

Vibroacoustics of panels subjected to turbulent boundary layer excitation with pressure gradients

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ABSTRACT

Current research advancement in aircraft engine technology with significant noise reduction at the source (i.e. engine) has created renewed interest in turbulent boundary layer (TBL) generated cabin noise in aircraft. The recent studies using computational fluid dynamics (CFD) and wind tunnel tests are focused on getting the reliable spectra of TBL excitation in areas of aircraft with favorable pressure gradient (FPG) and adverse pressure gradient (APG), respectively. The aim of this paper is to increase the understanding of TBL excitation with adverse pressure gradients and quantify their impact on sound radiation from aircraft stiffened panels subjected to TBL excitation with such pressure gradients. Based on literature review, it is found that most of the developed models for TBL auto-spectra under pressure gradients fit quite well to the wind tunnel tests which are used to develop these models. However, recently proposed Rozenberg model seems to perform well when compared to different set of data obtained either by wind tunnel tests or obtained through numerical simulations using CFD. Therefore, this Rozenberg model is used to perform vibroacoustic studies addressed in this paper. For a reference case, Goody's TBL model for zero pressure gradients is also discussed.

Keywords: TBL, Vibroacoustics, APG, FPG and ZPG, CFD

1. INTRODUCTION

Flow induced noise and vibrations are of immense importance due to its widespread application in variety of industrial problems. Recent advancement in engine technology has paved the way for significantly quieter engine. Due to this low acoustic signature of current state of the art high bypass ratio engines, the cabin noise due to turbulent boundary layer pressure fluctuations has gained a renewed interest and lately lot of research efforts [1-7] have been put to further improve the existing empirical model of wall pressure fluctuations. On the outer surface of the aircraft fuselage, the turbulent boundary layer pressure fluctuations excite the aircraft structure which subsequently radiates sound in the cabin. In the literature, the noise generated by the turbulent boundary layer (TBL) is addressed in depth by Graham [8,9]. Graham [8] concluded that an increase in structural damping and a decrease in skin stiffness reduces the radiated sound in aircraft cabins.

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To consider vibroacoustics in the design stage, the pressure fluctuation on the outer surface of the aircraft needs to be understood. In this regard, Graham [10] compared various TBL models developed in the past and reported the results for sound radiation from a simply supported rectangular panel excited by flat-wall TBL pressure fluctuations. In a similar work, Allen and Vlahopoulos [11] developed a numerical algorithm to compute the uncoupled structural-acoustic system response when the structure is subjected to random excitation.

The algorithm developed by Allen and Vlahopoulos [11] combines the finite element method and the boundary element method (BEM) and integrates them with stochastic analysis. Chase [12], Corcos [13,14], and Howe [15] characterized the fluctuating wall pressure distribution and gave semi-empirical equations to define the random fluctuating pressure load. The research conducted by Han et al. [16] provides the auto-spectrum of the random pressure excitation generated by TBL. This auto-spectrum, together with the developed Corcos type TBL model to represent the cross-spectrum in the literature can be used to define TBL spectra. For understanding the impact of various TBL models on cabin noise, it is important and advisable to quantify the impact of various TBL models on sound radiation from relatively simple but representative structures such as ortho-grid stiffened panel with similar dimensions as that of the side wall of a fuselage without trim panels and insulation.

In most of vibroacoustic studies addressing sound radiation due to TBL it is assumed that the developed TBL is stationary, homogeneous with zero pressure gradient. One of the most widely used model for aircraft cabin noise prediction is developed by Goody [1]. Goody's model works well for aircraft areas with zero pressure gradient. Areas such as cockpit and the region of fuselage with belly fairings develops adverse pressure gradient and can significantly change the auto-spectra of applied TBL pressure fluctuation. This change in auto-spectra of applied load is now well known and there have been recent numerical studies (Lee [17], Klages et al. [18]) as well as wind tunnel tests (Hu [19]) to improve the auto-spectra which includes the flow physics of TBL pressure fluctuations with adverse pressure gradient. The Rozenberg model [3] for auto-spectra of TBL pressure fluctuations includes the impact of APG and used in this work. To assess the impact of APG on radiated sound from a stiffened panel, Goody's ZPG model [1] is used as a reference.

