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Vibration analysis of the subsea dynamic umbilical under parametric excitations

Yao Jianjun[1] Sheng Tang[2] Wang Xiancheng[3], Wang Jiaqi[4], Wan Zhenshuai[5] Xiao Chenguang[6]

[1-2,4-6]College of Mechanical and Electrical Engineering, Harbin Engineering University, China.

[3]Ningbo Institute of Technology, Zhejiang University, China

ABSTRACT

As a key component of the submarine production, the subsea dynamic umbilical is a multilayer composite structure which connects the offshore facilities with the submarine production control system. However, due to complex dynamics characteristics of the umbilical, its non-linear dynamic response in the transverse direction may cause a parametric vibration at a dangerously high level which may lead to disastrous accidents, such as environmental pollution, property losses and even fatalities. Therefore, the parametric instability analysis of the umbilical is deserved to be studied further. In this paper, based on the deduced governing motion equation and the Hill's infinite determinant, the parametric instability behavior of the umbilical is investigated. Numerical simulations for the lateral response of the umbilical are performed under several typical cases with consideration of the coupling of modes. Besides, the influences of the umbilical damping variation on the system parameter stability regions are emphatically analyzed. The simulation results demonstrate that enhancing the damping of system can effectively reduce unstable regions. Furthermore, several useful guidelines are also proposed to optimize the design of the umbilical.

Keywords: Subsea dynamic umbilical, Parametric vibration, Mathieu instability domains
I-INCE Classification of Subject Number: 76

1. INTRODUCTION

The umbilical, the key component of submarine production, is deployed on seabed to supply necessary control signal, energy(electric, hydraulic) and chemicals to subsea oil and gas wells. And it is also called "lifeline" in marine engineering because of its importance[1]. As shown in Figure 1, the umbilical is divided into

[1] travisyao@126.com
[2] heu_st@126.com
[3] wxc@nit.zju.edu.cn
[4] 845799710@hrbeu.edu.cn
[5] 18242311944@163.com
[6] xcg911030@hrbeu.edu.cn

dynamic and static umbilical. The dynamic umbilical is used for connecting floating body with submarine equipment, the static umbilical is fixed to seabed which connects the different submarine equipment.

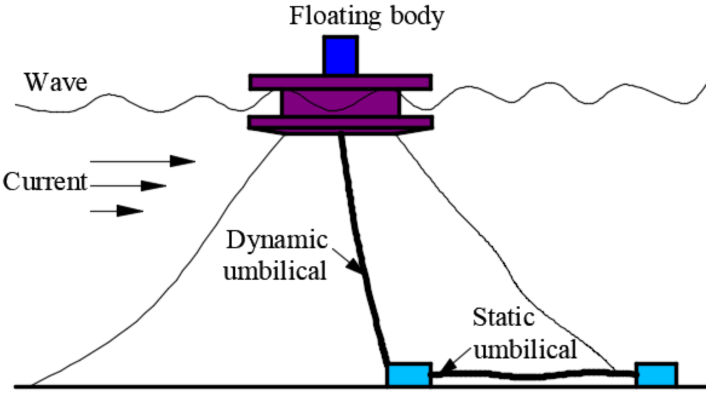


Figure 1: Typical schematic diagram of umbilical

The service condition of dynamic umbilical is very harsh. It is not only affected by the loads of marine environment, but also by the load generated by the movement of floating body, the load of wind-wave and the lifting force, and so on. Under the action of these multi-loads, the dynamic umbilical is extremely likely to break, which result in serious environmental pollution, property losses and even fatalities[2]. Among them, parametric vibration is a significant factor leading to fracture. Parametric resonance is a common phenomenon in springing vibration, even though the excitation frequency is not significantly equal to the linear natural frequency of the system, the umbilical can still generate a large response by a small excitation in the axial direction. Under working condition, the heave motion caused by the action of floating body leads to the axial tension of umbilical, which causes the horizontal dynamic response of the umbilical structure[3]. According to the relationship between parameters of umbilical when the system is unstable, the unstable region where parametric resonance occurs is plotted, as shown in Figure 2, these regions appear as one continuous parameter interval after another. Because of the large and irregular distribution of these regions, it is much more difficult for dynamic umbilical to avoid parametric resonance than to avoid ordinary vibration caused by forced vibration[4]. In consequence, optimizing the parameters of umbilical to avoid the parametric resonance is crucial in the design process.

