MADRID
inter.noise 2019
June 16 - 19
NOISE CONTROL FOR A BETTER ENVIRONMENT

Sound absorption prediction for an arbitrarily shaped microperforated membrane space absorber

Li, Xianhui[1]
Wang, Yueyue
Cai, Zenong
Zhao, Junjuan
Gai, Xiaoling
Beijing Municipal Institute of Labor Protection,
Taoranting Road 55, Xicheng District, Beijing 100054, PR China

ABSTRACT

A theoretical framework is proposed to predict the absorption of a space sound absorber formed by a single layer of microperforated membrane. Based on a coupled FEM-BEM formulation, the impedance model of the interior cavity is obtained and coupled to that of the microperforated membrane, which includes the contributions from the microperforations and the sound-induced vibrations. The diffuse field is modeled by a reverberant blocking force. An explicit formula for the absorption is derived using the reciprocity relationship between direct field radiation and diffuse reverberant loading. Numerical simulation on two cylindrical absorbers of different lengths validates the present method.

Keywords: Space sound absorber, Diffuse field reciprocity, Microperforated membrane
I-INCE Classification of Subject Number: 35

1. INTRODUCTION

Microperforated panels/membranes (MPPs/MPMs) are typically used in a double-leaf configuration or backed by a rigid wall for the sound absorption purpose. Sometimes such a panel-like shape is not suitable for the applications in actual rooms or buildings. Recently lightweight three-dimensional MPP/MPM space sound absorbers (MSAs) have been proposed to overcome these limitations. According to the work by Toyoda et. al. [1], the cylindrical and rectangular MSAs demonstrate moderate sound absorption performances similar to the double-leaf configuration. The absorption characteristics of a cylindrical MSA with a rigid cylindrical core are also investigated [2]. It is shown that the core diameters can be used to effectively adjust the sound absorption at the target

---

[1] lixianh@vip.sina.com

frequencies. Furthermore, the absorptions of MSAs can be greatly improved by filling porous materials in the cavity [3].

Numerical methods are often employed to predict the absorption characteristics of MSAs. For cylindrical and rectangular MSAs, a two-dimensional model is considered to be sufficient because microperforated structures generally have a higher absorption performance under plane waves with a normal incidence rather than other angles [1]. Although employing a two-dimensional model drastically reduces the required computational resources, it does not work for an arbitrarily shaped MSA. In this paper, a theoretical framework is proposed to predict the absorption of an arbitrarily shaped MSA. The cavity inside the MSA is modeled with an FEM model, from which an impedance model is obtained. Then the impedance model is coupled to that of the MPM, which includes the contributions from the microperforations and the sound-induced vibrations [4, 5]. The diffuse field is directly modeled by a reverberant blocking force, instead of the average of the incident plane waves from all directions. An explicit formula for the absorption is derived using the reciprocity relationship between direct field radiation and diffuse reverberant loading [6, 7]. Numerical simulation is conducted on two cylindrical MSAs of different lengths. When the length of the MSA is much larger than its diameter, the sound absorptions are in good agreement with the measured data and those from the two-dimensional model. It is also shown that the shorter MSA has similar absorption characteristics but with higher absorption efficiencies.

2. METHOD

Consider an arbitrarily shaped space absorber formed by a single layer of MPM. For simplicity, neither sound sources nor absorbent materials are placed inside the absorber.