Following introduction, section-II addresses two TBL models, i.e. Goody and Rozenberg. Section-III addresses the numerical approach used in this paper to compute vibroacoustic response of ortho-grid stiffened panel subjected to stochastic loads such as TBL pressure fluctuations. Results of the numerical study performed in this paper are addressed in section-IV. Conclusions and the future work of the study are discussed at the end of this paper in section-V.

2. TBL EMPIRICAL MODELS

TBL empirical model development is typically based on either wind tunnel tests or flight tests where the concept of self-similarity is used to compute various parameters in a function which has been fitted to measured data. It is well known that an experimental wall pressure spectrum exhibits three slopes (Goody [1]). A positive slope at low frequencies and a slight negative slope in overlap region with a sharp negative slope in higher frequency. Chase and later Howe presented TBL auto-spectra which captures slopes of low frequency and overlap region. However, Chase-Howe model showed discrepancies at higher frequencies. Goody [1] enhanced the mathematical form of Chase-Howe spectra by adding an additional

term in the denominator of the model for pressure spectra which mitigates the discrepancy in high frequencies. Goody's [1] functional form of the pressure measurements in a flow with zero pressure gradient was further extended by Rozenberg [3] to capture adverse pressure gradient flows. Figure 1 represents TBL flow over fuselage and it can be observed that the TBL flow will experience pressure gradients due to cockpit geometry. Therefore, it is important to study pressure gradient in TBL flows. Following subsection discusses Goody and Rozenberg model used in this study.

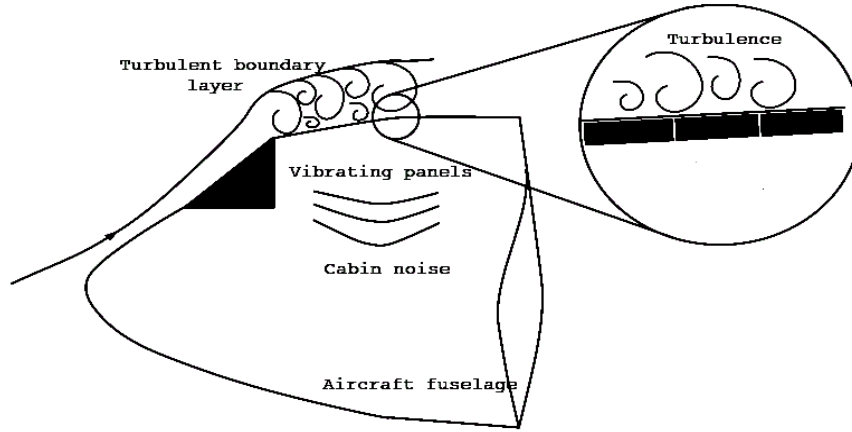


Figure 1 Representation of turbulent boundary layer over fuselage [20]

2.1 Goody Model

Goody enhanced the Chase-Howe model and reported better agreement with measurement data. In particular, Goody's model captured Reynolds number trends as seen in the measurement data with sharp decay at higher frequencies. The mathematical form of the Goody model is given as:

$$\frac{\varphi(\omega)U_e}{\tau_w^2\delta} = \frac{C_2(\omega\delta/U_e)^2}{[(\omega\delta/U_e)^{0.75} + C_1]^{3.7} + [C_3(\omega\delta/U_e)]^7} \quad (1)$$

