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Extraction of target sources from incoherent and partially coherent background noise using low-rank and sparse decomposition of the cross-spectral matrix

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ABSTRACT

Beamforming in acoustic measurements suffers from strong and complicated background noise, which is usually assumed to be incoherent and contributes to the diagonal part of the cross-spectral matrix (CSM). However,n in some measurements the background noise could frequently be partially coherent and contributes to the off-diagonal part of the matrix as well. The complicated background noise would also lead to uninterpretable beamforming results, especially in aeroacoustic measurements. A method was proposed in this study to extract the target sources from incoherent and partially coherent background noise in acoustic measurements. Different characteristics between the target sources and background noise were analyzed. The sources and noise were modeled as low-rank and weak sparsity, and separated by minimization of a rank and L0-norm function. The number of eigenvalues was reduced in the extracted source containment of CSM. The eigenvalue and related beamforming results showed that the method was robustness in the extraction of target sources from complicated background noise.

Keywords: beamforming, noise, partially coherent
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1. INTRODUCTION

Beamforming in acoustic measurements suffers a lot from background noise [5]. The sound level of strong and complicated background noise can sometimes be up to approximately 20 dB greater than sources of interest, especially in aeroacoustic measurements [7]. Moreover, deconvolution approaches such as the DAMAS [3] and CLEAN-SC [12] are very sensitive to background noise for the reason that noise is not considered in the point spread function [2]. It is a great challenge to provide a source map by beamforming in noisy environment.

The containment of background noise in CSM could be highly removed if a reference noise spectral matrix was measured in advance [2]. The processing of diagonal removal (DR) [7], which sets the diagonal elements of CSM to zero, can effectively remove effects of incoherent background noise without a prior knowledge or procedure in beamforming. However, Lacking of diagonal part in the CSM would lead to an underestimation of strength and negative power output. It has been shown that a full CSM (with both diagonal and off-diagonal part) is necessary for algorithm like the Functional Beamforming [8], and deconvolution method like the CLEAN-SC would become more complicated without diagonal part of CSM [12]. The diagonal part of CSM could be estimated from the off-diagonal elements [9, 11], but the background noise can be distributed randomly in CSM sometimes [10], which is beyond the assumption of incoherent noise. The concept of partially coherent noise was introduced in measurement of propellers in a hydrodynamic tunnel, in which the boundary layer noise was molded as partially coherent [1], and was compared with other methods [6]. The work in this paper can be seen as a further discussion of the pioneering work of Ref. [1].

The aim of this study was to develop a CSM decomposition approach in extraction of target sources from complicated background noise, and obtain a de-noised beamforming result in a noisy environment or in wind tunnel. The background noise is assumed to be incoherent and partially coherent, which means the containment would concentrate on not only the diagonal but also off-diagonal of CSM. The target source and background noise are molded as low-rank and weak sparsity, and could be separated by minimization of a rank and L0-norm function.

2. ACOUSTIC SOURCE PROPAGATION AND BEAMFORMING

2.1. The cross-spectrum matrix

Consider a microphone array of M microphones, the measured sound pressure in the frequency domain is notated as

$$\mathbf{p} = \mathbf{A}\mathbf{q} + \mathbf{e}, \tag{1}$$

where $\mathbf{q} = [q(\mathbf{x}_1, f), \cdots, q(\mathbf{x}_s, f), \cdots, q(\mathbf{x}_S, f)]^T$ stands for the incoherent amplitude of sources, and $\mathbf{e} \in \mathbb{C}^{M\times 1}$ is the background noise measured by the array. The M-by-S matrix $\mathbf{A}$ describes the propagation from all sources to the array, entry of the matrix is

$$a_{ms} = \frac{1}{r_{ms}} e^{-ikr_{ms}}, \tag{2}$$

the wavenumber $k = 2\pi f/c$ with f and c denote the analysis frequency and sound speed, r_{ms} is the distance from s-th source to m-th microphone.