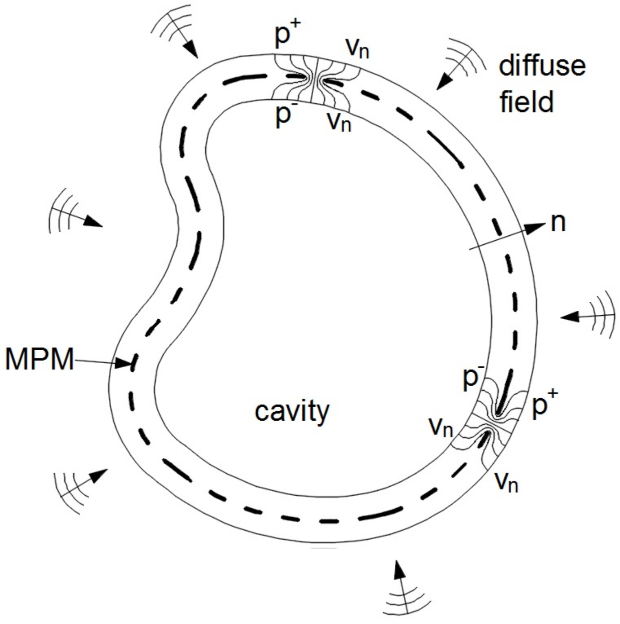

Figure 1: Diagram of an arbitrarily shaped MSA

In the frequency domain, the FE model for the acoustic cavity is

$$\left[\mathbf{H} - \omega^2 \mathbf{Q}\right] \mathbf{p}^- = -i\omega \mathbf{A} \mathbf{v}_n \,, \tag{1}$$

where ω is the radian frequency, $\mathbf{p}^-$ the nodal pressure vector of the inner surface, $\mathbf{v}_n$ the nodal normal velocity vector, $\mathbf{H} = \int_V \frac{1}{\rho_0}\nabla\mathbf{N}\,\nabla\mathbf{N}^T \mathrm{d}V$, $\mathbf{Q} = \int_V \frac{1}{\rho_0 c_0^2}\mathbf{N}\,\mathbf{N}^T \mathrm{d}V$, $\mathbf{A} = \int_S \mathbf{N}\,\mathbf{N}_s^T \mathrm{d}S$, $\mathbf{N}$ and $\mathbf{N}_s$ are the shape functions for the acoustic cavity and the MPM surface, and the superscript T denotes the matrix transpose.

The impedance model for the MPM includes the contributions from the microperforations and the sound-induced vibrations, which is given by

$$Z_{MPM} \equiv \frac{p^- - p^+}{v_n} = \frac{1}{(Z_h/\sigma)^{-1} + (i\omega\rho_s)^{-1}}\,, \tag{2}$$

where $p^\pm$ are the pressures on the outer and inner surfaces, v_n the average normal velocity of the acoustic medium, Z_h the impedance of a single perforation hole, σ the perforation ratio, and ρ_s the surface density of the MPM. Using the shape functions mentioned above, Equation 2 can be written in a matrix form as

$$\mathbf{A}^T\mathbf{p}^- = \mathbf{B}\mathbf{p}^+ + Z_{MPM}\mathbf{B}\mathbf{v}_n\,, \tag{3}$$

where $\mathbf{p}$+ is the nodal pressure vector of the outer surface and $\mathbf{B} = \int_S \mathbf{N}_s\,\mathbf{N}_s^T \mathrm{d}S$.

Under the diffuse field excitation, the outer surface pressure can be decomposed to

$$p^+ = p_{rev} + p_{rad}\,, \tag{4}$$

where p_{rev} denotes the reverberant loading and p_{rad} the radiation pressure. The latter is determined by the normal velocity of the outer surface. By usig a direct BEM formulation for the exterior problem, its nodal value can be written as

$$\mathbf{p}_{rad} = \mathbf{Z}_{rad}\mathbf{v}_n\,, \tag{5}$$

where $\mathbf{Z}_{rad}$ is the radiation impedance matrix.

From Equations 1, 3, and 5, a coupled impedance model is obtained as

$$\mathbf{Z}_{tot}\mathbf{v}_n = \left[\mathbf{Z}_{cav} + Z_{MPM}\mathbf{B} + \mathbf{B}\mathbf{Z}_{rad}\right]\mathbf{v}_n = -\mathbf{B}\mathbf{p}_{rev}\,, \tag{6}$$

where $\mathbf{Z}_{cav} = i\omega\mathbf{A}^T\left[\mathbf{H} - \omega^2\mathbf{Q}\right]^{-1}\mathbf{A}$.

Due to the diffuse reciprocity relationship [6, 7], the cross-spectrum of the block reverberant loading is associated with the radiation impedance by

$$\mathbf{S}_{rev} = \mathbf{B}\left\langle\mathbf{p}_{rev}\mathbf{p}_{rev}^T\right\rangle\mathbf{B}^T = \left\langle\bar{p}^2\right\rangle\frac{8\pi c_0}{\rho_0\omega^2}\mathrm{Re}\left\{\mathbf{B}\mathbf{Z}_{rad}\right\}\,, \tag{7}$$

where $\left\langle\bar{p}^2\right\rangle$ is the mean square reverberant pressure in the farfield and Re{ } the real part of a matrix.