Where, $\varphi(\omega)$ is one sided auto-spectrum of surface pressure fluctuations, U_e is the velocity at the boundary layer edge, τ_w is shear stress at the wall, δ is boundary layer thickness and ω is angular frequency. In Eq. 1, C_1 , C_2 and C_3 are three parameters which vary with Reynolds number and given as follows:

$$C_1 = 0.5, \quad C_2 = 3.0, \quad C_3 = 1.1 * R_T^{-0.57} \quad (2)$$

The time scale ratio, R_T is defined as:

$$R_T = (u_\tau\delta/\nu)\sqrt{C_f/2} \quad (3)$$

In Eq. (3), $u_\tau = \sqrt{\tau_w/\rho}$ is friction velocity, $\nu = \mu/\rho$ is kinematic viscosity and ρ is mass density of fluid. It is important to note that the auto-spectrum, $\varphi(\omega)$ in Eq. 1 has been normalized using τ_w as the pressure scale and δ/U_e as the time scale. Similarly, angular frequency, ω is also normalized using the time scale, δ/U_e . Hwang et al. [21] reviewed empirical spectral models and concluded that Goody's model shows the best agreement with the measurement data for zero pressure gradient turbulent boundary layer flows. Schloemer [22] reported through measurements that the mean pressure gradient has significant impact on wall pressure fluctuations. Therefore, Rozenberg further improved Goody's model to incorporate the impact of adverse pressure gradient on wall pressure fluctuations. The following subsection briefly discusses the Rozenberg model.

2.2 Rozenberg Model

There are many practical applications where TBL experiences adverse pressure gradients. For example, flow over the suction side of an airfoil, flow over cockpit and mid-fuselage region near wing and belly fairings experiences adverse pressure gradients. Rozenberg selected six reference spectra from numerical and experimental studies in the literature to characterize parameters which will add APG effect into Goody's model. The following equation shows the updated Goody model by Rozenberg and incorporates the APG effect in TBL flow:

$$\frac{\varphi_{pp}(\omega)U_e}{\tau_{max}^2\delta^*} = \frac{[2.82\Delta^2(6.13\Delta^{-0.75}+F_1)^{A1}][4.2(\frac{\pi}{\Delta})+1]\tilde{\omega}^2}{[4.76\tilde{\omega}^{0.75}+F_1]^{A1}+[C'_3\tilde{\omega}]^{A2}} \quad (4)$$

Detailed description of all the terms in Equation 4 can be found in Rozenberg et al. [3]. However, three parameters in Rozenberg model which incorporate the impact of APG on surface wall pressure spectra are discussed here. First parameter, $\beta_c = (\theta/\tau_w)(dp/dx)$ is Clauser's [23] pressure gradient parameter. The second parameter is wake strength parameter, Π as defined in Cole [24]. The third parameter is defined as $\Delta = \delta/\delta^*$ with δ^* being the boundary layer displacement thickness. Rozenberg pointed out that both Π and Δ are influenced by the boundary layer history whereas β_c is a local parameter. Durbin and Reif [25] used an empirical formula, $\Pi = 0.8(\beta_c + 0.5)^{3/4}$ which shows the correlation between Π and β_c .

Fig. 2 compares Goody and Rozenberg model with measurement performed at Ecole Centrale de Lyon (ECL). The developed TBL flow in this measurement result experiences APG and thus shows a very poor match with Goody model which is applicable for TBL flows with ZPG. However, Rozenberg model incorporates the impact of APG and shows a good match with the measurement. A more comprehensive comparison of Goody and Rozenberg model with various measurement can be found in Rozenberg et al. [3] and it can be concluded that for a reliable Aircraft cabin noise prediction at design stage, it is important to understand and perform vibroacoustics studies using TBL models which capture APG in TBL flows. Therefore, the following section addresses a numerical vibroacoustic studies on representative aircraft stiffened panel with such TBL models.