In frequency domain beamforming, data measured by the array is post-processed into CSM at first. In practical use, the matrix is obtained by averaging Fourier transformed signal of different sampling blocks. Theoretically, the matrix consists of two parts $\mathbf{G} = \mathbf{G}_\mathcal{S} + \mathbf{G}_\mathcal{N}$.

The containment of target source, which named as source spectral matrix in this paper, is expressed as

$$\mathbf{G}_\mathcal{S} = \mathbf{A}\mathbf{R}\mathbf{A}^H, \tag{3}$$

where $\mathbf{R} = \mathbb{E}\{\mathbf{q}\mathbf{q}^H\}$ and $\mathbb{E}\{\cdot\}$ represents the average of different sampling blocks.

The containment of background noise, which named as noise spectral matrix, is expressed as

$$\mathbf{G}_\mathcal{N} = \mathbb{E}\{\mathbf{e}\mathbf{e}^H\}. \tag{4}$$

2.2. Beamforming

The principle of beamforming is to steer the data measured by array and get a mapping result of the predefined source region. The source region is discretized into different scanning points, and the beamforming results would then be calculated at each scanning point

$$b(\mathbf{x}_n) = \mathbf{w}_n^H \mathbf{G} \mathbf{w}_n = \mathbf{w}_n^H \mathbf{G}_\mathcal{S} \mathbf{w}_n + \mathbf{w}_n^H \mathbf{G}_\mathcal{N} \mathbf{w}_n \tag{5}$$

where the first part indicates the point spread function from all S sources to the n-th scanning point, and the second part indicates influence of the background noise. $\mathbf{x}_n$ is the coordinate of scanning point with $n = 1, 2, \cdots, N$, and $\mathbf{w}_n$ is the steering vector which is normalized as $\mathbf{w}_n^H \mathbf{w}_n = 1$.

In beamforming, the background noise measured by array is usually assumed as incoherent, and thus the noise spectrum matrix would be an identity matrix multiplied by the average strength of background noise σ_e^2

$$\mathbf{G}_\mathcal{N} = \mathbb{E}\{\mathbf{e}\mathbf{e}^H\} \approx \sigma_e^2 \mathbf{I}. \tag{6}$$

However, the approximation of incoherent background noise in Equation 6 may not be applicable towards strong and complicated background noise: The noise would contribute to not only diagonal but also off-diagonal parts of the CSM. The aim of the paper is to extract the matrix $\mathbf{G}_\mathcal{S}$ (induced by target sources) from strong and complicated background noise and use the extracted matrix in beamforming

$$b(\mathbf{x}_n) = \mathbf{w}_n^H \mathbf{G}_\mathcal{S} \mathbf{w}_n. \tag{7}$$

3. EXTRACTION OF TARGET SOURCES FROM BACKGROUND NOISE IN CSM

Assume that the number of sources is much smaller than the number of microphones in beamforming, and it has been known that lower rank of CSM corresponds to fewer independent sources. Thus the source spectral matrix $\mathbf{G}_\mathcal{S}$ would be modeled as low-rank in mathematics. The absolute value of each entry in $\mathbf{G}_\mathcal{S}$ is

$$g_{s,ij} = \left| \sum_s q_s^2 \frac{1}{r_{is}} \frac{1}{r_{js}} e^{-ik(r_{is} - r_{js})} \right| \tag{8}$$

where $i, j = 1, 2, \cdots, M$. Note that the subscript "$\mathcal{S}$" and "$\mathcal{N}$" in $\mathbf{G}_\mathcal{S}$ and $\mathbf{G}_\mathcal{N}$ means source and noise, which is different from other subscripts (e.g., $g_{s,ij}$).

Obviously the adjacent entries of $\mathbf{G}_\mathcal{S}$ are related to the source position and array design (multi-arm spiral, square grid, *etc.*), and as a result the image of matrix $|\mathbf{G}_\mathcal{S}|$ would exhibit an evident periodicity of structures (see Figure 1 (c)).

