12 at one frequency band under each condition to realize utilization of this optimization method.**

**Keywords:** SEA, Shape Design, FEM

**I-INCE Classification of Subject Number:** 75, 76

### **1. INTRODUCTION**

Automotive industry requires for improving the fuel consumption is lighting the weight of car. Accordingly, vibro-acoustic analysis to the high frequency is desired by using high stiffness thin plate which is thinner than conventional one. When implementing the structural optimization considered the energy flow or power flow between structural subsystems for attempting to reduce structure-borne sound radiated from machinery, it is difficult to examine how the energy flow changes structural subsystems with conventional structural optimization methods. The conventional method using the peak value of FRF is not easy to set the objective function because of considering the peak of the magnitude in the discrete frequency for the case of existing the plural natural frequencies in the target frequency.

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Conversely, statistical energy analysis (SEA) is a method for vibro-acoustic analysis which regards the system as composed of high modal density and focuses on the power equilibrium between the subsystems [1]. In SEA, the coupling loss factor (CLF) denotes the energy flow between the subsystems, and power flow (PF) denotes the power flow between the subsystems during machine operation.

Therefore, it is considered that setting the CLF or PF to the objective function is easy to realize the structural optimization which considers the energy flow or power flow between the subsystems. In addition, the subsystem is averaged over space and frequency, so it is possible to become the uniformly thickness distribution of subsystem structure and decrease the number of objective function compared with the conventional method.

Accordingly, the authors developed a formulation of a structural optimization method for SEA subsystems for which the realization of the desired value of the loss factors is necessary [2]. This method is based on a combination of SEA and FEM calculation, calculating repeatedly until satisfying the value of objective functions under arbitrary constraints. As a result of applying the proposed method to an L shaped configuration, the design variable is taken as the thickness of the FEM element, a subsystem structure with the desired value of the CLF [2] or power flow [3] between subsystems for the one or multi frequency bands were constructed. However, it is difficult to apply the optimal results to real machine structure because of setting the thickness of the FEM element as the design variable.

So, in this paper, the aim is expanding the formulation of a structural optimization method for SEA subsystems to conduct them in which the design variable is grouped FE element, which should take a discrete value (in the form of either a damping rubber or original thickness) and adopt large-mass method instead of rain-on-the roof excitation. As test structures, a flat plate and an L-shaped plate are considered, where the total mass is taken as a constraint function in order to minimize the subsystem energy or CLF12 at one frequency band under each condition to realize utilization of this optimization method.

## 2. BASIC THEORY OF SEA AND STRUCTURAL OPTIMIZATION METHOD

### 2.1 SEA Power Balance Equation

In SEA, a system is regarded as an assembly of subsystems. If the system has  $r$  subsystems, consideration of the power balance between them leads to a basic set of SEA equations [1]:

$$\mathbf{P} = \mathbf{L}\mathbf{E}. \quad (1)$$

$$\mathbf{L} = \omega \begin{pmatrix} \eta_{1,1} + \sum_{i \neq 1}^r \eta_{1,i} & -\eta_{2,1} & \cdots & -\eta_{r,1} \\ -\eta_{1,2} & \eta_{2,2} + \sum_{i \neq 2}^r \eta_{2,i} & \cdots & -\eta_{r,2} \\ \vdots & \vdots & \ddots & \\ -\eta_{1,r} & -\eta_{2,r} & & \eta_{r,r} + \sum_{i \neq r}^r \eta_{r,i} \end{pmatrix}. \quad (2)$$

Here,  $\omega$  is the center angular frequency of the band,  $\mathbf{E}$  is a vector containing the subsystem energies, and  $\mathbf{P}$  is the external input power vector. The loss factor matrix,  $\mathbf{L}$ , comprises Internal Loss Factors (ILFs),  $\eta_{i,i}$ , and Coupling Loss Factors (CLFs),  $\eta_{i,j}$ . Estimation of the ILFs and CLFs is referred to as the construction of the SEA model.