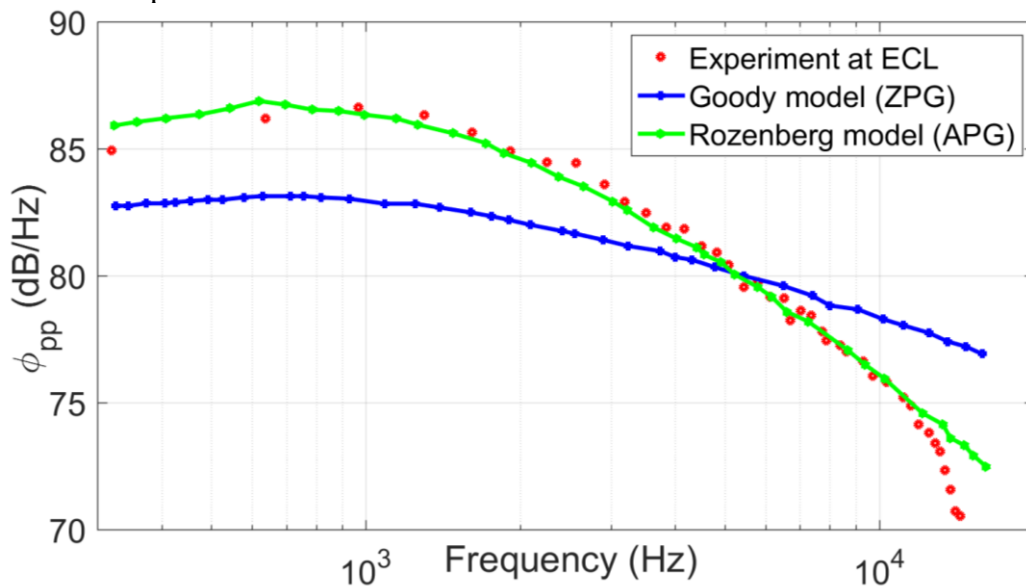


Figure 2 Comparison of wall pressure spectra to measurement (Rozenberg et al.[3])

3. VIBROACOUSTIC RESPONSE UNDER TBL LOADS

The equation of motion for a multi-dimensional vibrating structure can be represented by Eq. 5 in Fahy [26]. Where $\{u\}$ is the nodal displacement vector, $[M]$ the mass matrix, $[B]$ the damping matrix, $[K]$ the stiffness matrix, and $\{F\}$ is the nodal forcing vector.

$$[M]\{\ddot{u}\} + [B]\{\dot{u}\} + [K]\{u\} = \{F\} \quad (5)$$

If we use harmonic input, Eq. 5 can be written in the form of Eq. 6.

$$-\omega^2[M]\{u\} + i\omega[B]\{u\} + [K]\{u\} = \{F\} \quad (6)$$

Normal velocity components can be obtained using transformation matrix $[T]$ and can be written as:

$$\{v_n\} = [T]\{u\} = [T][S]^{-1}\{F\} \quad (7)$$

For the problem at hand we have used Altair HyperWorks for velocity calculation. Once we have the normal velocity on the surface of the structure, we can calculate transfer function $[H]$ for radiated acoustic pressure from stiffened panel excited by turbulent boundary layer pressure. Rayleigh integral (Fahy [26]) approach is used to compute radiated acoustic pressure from stiffened panel velocities. The output power spectral density for acoustic pressure at far-field locations (i.e. hemisphere points) can be calculated using this transfer function and Corcos type cross-correlation model (Corcos [13]). Acoustic pressure at the k^{th} data recovery point can be calculated as:

$$AP_k = H_{ki} \cdot F_i \quad (8)$$

This equation can be utilized to relate the power spectral density of the response S_{AP} to the power spectral density of the load S_F and can be written as:

$$S_{AP_k} = |H_{ki}|^2 S_{F_i}(\omega) \quad (9)$$

Transfer function H_{ki} is calculated using frequency response analysis. When multiple sources are applied on the system, the cross spectral density $S_{F_i F_l}$ can be used to define degree of correlation between them. This information can be used to calculate autocorrelation of the response which is radiated acoustic pressure on a hemisphere in far field in the present work. Eq. 10 shows this autocorrelation.

$$S_{AP_k}(\omega) = \sum_i \sum_l H_{ki} \cdot H_{kl}^* S_{F_i F_l}(\omega) \quad (10)$$