On the other hand, the background noise correspond to matrix $\mathbf{G}_\mathcal{N}$ could be classified as two categories:

- incoherent: the background noise measured by different microphones are incoherent, nonzero entries of $\mathbf{G}_\mathcal{N}$ concentrate on the diagonal and the off-diagonal part equal to zero.

- partially coherent: the background noise measured by different microphones are partially coherent, which means the nonzero entries of $\mathbf{G}_\mathcal{N}$ mainly concentrate on diagonal with some others randomly concentrate on off-diagonal parts.

A weak symmetric sparse matrix ($\mathbf{G}_\mathcal{N}$) would thus appear due to the two kinds of background noise discussed above. The two kinds of background noise was discussed separately in this study because of the different influence in mapping result of beamforming.

Matrix of different characters (low-rank and sparse) can be visualized: the periodicity of structures and symmetric sparse in Figure 1 correspond to low-rank and weak sparsity respectively. Note that the matrix used for visualization has been normalized by

$$v_{ij} = \frac{g_{ij} g_{ij}^*}{g_{ii} g_{jj}}. \tag{9}$$

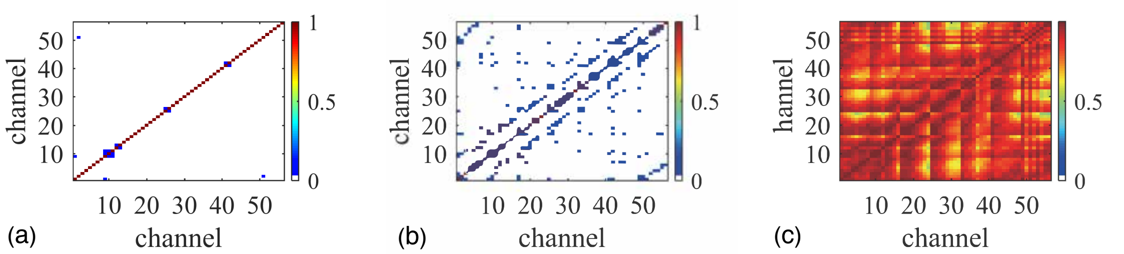

Figure 1: Visualization of different kinds of matrix: (a) weakly sparse induced by incoherent background noise; (b) weakly sparse induced by partially coherent background noise (c) combination of low-rank and weakly sparse induced by sources and partially coherent background noise

The source spectral matrix $\mathbf{G}_\mathcal{S}$ and noise spectral matrix $\mathbf{G}_\mathcal{N}$ could be separated based on the low-rank and sparsity of matrix [1]

$$\begin{aligned} \text{minimize} \quad & \operatorname{rank}(\mathbf{G}_\mathcal{S}) + \lambda \|\mathbf{G}_\mathcal{N}\|_{l_0} \\ \text{subject to} \quad & \mathbf{G}_\mathcal{S} + \mathbf{G}_\mathcal{N} = \mathbf{G} \end{aligned} \tag{10}$$

where rank and l_0 norm of matrix are selected to represent the low-rank and sparsity of two objective matrices, λ is a trading-off parameter between the low-rank (strong coherence from sources in CSM) and the sparsity (weak coherence from background noise), and the parameter is recommend as $1/M$ with M to be the number of microphones. The NP-hard problems can be effectively solved after relaxing into a convex optimization problem [4].

4. SIMULATION RESULTS

The simulation was conducted using a planar array and two monopole sources. The multi-arm spiral array of 1 meter aperture can be found in Figure 2 (a), the array was placed in the x-y plane and centered at the origin of coordinate system. Two sources of same strength were simulated at coordinates $(-0.2, -0.2, 1.5)$ and $(0.2, 0.1, 1.5)$, the scanning plane is a 1×1 meter square with a grid size of 0.015×0.015 meter and parallels to the array (see Figure 2 (b)), all sources were positioned at the grid point of the scanning plane.