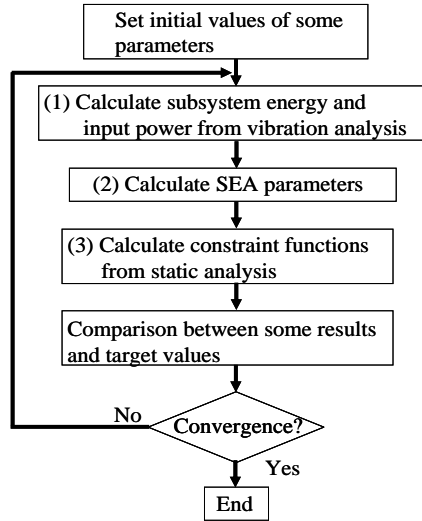


Figure 1: Flowchart of optimization procedure.

## 2.2 Structural Optimization Method

The flowchart of the developed structural optimization method was shown in Fig. 1. First, calculating the subsystem energies and input power of subsystem by applying large-mass method [4] instead of rain-on-the-roof-excitation [5] on the basis of initial value of the design variables for reducing analytical cost. The design variables are the density, Young's modulus, the damping values associated with the material properties, the thickness of the plate elements, the shape, and the coupling between the subsystems related to the structures, and so on. Second, calculating the SEA parameters on the basis of the power injection method [6] of the objective functions using the calculated subsystem energies and input powers. Finally, calculating the constraints functions by performing static analysis. The optimization algorithm defines new value for the design variables, and a new set of SEA parameter and constraints functions calculation are performed until satisfying the value of objective functions.

For the case of calculating the subsystem energies and input powers, rain-on-the-roof excitation was a method for satisfying the SEA assumptions that the excitation force applies to all frequency components uniformly and that all vibration modes were excited at the target frequency. In this study, we define rain-on-the-roof excitation by invoking a point force at each FEM node and then calculating and integrating the response of each plate to these forces over the plates. Large-mass method is a kind of displacement excitation methods, it calculates the vibration response over a frequency range including the magnitude and phase of the vibration response. In the large-mass method, the excitation point is treated as a rigid body and so the response at the excitation point is that for a rigid body. Therefore, it is difficult to identify the input power. Instead, the input power is evaluated [7] from

$$P = \frac{1}{2} m \omega \eta v^2. \quad (3)$$

where  $m$  is the mass of the subsystem,  $\eta$  is the internal loss factor, and  $v^2$  is the mean-square velocity. The analytical cost of base excitation is lower than that for rain-on-the-roof excitation because there is only one excitation per subsystem.

## 2.3 Formulation of the Structural Optimization Problem by SEA

The formulation of the optimization problem by taking into account the subsystem structure is considered together with past structural optimization problems. The structure for which the objective function is maximized (minimized) or satisfies the target value is generated using a numerical method such as FEM. For example, the objective function is

assumed to be CLF at an arbitrary frequency band and is used to formulate the minimization of the objective function. In the case of the minimization of the objective function  $CLF_i(\{x_j\})$  at multiple frequency bands ( $i=1, \dots, n$ ) on the basis of the constraint function  $g(\{x_j\})$  in a feasible design region  $D$ , the following equations can be written by,

$$\text{Minimize } \sum_i (CLF_i(\{x_j\})). \quad (4a)$$

$$\text{Subject to } g(\{x_j\}) - g_{\max} \leq 0. \quad (4b)$$

$$\{x_j\}^L \leq \{x_j\} \leq \{x_j\}^U \quad (j = 1, \dots, n). \quad (4c)$$

Here,  $g_{\max}$  is the upper limit of the constraint function  $g(\{x_j\})$ , and  $\{x_j\}^L$  ( $\{x_j\}^U$ ) is for lower limit (upper limit) on design variables  $\{x_j\}$ .