$S_{F_i F_l}$ is the power spectral density of the excitation and it can be expressed as follows:

$$[S_{F_i F_l}] = \begin{bmatrix} S_{F_i F_i} \dots S_{F_i F_i} \\ \cdot \\ \cdot \\ \cdot \\ S_{F_i F_l} \dots S_{F_i F_l} \end{bmatrix} \quad (11)$$

The crosses spectral terms are generated using Corcos model where cross spectral terms are given as:

$$S_{F_i F_l}(\xi_1, \xi_2, \omega) = S_{F_i F_l}(\omega) e^{-\gamma_1 \left| \frac{\omega \xi_1}{U_c} \right| - \gamma_2 \left| \frac{\omega \xi_2}{U_c} \right| - j \frac{\omega \xi_1}{U_c}} \quad (12)$$

U_c is turbulence convection speed, $U_c = 0.70U_0$, where U_0 is mean flow velocity. ξ_1 and ξ_2 are separation distances in the stream wise and span wise direction as shown in Fig. 3. γ_1 and γ_2 are decay rates. Auto power spectral density $S_{F_i F_i}$ of turbulent boundary layer generated fluctuating pressure can be given by two empirical models discussed in previous section. The TBL spectra, $S_{F_i F_i}$ is equal to $\varphi(\omega)$ as defined in Eq. 1 for Goody model and in Eq. 4 for Rozenberg model.

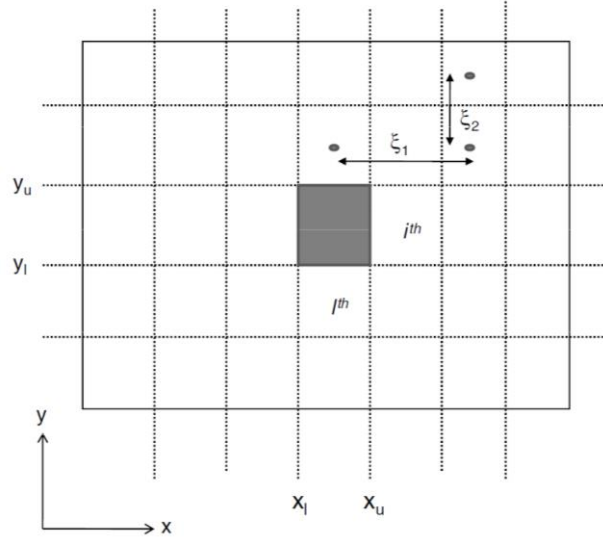


Figure 3 Subpanels to capture pressure correlation on stiffened panel Surface subjected to TBL excitation (Allen et al. [27])

Fig. 4 shows the geometry of a representative ortho-grid stiffened panel of a typical sidewall of an aircraft. Table 1 shows all geometric, material and finite element model parameters. Location of frame in stiffened panel is at $x = 500$ mm and the first seven bays between stringers along x-direction are of equal width of 120.0 mm. Fig. 5 shows the finite element model of the ortho-grid stiffened panel. A simply supported boundary condition on all four edges of the stiffened panel is used while performing dynamic analysis using OptiStruct solver in HyperWorks. Fig. 5 shows the finite element model of ortho-grid stiffened panel. While developing the finite element model, it was the aim of the author that the developed model is a good candidate to represent the fuselage sidewall dynamics without complicating the developed finite element model with minute details.