Figure 2: Parametric stability diagram

The mechanism researched of parametric resonance generally concentrates on the mathematical theory of the Mathieu equation and its stability. Hsu[5] is the first person who studied the parametric resonance of ocean cables. Geoffrey[6] analyzed its instability, and drew dimensionless instability diagrams of parametric resonance based on Floquent theory. Mathieu instability is also discussed in the literature on marine engineering, Haslum and Faltinsen[7] showed the stability diagram of the Mathieu equation without considering the pitching damping effect. Zhang Jie et al.[8-9] analyzed the variation law of natural vibration characteristics of the riser and compared the influence of design parameters on natural vibration characteristics. Tang Yougang et al.[10] studied the parametric-vortex-induced coupled vibration of risers in shear flow, and analyzed the influence of parametric excitation on the transverse vortex-induced vibration. Kuiper[11] analyzed the effects of fluid pressure difference, structural damping, section torsion and shear deformation on the parametric vibration of risers. Liu et al.[12] concluded the dynamic response of a deepwater platform under nonlinear vibration based on the stability diagram drawn. At the same time, the dynamic umbilical studied in this paper belongs to slender flexible structure, Deepark and Radhakrishnan[13] proposed a system parameters definition for parametric resonance of such structure. However, there are few studies on parametric resonance of the umbilical.

In order to obtain the parametric vibration instability region of umbilical, firstly, the parameter instability of dynamic umbilical under parameter excitation is analyzed, and the vibration model is established. Furthermore, the Mathieu instability of umbilical is analyzed by using the Hill infinite determinant method, and the region of instability is solved. Finally, the relationship between parameters of the vibration equation is analyzed, and the dynamic responses of dynamic umbilical under different working conditions is discussed and analyzed.

2. THE PARAMETRICALLY EXCITED SYSTEM

The dynamic umbilical is a multi-layer composite structure integrated by wire unit, cable unit, soft/rigid tube unit, filling unit, sheath unit and armored unit. since its structure is complicated, for the convenience of analysis, it is considered as the flexible and long tube, and following assumptions in this study:

1) The interaction between the internal units of umbilical is ignored.

2) Considering only the elastic stiffness of umbilical, excluding flexural stiffness, torsional stiffness and shear stiffness.

3) The constitutive relationship of the umbilical’s deformation obeys Hooke's law and the force is evenly distributed at each point.

The dynamic force diagram of the umbilical is shown in Figure 3. The link between the end of the umbilical and floating body and shock absorber can be simulated as hinge[4]. This hypothesis can analyze the essential characteristics of the umbilical’s parametric resonance more widely.

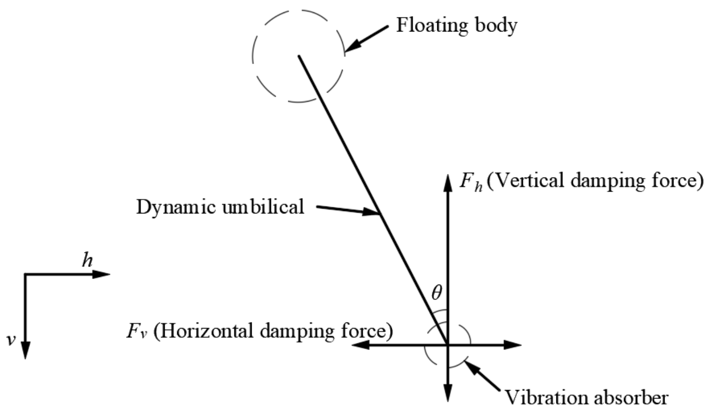


Figure 3: Schematic of the dynamic umbilical under parametric excitation

In Figure 3, the length of umbilical is L, where L is a fixed constant, and θ is the angle between the umbilical and the vertical direction, which is a small value. The tension of shock absorber subjected to umbilical is F, that is, the tension of umbilical.