### 3. APPLICATION TO THE SIMPLE STRUCTURE MODEL

In this section, the validity of the structural optimization method was verified through numerical FEM analyses of a simple two types of model, (i) simple flat plate consisting of one subsystem, and (ii) an L plate consisting of two subsystems.

#### 3.1 Test Structure for One Subsystem and Problem Settings

Here, we applied the structural optimization method to a simple flat steel plate whose lateral dimensions were 0.6 m by 0.3 m and whose thickness was 1.6 mm. All the edges of the plate were free supported. In this work, the FEM software package ANSYS Ver. 16.1 for constructing the model together with ANSYS Parametric Design Language (APDL), the SEA parameters were calculated using MATLAB, and the optimization results were obtained using OPTIMUS 10.18, which is software for automation, integration, and optimization. An elastic shell element (shell 181) was used that consists of 4 nodes. The size of each element in the mesh is  $0.02 \text{ m} \times 0.02 \text{ m}$ , which was sufficient to contain six nodes per bending wavelength up to 1k Hz, the total numbers of nodes and elements are 648 and 450, respectively. The material properties of the plate were as follows: Young's modulus,  $E = 210 \text{ G Pa}$ , and Poisson's ratio,  $\nu = 0.3$ . The objective function was subsystem energy which was calculated and integrated over the plate at the range of 25-1k Hz in 5 Hz steps, after that, the one-third octave frequency band characteristics were calculated in the range of 50-800 Hz. For the large-mass method, a mass equal to the plate's mass multiplied by  $10^6$  was set through a rigid body with a length of 0.3 m below the node position (0.04 m, 0.04 m) of the plate from Fig. 4, and the edge of the rigid body was excited by a vibration acceleration of  $9.8 \text{ m/s}^2$ .

The subsystem was grouped into 18 elements, as shown in Fig. 4, and the each grouped element was set as a design variable, which should take a discrete value (in the form of either a plate with damping rubber (2mm) or original plate) except for the excitation area. Thus, there were 17 design variables. The material density of the modified plate with damping rubber was  $\rho = 9651 \text{ kg/m}^3$ , and the loss factors were assumed to be 0.05 from the experimental test. The material density of the original one was  $\rho = 7542 \text{ kg/m}^3$ , and the loss factors were assumed to be 0.001. The mass of the original plate was 2.17 kg. The total mass was taken as a constraint function. The upper limit for the design variable was 2.31 kg, that was the mass addition by rubber sticking was permitted to four area. The target objective function was the minimization of subsystem energy at 125 Hz in 1/3 octave band from Fig. 3.

After the setup of the above mentioned, the optimization algorithms were set in the OPTIMUS software. The Self-Adaptive Evolution (SAE) method, which is a kind of

global optimization method applicable to discrete problems, was chosen here. The population size was chosen to be 5 times the number of design variables. That is, 85 values of the population size were generated in each iteration. Since the time required for obtaining the optimization results was rather long as compared to the local optimization method, the number of iterations was set to nine times in Section 3.1 and twenty times in Section 3.2.

### 3.1.1 Analysis Results

The iteration history of the objective function subsystem energy is shown in Fig. 2. Here, only the minimum value is indicated in each iteration. Figure 3 shows the comparative values of the subsystem energy between the initial value and the optimization results at the 6th iteration, which reaches the minimum value from 9th iterations. From Fig. 2, the value of the subsystem energy at first iteration decreased by about 97.7% as compared with the initial value. From Fig. 3, the value of the subsystem energy at target frequency band decreased by about 98.7% and become  $3.03 \times 10^{11}$  as compared with the initial value of  $2.30 \times 10^{13}$ . The mass is 2.31 kg. The optimization results in this case indicate that the all the values of subsystem energy

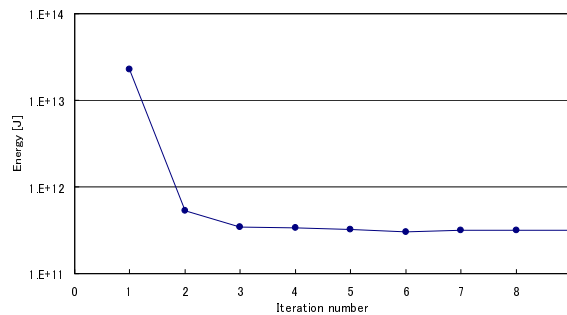


Figure 2: Iteration history for the objective function subsystem energy in the 125 Hz band.