Table 1 Geometric and finite element model parameters for stiffened panel

Stiffened plate length	1000.0 mm
Stiffened plate width	1000.0 mm
Stiffened panel thickness	2.5 mm
Frame height	120.0 mm
Frame thickness	2.5 mm
Stringer height	48.0 mm
Stringer thickness	2.5 mm
Stringer pitch (along y-axis)	120 mm
Young's Modulus of the material	73.0 GPa
Density	2700.0 kg/m ³
Poisson's ratio	0.33
Damping loss factor	2%
Element size	10.0 mm

Table 2 General turbulent boundary layer flow parameters for an Aircraft in cruise

Flight altitude	30000 <i>ft</i>
Length scale	10000 <i>mm</i>
Mach number	0.75
Free stream velocity	225.0 <i>m/s</i>
Convection velocity	0.7*U
Turbulent boundary layer thickness	0.1 <i>m</i>
Streamwise decay rate of coherence	0.10
Spanwise decay rate of coherence	0.77
External air density	0.44 <i>kg/m³</i>
External speed of sound	300 <i>m/s</i>
Internal air density	1.2 <i>kg/m³</i>
Internal speed of sound	340 <i>m/s</i>

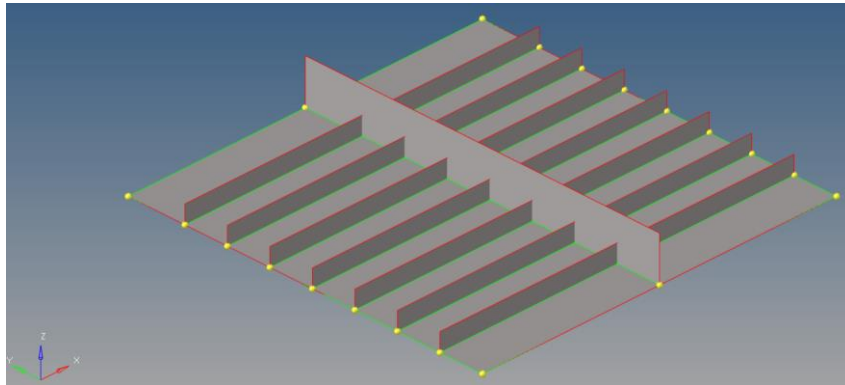


Figure 4 Ortho-grid stiffened panel

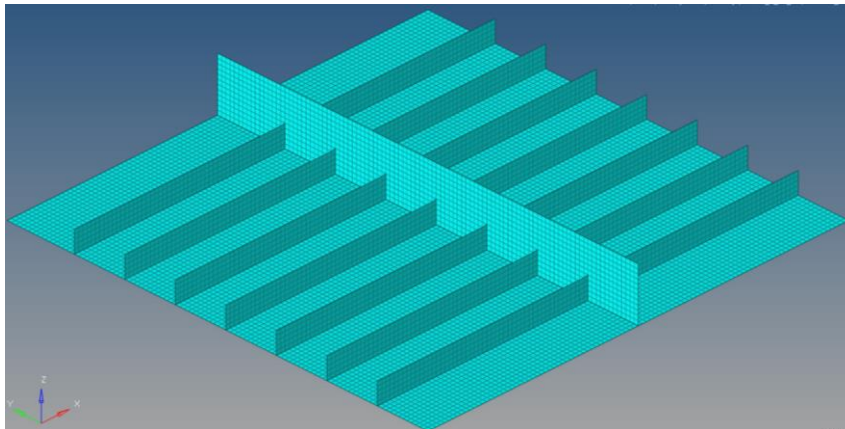


Figure 5 Finite element model of stiffened panel

4. RESULTS

This section discusses vibroacoustic results of the implemented transfer function-based approach as discussed in previous section. The stiffened panel is assumed to be in an infinite baffle with stiffened side of the panel radiating sound in far-field. Far-field radiation is computed over a hemisphere of 5.0 m radius. Fig. 6 shows the radiated sound pressure (rms) on a hemisphere due to vibration of stiffened panel subjected to turbulent boundary layer pressure fluctuation with zero pressure gradient. The maximum RMS pressure on a hemisphere

is 81.8 dB (see Fig. 6). Similarly, the radiated sound pressure on a hemisphere for the case where TBL auto-spectra is represented using Rozenberg model is shown in Fig. 7. For this case, the maximum RMS value of radiated sound pressure on a hemisphere is 92.7 dB. Therefore, from Fig. 6 and Fig. 7, it can be concluded that the adverse pressure gradients in TBL flow play significant role in modifying the sound radiation characteristics of sidewall of the fuselage. Which in turn, will change the cabin noise levels. For a representative stiffened panel (as shown in Fig. 5), the maximum radiated sound pressure for APG in TBL is 10.9 dB higher than the case of ZPG in TBL flow.