Then the longitudinal tension of umbilical to shock absorber, i.e., the longitudinal tension of umbilical is as follows:

$$F_v = -F\cos\theta = -F \tag{1}$$

The transverse tension of umbilical to shock absorber, i.e., the transverse tension of umbilical is as follows:

$$F_h = -F\sin\theta = -F\frac{h}{L} \tag{2}$$

In order to study the parametric vibration of umbilical, the transverse dynamic equation is established with consideration of its inertia force.

$$(M+m)\ddot{h} + F_h = (M+m)\ddot{h} + F\frac{h}{L} = 0 \tag{3}$$

where M is the mass of the umbilical, m is the added mass.

Due to the axial tension and compression of dynamic umbilical cord cable caused by the floating body, the change of the axial tension of the umbilical in time domain is a periodic excitation. Supposing the variation of the tension F is:

$$F = F_0 + \Delta F\cos\omega \mathrm{t} \tag{4}$$

where F_0 is the static tension component of umbilical, $\Delta F\cos\omega \mathrm{t}$ is the dynamic tension component, ΔF is the changing amplitude of dynamic tension, ω is the frequency of tension change.

By substituting Equation 4 into Equation 3, the undamped non-linear vibration equation of the dynamic umbilical is yields:

$$(M+m)\ddot{h} + (F_0 + \Delta F\cos\omega \mathrm{t})\frac{h}{L} = 0 \tag{5}$$

Considering the damping effect, the nonlinear vibration equation of umbilical is:

$$(M+m)\ddot{h} + C\dot{h} + (F_0 + \Delta F\cos\omega \mathrm{t})\frac{h}{L} = 0 \tag{6}$$

where C is the system damping of the dynamic umbilical, including both structural and fluid damping.

Nondimensionalizing Equation 6:

$$\ddot{h} + \frac{C}{M+m}\dot{h} + \left(\frac{F_0}{(M+m)L} + \frac{\Delta F\cos\omega \mathrm{t}}{(M+m)L}\right)h = 0 \tag{7}$$

3. ANALYSIS OF THE MATHIEU STABILITY

By replacing the parameters, Equation 7 is transformed into the form of the Mathieu equation:

$$\ddot{h}+c\dot{h}+\left(\delta+\varepsilon\cos\omega t\right)h=0 \tag{8}$$

where $\delta+\varepsilon\cos\omega t$ is the dimensionless variable stiffness of the dynamic umbilical; c is the dimensionless damping, $c=\frac{C}{M+m}$; δ is the dimensionless stiffness of static umbilical, $\delta=\frac{F_0}{\left(M+m\right)L}$; ε is the dimensionless amplitude of the axially excited vibration, $\varepsilon=\frac{\Delta F}{\left(M+m\right)L}$.

It is impossible to obtain exact solutions for the non-linear equation like Equation 8, but its stability can be studied by parameters, and analyzed by perturbation method and the Hill infinite determinant method. The δ-ε plane can be divided into a stable region and an unstable region by the transition curve of a linear differential equation by based on Floquent theory. The transition curve Equation 8 with at least one solution is periodic with a period of 2π or 4π[15-17].

In Equation 8, it can be found out that there is a certain relationship among stiffness δ, damping c and frequency ratio. For a given δ, the response ε can be obtained by using the Hill infinite determinant method. Given a damping c, the δ-ε plane is divided into stable region and unstable region by the unstable critical point of Equation 8. In unstable region, parametric resonance will occur in umbilical cord cable, but not in stable region.