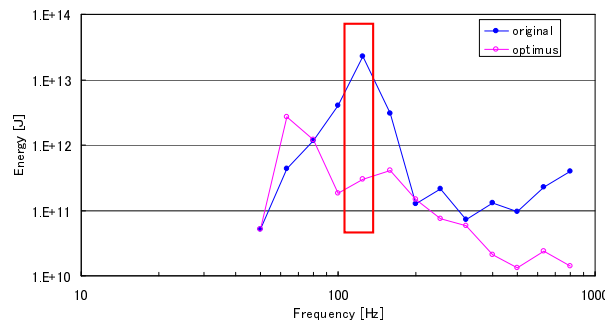


Figure 3: Comparison between the initial values and optimum values of the subsystem energy by large-mass method.

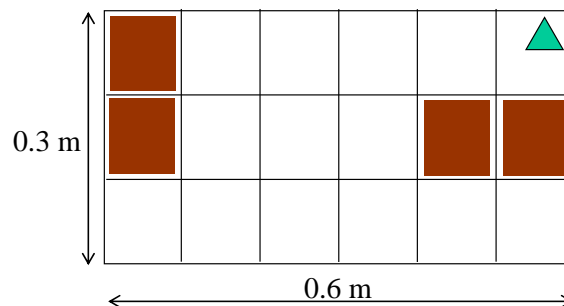


Figure 4: Test-plate structure 1: points marked “▲” is rigid-body points for large-mass method and excitation location, and “■” are structure modified location.

over 100 Hz band except for 200 Hz band are smaller compared with the initial value.

### 3.1.2 Comparison with Point Excitation Results and Discussions

In this section, the proposed method is validated through the comparison of the results obtained by the point excitation instead of the large-mass method. The difference of these two types of method is presence or absent of rigid body that attached to the system. In case of the point excitation, there is no rigid body in the target structure, and the designated node is excited by unit force.

Figure 5 shows the comparative values of the subsystem energy between the initial value and the optimization results by point excitation. From Fig. 5, it is different from the trend in Fig. 3, the value of the subsystem energy at target frequency band decreased by about 98.6% and become  $4.11 \times 10^{-4}$  as compared with the initial value of  $3.28 \times 10^{-2}$ .

Table 1 shows a comparison between the initial and optimum values of the first tenth natural frequencies except for the rigid mode. The sixth, the seventh and the eighth natural frequencies influence on the target frequency 125 Hz band (from 112 Hz to 141 Hz) in the initial and optimum conditions in Table 1. Replacement was observed at the initial and optimum conditions in the 6<sup>th</sup> and the 7<sup>th</sup> mode shapes.

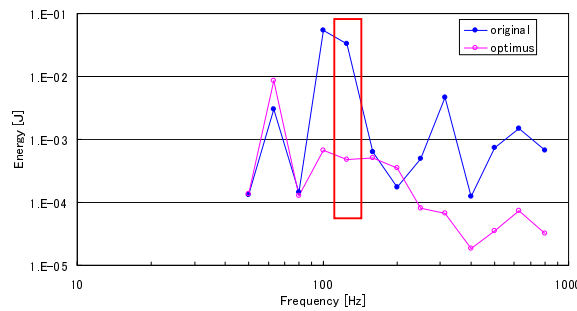


Figure 5: Comparison between the initial values and optimum values of the subsystem energy by point excitation.