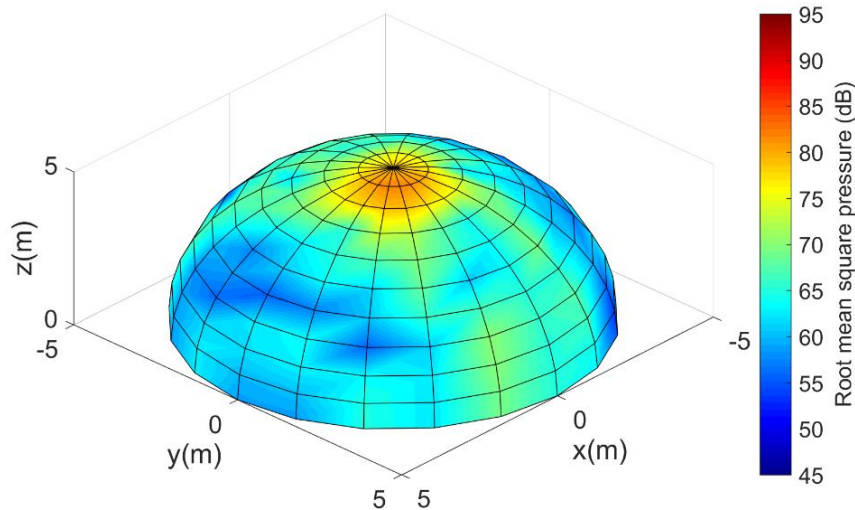


Figure 6 Pressure (rms) on a hemi-sphere (radius = 5 m) covering stiffened panel using Goody spectra: Ref. pr. = $20 \mu\text{Pa}$; Maximum pressure (rms)=81.8 dB

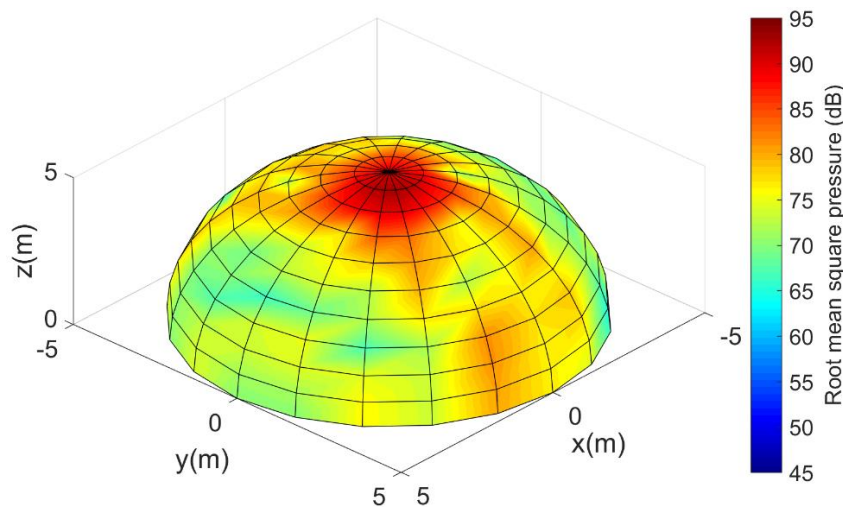


Figure 7 Pressure (rms) on a hemi-sphere (radius = 5 m) covering stiffened panel using Rozenberg spectra: Ref. pr. = $20 \mu\text{Pa}$; Maximum pressure (rms)=92.7 dB

Furthermore, if we compare the auto-spectra of radiated sound pressure on a hemisphere point ($x = 0.908 \text{ m}$, $y = 1.25 \text{ m}$, $z = 4.75 \text{ m}$) we can observe in Fig. 8 that the sound pressure spectra for APG case is more than 8.0 dB higher in 50 Hz to 1000 Hz frequency band when compared to the auto-spectra of radiated sound pressure at the same point with ZPG in TBL flow.