The Mathieu equation is time-varying, but the occurrence of parametric resonance is independent of time, it is related to parameters δ and ε. In order to obtain different stability periodograms of (δ, ε), the solution of equation 8 is taken as Fourier series with a period of 2π:

$$h\left(t\right)=\sum_{n=-\infty}^{+\infty}s_n e^{int} \tag{9}$$

Substituting Equation 9 into Equation 8, and Euler formula is applied to obtain equation as follows by using variable substitution.

$$\sum_{n=-\infty}^{+\infty}e^{int}\left\{\frac{1}{2}\varepsilon s_{n+1}+\left(\delta+inc-n^2\right)s_n+\frac{1}{2}\varepsilon s_{n-1}\right\}=0 \tag{10}$$

Only if all the coefficients in Equation 10 are zero, the equation make sense, i.e.:

$$\frac{1}{2}\varepsilon s_{n+1}+\left(\delta+inc-n^2\right)s_n+\frac{1}{2}\varepsilon s_{n-1}\text{,}\quad n=0,\pm1,\pm2,\ldots \tag{11}$$

If there is a non-zero solution for Equation 10, the determinant of its infinite coefficient matrix must be zero. Assuming $\delta+inc-n^2\neq0$, then:

$$\begin{vmatrix}\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\ \cdots & \gamma_1 & 1 & \gamma_1 & 0 & 0 & \cdots\\ \cdots & 0 & \gamma_0 & 1 & \gamma_0 & 0 & \cdots\\ \cdots & 0 & 0 & \gamma_{-1} & 1 & \gamma_{-1} & \cdots\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\end{vmatrix}=0 \tag{12}$$

where $\gamma_n=\frac{\varepsilon}{2\left(\delta+inc-n^2\right)}$, $n=0,\pm1,\pm2,\ldots$.

Similarly, the solution of Equation 8 is defined as a period of 4π as follows:

$$h(t)=\sum_{n=-\infty}^{+\infty} s_n e^{\frac{1}{2}int} \tag{13}$$

Substituting Equation 9 into Equation 8, and Euler formula is applied to obtain equation as follows by using variable substitution.

$$\sum_{n=-\infty}^{+\infty} e^{\frac{1}{2}int}\left\{\frac{1}{2}\varepsilon s_{n+2}+\left(\delta+\frac{1}{2}inc-\frac{1}{4}n^2\right)s_n+\frac{1}{2}\varepsilon s_{n-2}\right\} \tag{14}$$

Only if all the coefficients in Equation 10 are zero, the equation make sense, i.e.:

$$\frac{1}{2}\varepsilon s_{n+2}+\left(\delta+\frac{1}{2}inc-\frac{1}{4}n^2\right)s_n+\frac{1}{2}\varepsilon s_{n-2}=0 \tag{15}$$

To make Equation 15 have a non-zero solution, the determinant value of this infinite coefficient matrix must be zero. Assuming $\delta \neq n^2$, its corresponding determinant is:

$$\begin{vmatrix} \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \gamma_1 & 0 & 1 & 0 & \gamma_1 & 0 & 0 & \cdots \\ \cdots & 0 & \gamma_0 & 0 & 1 & 0 & \gamma_0 & 0 & \cdots \\ \cdots & 0 & 0 & \gamma_{-1} & 0 & 1 & 0 & \gamma_{-1} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{vmatrix}=0 \tag{16}$$

where $\gamma_n=\dfrac{\varepsilon}{2\left(\delta+\frac{1}{2}inc-\frac{1}{4}n^2\right)}$, $n=0,\pm1,\pm2,\ldots$.

The stability charts of the Mathieu equation with different damping ratios by Equation 12 and Equation16 are obtained, as shown in Figure 4. In order to analyze the first-order unstable region more clearly, the red circle marked part in Figure 4 is magnified as shown in Figure 5.