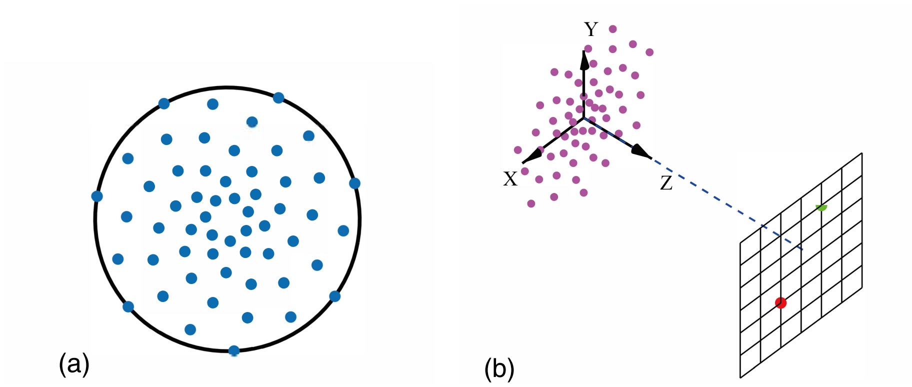

Figure 2: Simulation setup: (a) the multi-arm spiral array of 56 microphones; (b) setup of the array and the scanning plane.

Bandwidth Gaussian white noise signal was simulated at sources, and the background noise were added to each channel. Incoherent and partially coherent background noise were investigated separately, although the two kinds of noise are superimposed in real measurements. To simulate the partially coherent background noise, totally 21 channels were selected with the intention of adding coherent (same) noise signal, and incoherent noise signal were added to the other 35 channels. The simulated noise spectral matrix $\mathbf{G}_\mathcal{N}$ can be visualized in Figure 3.

The sound pressure were simulated at a sampling frequency of 16384 Hz, and the CSM was calculated by standard array processing procedure. The source spectrum matrix $\mathbf{G}_\mathcal{S}$ was extracted according to Equation 10, and applied to beamforming as in Equation 7.

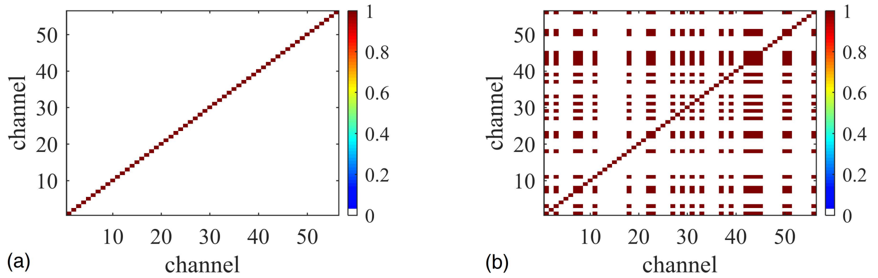

Figure 3: Visualization of the generated noise spectrum matrix with incoherent (a) and partially coherent (b) background noise in simulation

The beamforming results toward incoherent and partially coherent background noise were compared to the DR as indicated in Figure 4 and Figure 4 .

The mapping results toward incoherent noise are nearly identical for the two methods. The incoherent background noise would concentrate on diagonal of CSM, and could be totally removed by the processing of DR. The unit matrix is still a weakly sparse matrix, and thus the noise could also be removed by the proposed method (see Figure 4).

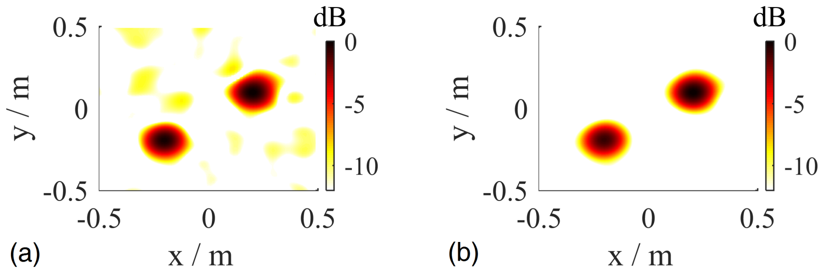

Figure 4: Beamforming results toward incoherent background noise using CSM induced by DR (a) and the proposed method (b).