The power dissipated by the MSA is

$$P_{diss} = \frac{1}{2}\mathbf{v}_n^H\mathrm{Re}\left\{Z_{MPM}\mathbf{B}\right\}\mathbf{v}_n = \frac{4\pi c_0}{\rho_0\omega^2}\left\langle\bar{p}^2\right\rangle\mathrm{Tr}\left(\mathrm{Re}\{Z_{MPP}\mathbf{B}\}\mathbf{Z}_{tot}^{-1}\mathrm{Re}\{\mathbf{B}\mathbf{Z}_{rad}\}\mathbf{Z}_{tot}^{-H}\right)\,, \tag{8}$$

where Tr denotes the trace of a matrix and the superscript H the Hermitian transpose.

Since the intensity in the diffuse field is

$$I = \left\langle\bar{p}^2\right\rangle/4\rho_0 c_0\,, \tag{9}$$

the absorption of the MSA is obtained as

$$S_{abs} \equiv P_{diss}/I = \frac{4\lambda^2}{\pi}\mathrm{Tr}\left(\mathrm{Re}\{Z_{MPP}\mathbf{B}\}\mathbf{Z}_{tot}^{-1}\mathrm{Re}\{\mathbf{B}\mathbf{Z}_{rad}\}\mathbf{Z}_{tot}^{-H}\right), \tag{10}$$

where λ denotes the wavelength.

The absorption coefficient is easily obtained as the ratio between the absorption and the surface area of the MSA, $\alpha = S_{abs}/S_{MSA}$. When the size of the absorber is small as compared with the wavelength, the system can be projected onto the subspace spanned by the first few acoustic radiation modes [8, 9]. These modes are obtained by a modal decomposition of the discretized radiation operator $\mathrm{Re}\{\mathbf{B}\mathbf{Z}_{rad}\}$, which is symmetric positive definite by nature. Denote the subspace spanned by the acoustic radiation modes by $\mathbf{\Phi}$, and the corresponding eigenvalue matrix by $\mathbf{\Lambda}$, then Equation 10 can be reduced to

$$\hat{S}_{abs} = \frac{4\lambda^2}{\pi}\mathrm{Tr}\left(\mathbf{T}^H\mathrm{Re}\{Z_{MPP}\mathbf{B}\}\mathbf{T}\mathbf{\Lambda}\right), \tag{11}$$

where $\mathbf{T} = \mathbf{Z}_{tot}^{-1}\mathbf{\Phi}$.

3. VALIDATION

Two cylindrical MSAs with 1 m perimeter are investigated. The longer one is 1 m in length, while the shorter one 0.318 m. Both MSAs have rigid covers on the top and the bottom, forming a closed cavity inside. The MPM parameters are taken from those in Reference [1], with a 0.5 mm hole diameter, 0.5 mm thickness, 0.785% perforation ratio, and 0.6 kg/m^2 surface density. Figure 2 shows the absorptions of the long cylindrical MSA suspended from a ceiling. It demonstrates that the present method provides a better prediction than the two-dimensional model as compared with the measured data [1].

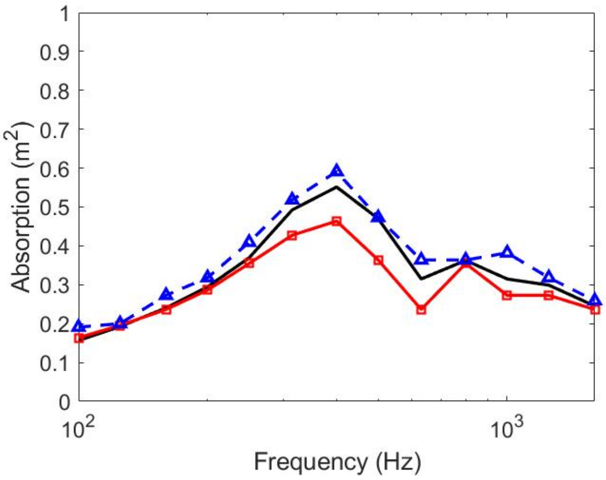

Figure 2: Absorption of the long cylindrical MSA suspended from the ceiling, black line: present method; red square line: two-dimensional model; blue triangular dotted line: measured data

Figure 3 shows the absorptions of the long cylindrical MSA placed on a floor. It is observed that the absorptions in the low frequencies are improved in this case. They agrees well with the measured data [1], while the two-dimensional model fails to capture the difference between the two installation conditions.