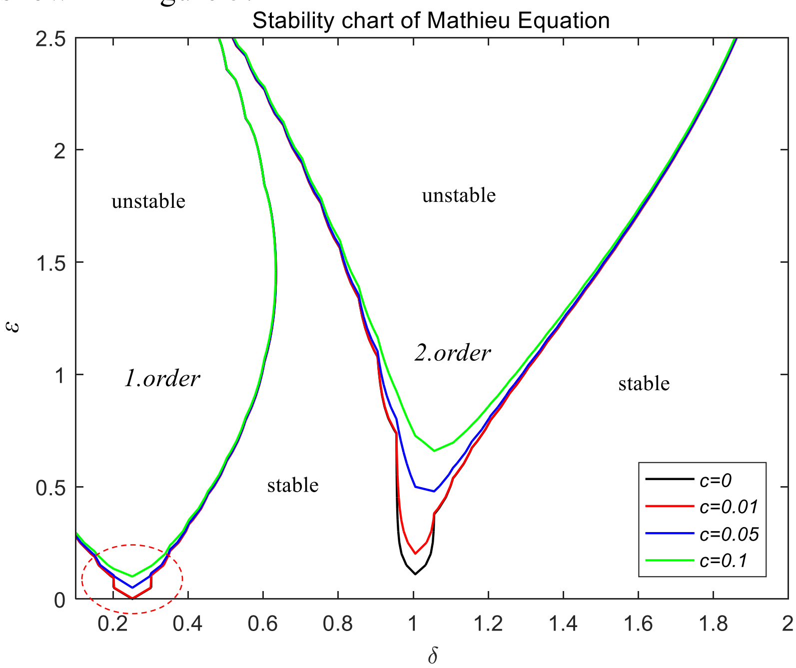


Figure 4: Stability chart of the Mathieu equation

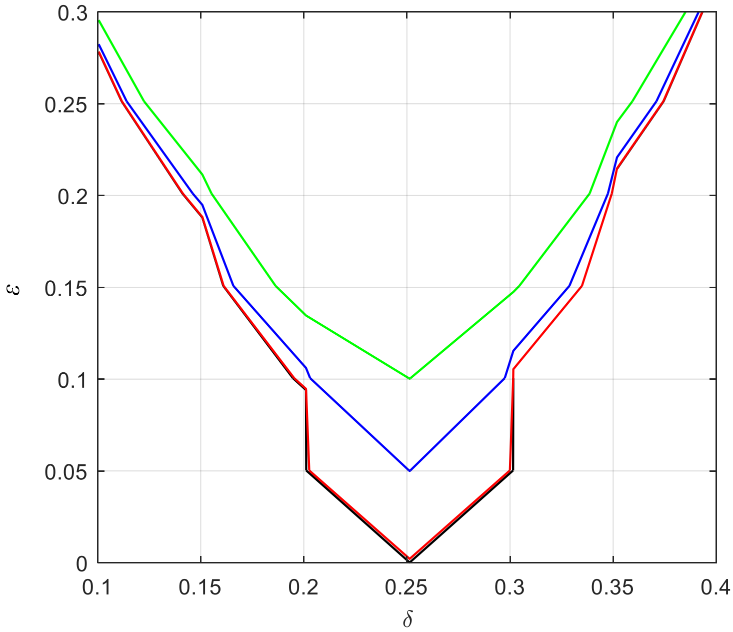


Figure 5: Enlarged drawing

The unstable regions where parametric resonance may occur are mainly concentrated on the first and second unstable regions. From Figure 4 and Figure 5, it can be seen that the unstable region is far away from the δ-axis when the system is added with damping, which indicates that the unstable region is became smaller with the damping increase, and the second unstable region is more significantly affected by the damping. Nevertheless, the influence is limited. When $\delta = 0.25$ (that is, the transverse natural frequency is half of the heave frequency), the unstable region is the largest. Therefore, the transverse natural frequency of the umbilical should be designed as far as possible away from half of the axial excitation frequency.