Table 1 Comparison between the initial and optimum values of the natural frequencies except for the rigid mode. Unit : Hz

Order	Initial	Optimal
1	24.2	23.2
2	29.8	29.1
3	65.5	64.1
4	66.9	65.1
5	98.1	95.2
6	113.4	110.0
7	115.7	112.1
8	133.1	128.9
9	159.9	155.2
10	178.3	175.7

### 3.2 Test Structure for Two Subsystems and Problem Settings

As shown in Fig.6, the target structure consisted of two rectangular steel plates coupled in an L-shaped configuration. The lengths of plates 1 and 2 were  $L_1 = 0.5$  m and  $L_2 = 0.3$  m, respectively. Both plates had a width of  $L_3 = 0.6$  m and a thickness of 1.6 mm. All the plate edges were free supported. The locations of the excitation points are depicted as black squares in Fig. 6. The element type and size were the same as those stated in section 3.1 for one subsystem. The total numbers of nodes and elements were 1578 and 1202, respectively. For the large-mass method, the analytical conditions were the same as those for one subsystem. Excluding the junction, at each frequency, the displacement response of each plate is calculated and integrated over the plates.

The subsystem was grouped into 48 elements, as shown in Fig. 9, and the each grouped element was set as a design variable except for the excitation area. Thus, there were 46 design variables. The mass of subsystem 1 and 2 were 3.60 kg and 2.17 kg. These masses were taken as a constraint function. The upper limit for the design variable were 3.86 kg and 2.31 kg, that were the mass addition by rubber sticking was permitted to seven and four area. The target objective function was the minimization of CLF12 at 500 Hz in 1/3 octave band from Fig. 8. Various optimum conditions were the same as in Section 3.1, and as a result, 230 values of the

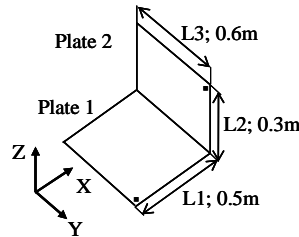


Figure 6: Test-plate structure 2: points marked “■” are rigid-body points for large-mass method.

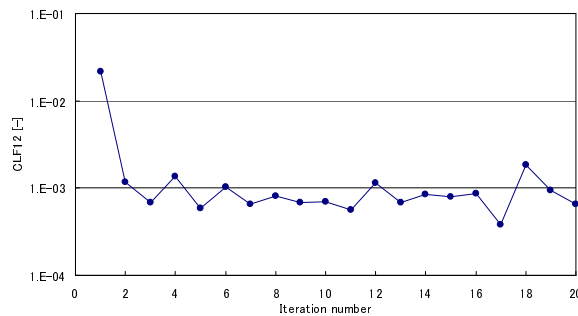


Figure 7: Iteration history for the objective function CLF12 in the 500 Hz band.

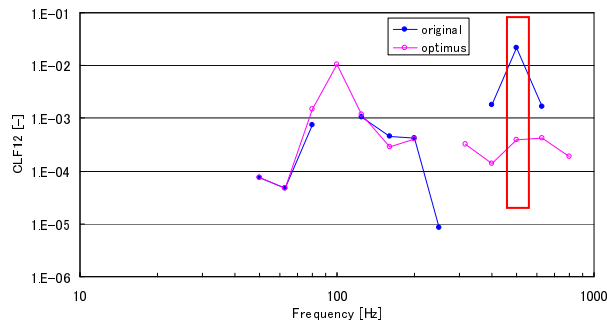


Figure 8: Comparison between the initial values and optimum values of the CLF12.

population size were generated in each iteration. The equation for two subsystems is expressed as

$$\begin{pmatrix} \eta_{1,1} \\ \eta_{1,2} \\ \eta_{2,1} \\ \eta_{2,2} \end{pmatrix} = \frac{1}{\omega} \begin{pmatrix} E_1^1 & E_1^1 & -E_2^1 & 0 \\ 0 & -E_1^1 & E_2^1 & E_2^1 \\ E_1^2 & E_1^2 & -E_2^2 & 0 \\ 0 & -E_1^2 & E_2^2 & E_2^2 \end{pmatrix}^{-1} \begin{pmatrix} P_1 \\ 0 \\ 0 \\ P_2 \end{pmatrix}. \quad (5)$$

where  $E_i^j$  is the energy of subsystem  $i$  when subsystem  $j$  is excited by input power  $P_j$ . The input power  $P_j$  is calculated by Eq. (6).