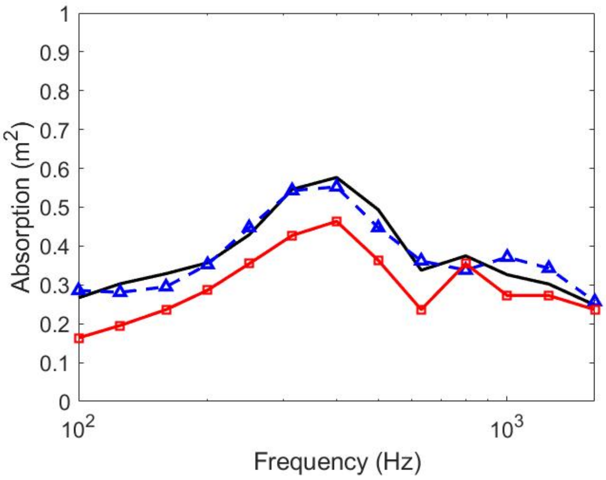

Figure 3: Absorption of the long cylindrical MSA placed on the floor, black line: present method; red square line: two-dimensional model; blue triangular dotted line: measured data

The acoustic radiation modes are computed at different center frequencies. The eigenvalues represent the radiation efficiencies of the corresponding acoustic modes, which are shown in Figure 4. For the center frequency at 160 Hz, only 5 acoustic modes are necessary to describe the interaction between the MSA and the surrounding medium. The necessary mode number is more than 50 when the center frequency is 1000 Hz. Projecting the system onto the subspace spanned by 50 acoustic modes, the calculated absorptions almost overlap those from the direct computation, which are not plotted in Figures 2 and 3 for simplicity.

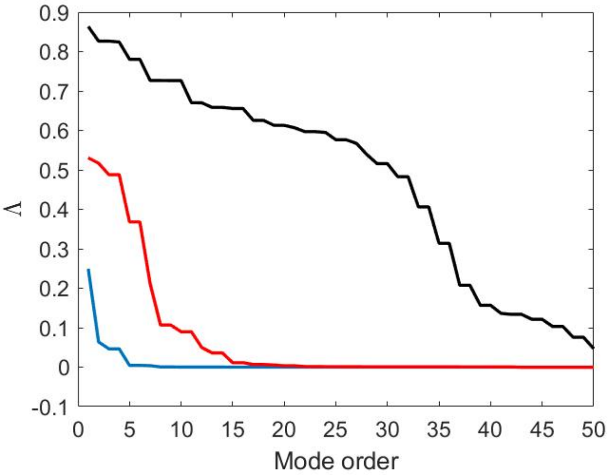

Figure 4: Radiation efficiencies of the acoustic radiation modes at different center frequencies, blue line: 160 Hz; red line: 400 Hz; black line: 1000 Hz

Figure 5 shows the absorptions of the short cylindrical MSA suspended from the ceiling and placed on the floor, respectively. Since the two-dimensional model works mainly for the MSA with a large height-to-perimeter ratio [1], its results are not included herein. Although the short cylindrical MSA has lower absorptions than the long cylindrical MSA, its absorption coefficients are relatively large for its surface area is about one third of the latter's.

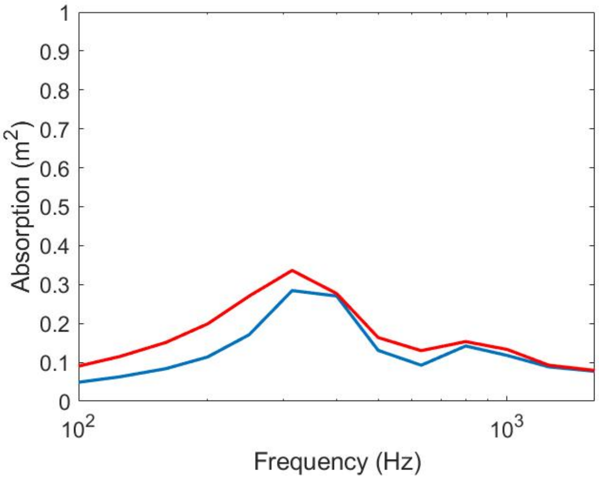

Figure 5: Absorption of the short cylindrical MSA, blue line: suspended from a ceiling; red line: placed on the floor

The eigenvalues of the acoustic modes of the short cylindrical MSA are shown in Figure 6. For the center frequency at 160 Hz, only 3 acoustic modes are necessary to describe the interaction between the MSA and the surrounding medium. The necessary mode number is about 30 when the center frequency is 1000 Hz. Projecting the system onto the subspace spanned by 30 acoustic modes yields the same results as shown in Figure 5.