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Locations of the two sources could hardly be confirmed in the blurry mapping result of DR in Figure 5 (a), in which the background noise were assumed as incoherent and concentrate on diagonal of CSM. As a comparison, sources could be properly located using the extracted matrix of target sources, the partially coherent noise has been effectively removed by the proposed method, as indicated in Figure 5 (b).

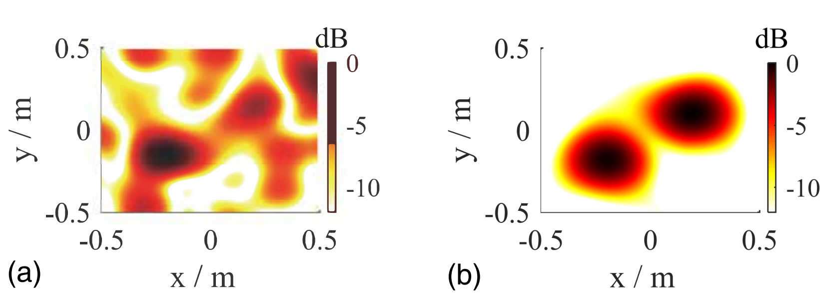

Figure 5: Beamforming results toward partially coherent background noise using CSM induced by DR (a) and the proposed method (b).

5. EXPERIMENTAL RESULT

The proposed method was validated by experiments using a microphone array in a noisy environment. Two 1 inch loudspeakers were used as target sources, and mounted on a wooden plate to emit sound. Backside of the two speakers were enclosed by five pieces of organic glass with absorbing materials. The experiment was performed in a noisy workshop with some other noisy machines operating, which leads to a strong and complicated background noise.

Bandwidth noise was emitted as signal of target sources and was measured by a microphone array at a distance of 1 meter. The sampling frequency was 16384 Hz with

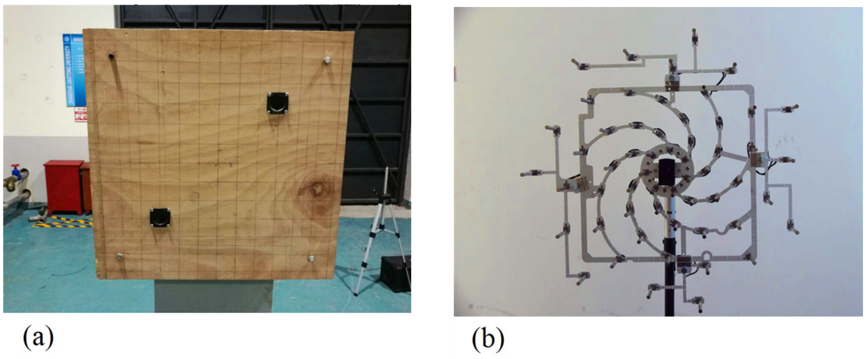

Figure 6: The two loudspeakers (a) and microphone array (b) used in the experiment.

a block size of 256 points. Totally 1600 blocks were sampled with 90% overlap. Other parameters were identical to those in the simulation.

The Signal-to-noise ratio (SNR) was around -15 dB in measurement, and the background noise presents to be partially coherent at some frequencies. The Beamforming result was generated at a frequency band of 2300 $\sim$ 2500 Hz and the ratio $\lambda = 0.19$. Results of the proposed method and the DR can be found in Figure 7. Generally speaking, the proposed method present an interpretable mapping result in locating sources. The strong and complicated background noise (consists of both partially coherent and incoherent background noise) was highly removed comparing to the result of DR. Note that the capability of locating sources at such low SNR comes from a combination of array and the proposed method: the microphone array of 56 microphones contributes a lot in noise attenuation, and the proposed method extended the capability.