$$P_j = \frac{\text{Re}(F_j v_j^*)}{2}. \quad (6)$$

where  $\text{Re}()$  is the imaginary part,  $F_j$  is impulsive power spectrum, and  $v_j$  is a velocity response spectrum in the vicinity of the excitation location from Fig. 9.

### 3.2.1 Analysis Results

The iteration history of the objective function CLF12 is shown in Fig. 7. Here, only the minimum value is indicated in each iteration. Figure 8 shows the comparative values of the CLF12 between the initial value and the optimization results at the 17th iteration, which reaches the minimum value from 20th iterations. When the value of the CLF12 at 100 Hz, 250 Hz, 315 Hz and 800 Hz are negative, points are not plotted in Fig. 8. From Fig. 7, the value of the CLF at first iteration decreased by about 94.6% as compared with the initial value. From Fig. 8, the value of the CLF12 at target frequency band decreased by about 98.2% and become  $3.84 \times 10^{-4}$  as compared with the initial value of  $2.16 \times 10^{-2}$ . The mass of subsystem 1

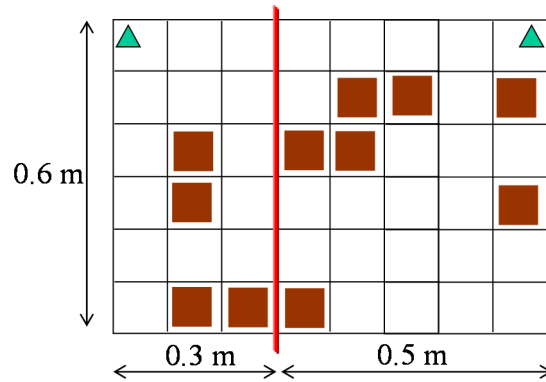


Figure 9: Test-plate structure 2 in development view: points marked “▲” is rigid-body points for large-mass method and excitation location, and “■” are structure modified location.

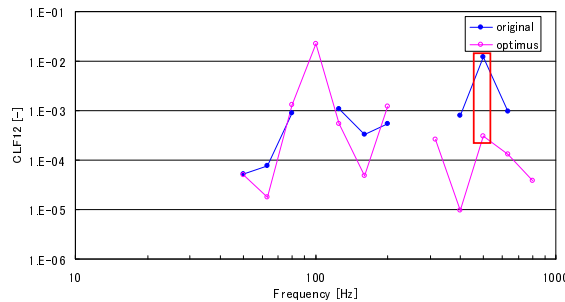


Figure 10: Comparison between the initial values and optimum values of the subsystem energy by point excitation.



and 2 were upper limit from Figure 9.

### 3.2.2 Comparison with Point Excitation Results and Discussions

Figure 10 shows the comparative values of the CLF12 between the initial value and the optimization results by point excitation. From Fig. 10, the value of the CLF12 at target frequency band decreased by about 97.4% and become  $3.07 \times 10^{-4}$  as compared with the initial value of  $1.19 \times 10^{-2}$ .

## 4. CONCLUSIONS

In this study, a structural optimization method on the basis of the large-mass method for SEA subsystem was applying to realize the desired value of subsystem energy and CLF for the one one-third octave bands frequency. As a result of applying the developed method to simple structures, a flat plate and an L-shaped plate, a subsystem structure with the desired value of subsystem energy or CLF12 for the target frequency band was constructed. The effectiveness of the proposed method has been verified for the structure with the desired values of the subsystem energy and CLF under arbitrary constraints obtained by applying a combination of large-mass method and the optimization procedure.

## 5. ACKNOWLEDGEMENTS

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