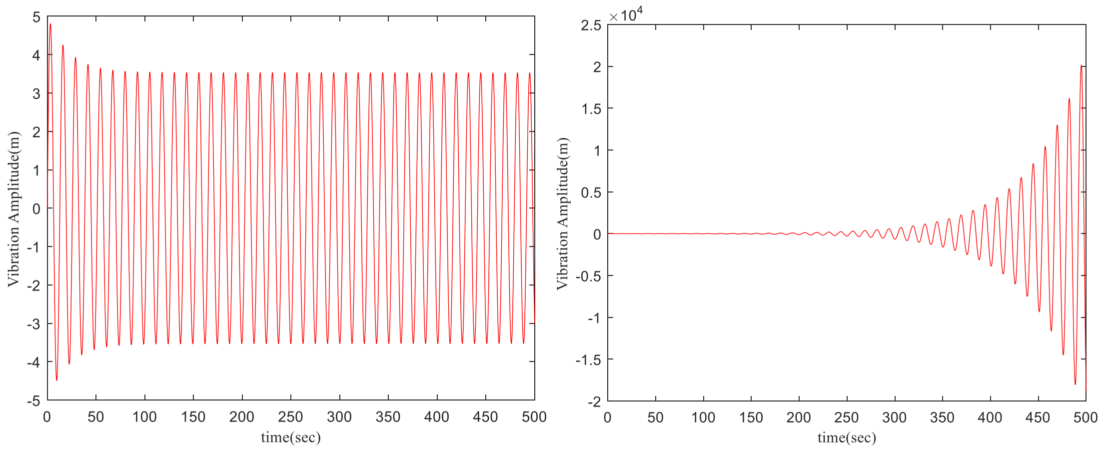
4. DYNAMIC RESPONSE ANALYSIS

It can be seen that different regions in Figure 4 have different effects on the dynamic response of umbilical, and four points are taken from the figure for analysis. Since the unstable region of order is greatly affected by the damping, the first few orders are mainly considered in real project, so the dynamic response analysis is performed where the frequency ratios and is around 0.25 and 1 respectively (i.e., $\delta = 0.25$ or $\delta = 1$), and two cases of stability and instability (a total of 4 points selected in Figure4) are compared, as shown in Table 1.

Table 1. System parameters under different case

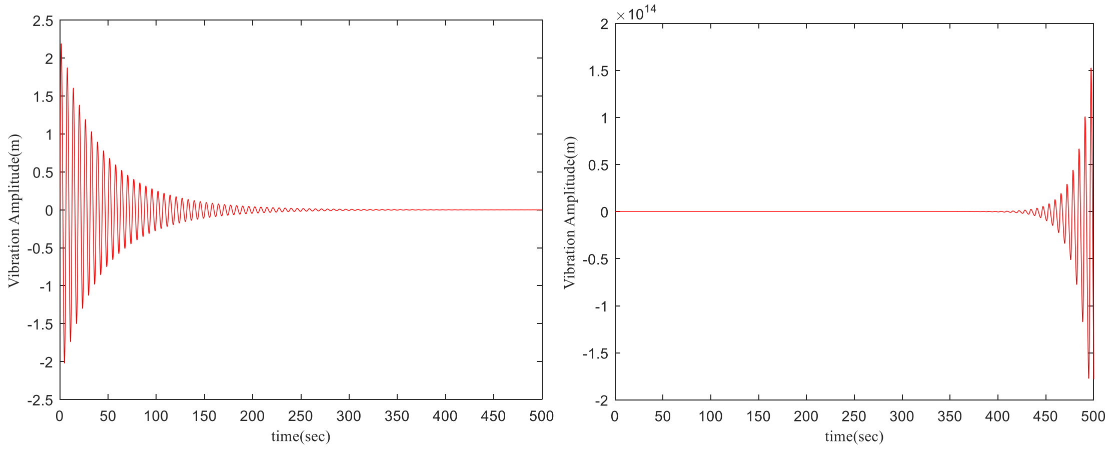
case	order	δ	ε	c
stable motion	1.order	0.25	0.025	0.025
Unstable motion	1.order	0.25	0.06	0.025
stable motion	2.order	1	0.3	0.05
Unstable motion	2.order	1	1	0.05