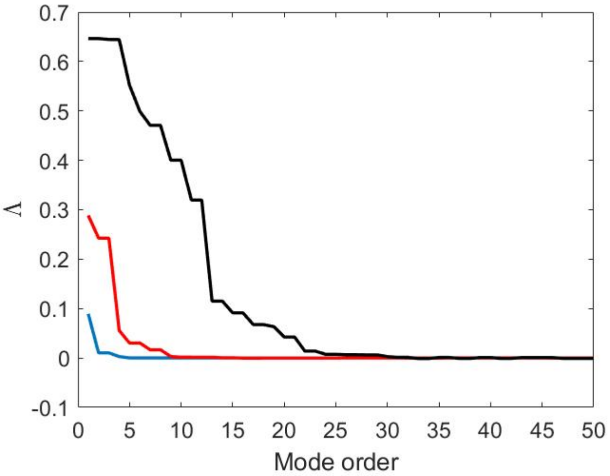

Figure 6: Radiation efficiencies of the acoustic radiation modes at different center frequencies, blue line: 160 Hz; red line: 400 Hz; black line: 1000 Hz

4. CONCLUSIONS

A theoretical framework is proposed to predict the absorption of an arbitrarily shaped MSA. The impedance model of the absorber is derived from a coupled FEM and BEM formulation. The diffuse field is modeled by a reverberant blocking force. An explicit formula for the absorption is derived using the diffuse field reciprocity. Numerical simulation is conducted on two cylindrical MSAs of different lengths. The predicted absorptions of the long cylindrical MSA agree well with the measured data and those obtained from the two-dimensional model. The short cylindrical MSA is demonstrated to have higher absorption coefficients, suggesting that several small MSAs are more efficient in sound absorption than one large MSA.

5. ACKNOWLEDGEMENTS

This work is supported by Beijing Natural Science Foundation (1172007, 1182011), Natural Science Foundation of China (11704035, 11604015, 11174041), Xicheng District Excellent Talent Project (20180029), Beike Scholar Project and Youth Core Plan (YC201807) of Beijing Academy of Science and Technology.

6. REFERENCES

[1] M. Toyoda, S. Kobatake, and K. Sakagami. Numerical analyses of the sound absorption of three-dimensional MPP space sound absorbers. *Applied Acoustics*, 79:69–74, 2014.

[2] M. Toyoda, S. Fujita, and K. Sakagami. Numerical analyses of the sound absorption of cylindrical microperforated panel space absorbers with cores. *Journal of the Acoustical Society of America*, 138(6):3531–3538, 2015.

[3] M. Toyoda, K. Sakagami, M. Okano, T. Okuzono, and E. Toyoda. Improved sound absorption performance of three-dimensional MPP space sound absorbers by filling with porous materials. *Applied Acoustics*, 116:311–316, 2017.

[4] D. Y. Maa. Potential of microperforated panel absorber. *Journal of the Acoustical Society of America*, 104(5):2861–2866, 1998.

[5] J. Kang and H. V. Fuchs. Predicting the absorption of open weave textiles and micro-perforated membranes backed by an air space. *Journal of Sound and Vibration*, 220(5):905–920, 1999.

[6] P. J. Shorter and R. S. Langley. On the reciprocity relationship between direct field radiation and diffuse reverberant loading. *Journal of the Acoustical Society of America*, 117(1):85–95, 2005.

[7] P. J. Shorter and R. S. Langley. Vibro-acoustic analysis of complex systems. *Journal of Sound and Vibration*, 288(3):669–699, 2005.

[8] S. J. Elliott and M. E. Johnson. Radiation modes and the active control of sound power. *Journal of the Acoustical Society of America*, 94(4):2194–2204, 1993.

[9] K. A. Cunefare and M. N. Currey. On the exterior acoustic radiation modes of structures. *Journal of the Acoustical Society of America*, 96(4):2302–2312, 1994.