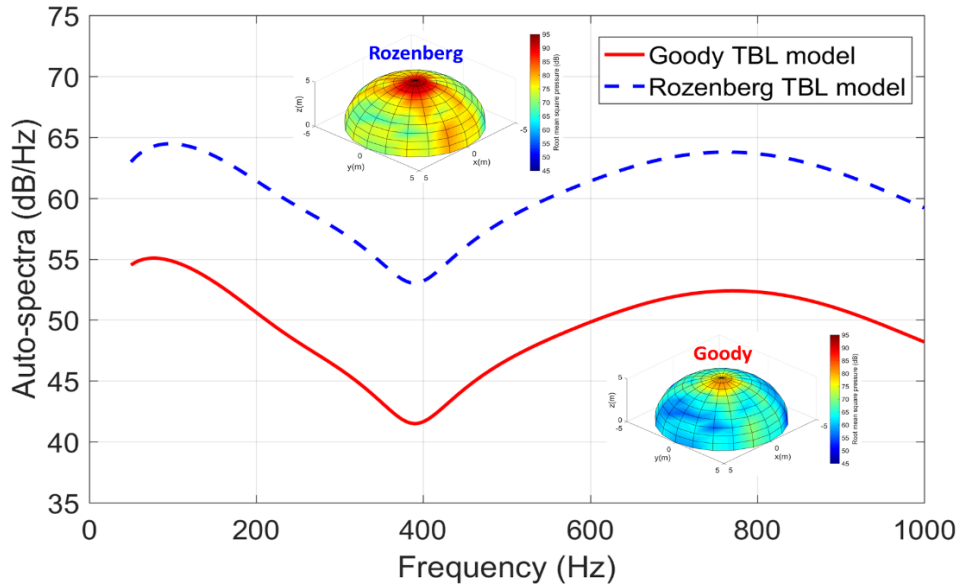


Figure 8 Auto-spectra at a point ($x = 0.908$ m, $y = 1.25$ m, $z = 4.75$) on hemisphere

5. CONCLUSIONS

Vibroacoustic numerical studies using finite element method are performed to quantify the impact of pressure gradient in applied turbulent boundary layer pressure fluctuations with and without pressure gradient. The spectrum of external Turbulent boundary layer pressure fluctuations with zero pressure gradient is represented using Goody model. For turbulent boundary layer pressure fluctuations with adverse pressure gradient, Rozenberg model of external pressure spectrum is implemented to perform vibroacoustic studies. Through the implementation of Goody and Rozenberg Model, it is shown that the impact of adverse pressure gradient on radiated sound from ortho-grid stiffened panels cannot be neglected and it is paramount to use the appropriate model for intended application. For aircraft application, TBL models which do not capture adverse pressure gradient in TBL flow can easily underestimate the cabin pressure by more than 5 dB while performing vibroacoustic studies at the design stage.

There are still many challenges to be addressed to have highly reliable sounds radiation estimate in low to mid frequencies under complex flows such as turbulent boundary layer pressure fluctuation with adverse pressure gradient. However, as far as future extension of this work is concerned, there are three areas which author thinks as a natural extension of the work in this manuscript.

- First one is to implement the current finite element approach to full scale validated finite element models of the aircraft and increase the reliability of cabin noise prediction using statistical energy analysis.
- Second one is to develop highly reliable low to mid frequency TBL model based on one or the combination of (a): the wind tunnel tests with low background noise; (b): numerical approaches using computational fluid dynamics; (c): flight tests.
- Third one is to decrease the computational time and improve the convergence of the implemented transfer function approach for performing design optimization studies for developing future noise control concepts for TBL excited structures.

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