According to the system parameters of Table 1, the dynamic response analysis results is performed as follows:



(*a*) *Stable motion response diagram* (*b*) *Unstable motion response diagram*

Figure 6: Motion response diagram of first-order region



(*a*) *Stable motion response diagram* (*b*) *Unstable motion response diagram*

Figure 7: Motion response diagram of second-order region

It can be seen from Figure 6(a) that the dynamic response of umbilical is in a stable region under the first case. For a given initial disturbance, the response amplitude of umbilical does not increase significantly over time. In the first-order region, since the influence of damping on the unstable region is small, the range in which the parameter resonates is large. Although the motion response of umbilical can be stabilized in the stable region, its convergence speed is slow and vibration can not be effectively suppressed. Therefore, the umbilical should be designed to avoid the first-order unstable region.

Figure 6(b) shows that the motion response of the umbilical is obviously in first-order unstable region, and the parametric resonance of the umbilical occurred. The motion amplitude increases continuously for a limited period of time until the umbilical is damaged.

From Figure 7(a), it can be seen that the motion response of the umbilical is in the second-order stable region under the third case. Due to the effect of damping, the response can stabilize in a short time when the initial disturbance is given, which is an ideal state for the design of umbilical to avoid parametric resonance.

It can be seen from Figure 7(b) that in the fourth case, after a long period of time, the umbilical cable suddenly undergoes intense parametric resonance, and the amplitude of motion increases sharply, which is extremely dangerous in practical engineering, because the degree and influence of its resonance is impossible to predict and control. In Case 2, the amplitude of the umbilical's movement gradually increases over time, so that its response can be judged within foreseeable time, and unstability of the motions for umbilical can be eliminated by adjusting the floating body motion or the structure of umbilical. In Case 4, the intense movement of the umbilical will cause damage to the umbilical if engineers fail to detect it in time. However, since the order instability is greatly affected by the damping, the larger the damping and the smaller the unstable region. Thus, reasonable design of umbilical cord cable can make the system far away from unstable area.

The dynamic response analysis verifies the phenomenon of parametric resonance of umbilical caused by floating body motion, and the influence of damping on it. Through the above analysis, in the design procedure of umbilical, improving the stiffness of the umbilical structure, changing its structure to increase the damping or adding damping device, and reducing the pendulum motion of floating body in operation can effectively avoid the parametric resonance.

5. CONCLUSIONS

In this paper, the parametric resonance of subsea dynamic umbilical was investigated. Numerical analysis was formulated, and the effects of different damping on the motion response of umbilical were discussed in combination with typical cases.

1) The dynamic umbilical is parametrically resonated in the horizontal direction by the axial excitation generated by the floating body. Significant parametric resonance occurs when the axial excitation frequency is double of the natural frequency of the subsea dynamic umbilical. Therefore, when designing the umbilical, the axial excitation frequency of the umbilical should be kept away from the twice times of the natural frequency, which could effectively avoid the occurrence of umbilical parametric resonance phenomenon.

2) With the increase of damping, the unstable region decreases gradually. The damping has a greater influence on the unstable regions with smaller orders, while the impact on the main instability regions is relatively small. Therefore, when designing umbilical, the system damping should be increased as much as possible.

3) The effect of damping on subsea dynamic umbilical is to reduce the unstable region of parametric resonance, which could not effectively suppress the large increase of the parametric resonance amplitude.

4) Once the parametric resonance happens, intense vibration often occurs in a short time, which is extremely destructive.

6. ACKNOWLEDGEMENTS

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