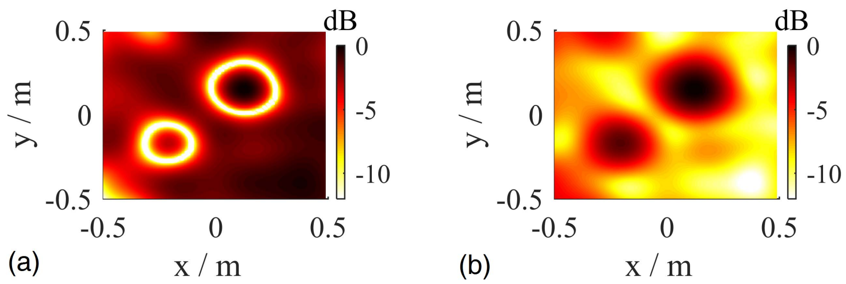

Figure 7: Beamforming results of experiment in a noisy environment by DR (a) and the proposed method (b).

A "ring" was appeared around the two target sources in result of DR (see Figure 7 (a)), which is uncommon and needs some more interpretation. Three parts may contribute to this mapping result (and CSM of this result): target sources, incoherent background noise and partially coherent background noise. Effects of the three parts can be be sketched using a schematic map, which presents the beamforming result in $1-$ dimensional scanning plane (see Figure 8).

The beamforming result of target sources (solid line in red) exhibit a peak at source position, which is determined by the array and beamforming algorithm. Results of incoherent background noise (dashed line in green) are same at all scanning points for the reason that the noise measured were incoherent. The partially coherent background noise, in which measured by array would have correlation between some channels, will lead to some ghost images in the mapping result. Besides, the beamforming results would be attenuated at central part of scanning plane if no source exists. Thus there is a tendency of dropping down in scanning points without sources and in front of array.

In Figure 8 (a) the target sources can not be located because of the low SNR. In result of DR (see Figure 8 (b)) the contribution of incoherent noise (dashed line in green, named "In") has been removed, but the contribution of partially coherent noise (dotted line in black, named "Pa") still exists and a "ring" appeared around target sources due to the uneven contribution of the noise. By this mean the "ring" in experimental result of DR (see Figure 7(a)) has been explained. As a comparison, sources were effectively located in the mapping result of the proposed method, in which the extracted source signal was used in beamforming. And this is in accordance with the experimental result of the proposed method (Figure 7 (b)).

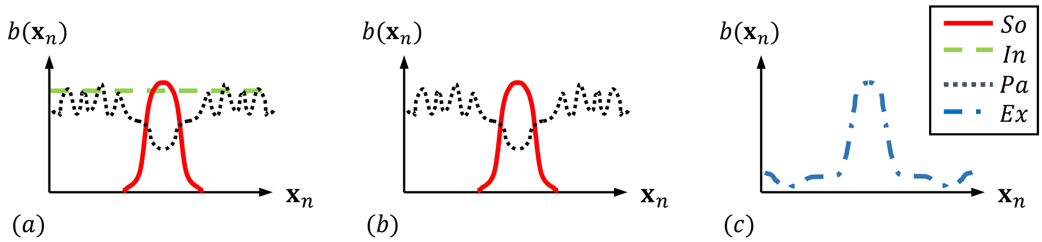

Figure 8: A sketch map describing contributions of different parts in beamforming result: (a) all parts exist, (b) the incoherent background noise removed by DR, (c) incoherent and partially coherent noise removed by the proposed method. $\mathbf{x}_n$ is the scanning points in the 1 dimensional space, and $b(\mathbf{x}_n)$ is the beamforming results at each position. Different names of lines indicate different parts: "So" indicates target source; "In" and "Pa" indicated incoherent and partially coherent background noise; the "Ex" indicates the extracted target sources from the entire backgrContributoround noise.

6. CONCLUSIONS

A method was proposed to extract the target sources from strong and complicated background noise in acoustic measurements. The background noise in CSM is molded as incoherent and partially coherent in the CSM, and the target source in CSM is molded as low-rank. The two parts can be separated by minimization of a rank and L0-norm function, beamfroming results using the extracted matrix from target source has proved the effectiveness of the method. The method extends the state of art denoising approaches to partially coherent background noise assumption, and could be applied to support deconvolution algorithms due to the entire CSM with highly removed noise.

7. ACKNOWLEDGEMENTS

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