

Topology Optimization Analysis of Leaf Vein Bionic Piezoelectric Energy Collection Device based on Vortex-induced Vibration

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ABSTRACT

Vortex-excited vibration induced by cylindrical flow around piezoelectric plates is the prototype of many piezoelectric energy collection devices. The increasingly lack of energy has put forward new requirements for the development of new energy sources. The vortex-induced vibration acts on the piezoelectric plate, and the energy conversion efficiency is affected by the piezoelectric plate structure. Therefore, the structure design of piezoelectric energy collection device is worth further studying. In this paper, utilizing the improved model based on SIMP modelling method and the MMA solution method, the topological optimization analysis of piezoelectric material structure is carried out. Optistruct simulation results show that, piezoelectric structure by the topology optimization is very similar with leaf shape in nature. Therefore, several bionic piezoelectric blade structures are boldly optimized. The experiment further proves that the piezoelectric energy collection efficiency has a significant improvement compared with that before the optimization.

Keywords: vortex-excited vibration, topological optimization, vein bionics

I-INCE Classification of Subject Number: 42

1. INTRODUCTION

The rapid development of productivity has new requirements for the utilization rate of energy and reserves. The massive use of oil, coal and natural gas has caused certain pressure on the environment, so it is particularly important for the development of new energy. Energy collection is to transform the energy in the environment (such as wind energy, light energy, water energy, etc.) into energy that can be used. It can get rid of the limitations of batteries and places, has the advantages of durability and unmanned control^[1], and realizes the improvement of battery miniaturization, wireless sensors and other technologies^[2]. Therefore, the

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collection of wind energy has strong research significance in environmental protection, sustainable development and building safety^[3].

In daily life, mechanical vibration can be seen everywhere, so these vibrational energy is a very promising energy. So far, there are three main methods to collect vibration energy : (a) electrostatic; (b) electromagnetic formula; (c) piezoelectric formula^[4]. On the contrary, the piezoelectric structure is simple and the energy collection efficiency is high. Therefore, piezoelectric energy collection system has become an important research field for scholars who have designed the mechanism of the system (piezoelectric blade mechanism, circuit design and layout, and overall device mechanism, etc.) through vorticity induced vibration, flutter and other wind-induced vibration mechanisms.

Vortex-induced vibration^[5] is a common phenomenon of wind-induced vibration under Reynolds number conditions. The stable fluid passes through the streamlined bluff body, which can produce alternating vortex shedding after the bluff body, that is, the Karman vortex street. The bluff body generates alternating vortex pressure, and the bluff body generates vibration, which further affects the subsequent eddy current. Figure 1 below shows the eddy current diagram around the cylinder.



Figure 1: Vortex transfer diagram at the rear of a cylinder around a flow

With the development of topology optimization technology^[6], more and more researchers begin to use topology optimization methods to design piezoelectric energy collection structures in dynamic and static environments. At present, the continuous topology optimization methods mainly include homogenization method, variable density method, progressive structure optimization method (ESO), and level set method^[7]. By setting different design variables and using different objective functions, the objective function can reach convergence after a certain iteration. In this process, the increase and decrease of material elements can be realized, so as to reduce and add materials, optimize the structure and maximize the efficiency of piezoelectric energy collection. In the early days, the homogenization method proposed by Silva^[8] in 1999 was basically used. Silva and Kikuch^[9] then extended the work to piezoelectric sensor design. They proposed a piezoelectric material with penalization (PEMAP) model to describe the problem of eigenvalue optimization of piezoelectric sensor, which was actually an extension of SIMP model. Based on PEMPAP, Naksone and Silva^[10] introduced a new design variable to describe the polarization direction of piezoelectric material, and proposed a new piezoelectric material with penalization and polarization direction (PEMAP-P) model. Zheng^[11] uses topological optimization technique to design a piezoelectric energy collecting device, which optimizes the distribution of piezoelectric materials. Zhao Guozhong and Gu Yuanxian^[12] studied the finite element analysis of piezoelectric truss structure based on piezoelectric constitutive equation.

Leaves in nature can be regarded as thin plates. In order to resist the pressure of the external environment, such as wind and rain, the leaf vein structure is naturally

formed. In a certain sense, plant tissue is a kind of structure rather than material, and its mechanism is safe and reliable.

Therefore, we can optimize the structure of the piezoelectric energy collection device through topology optimization, and obtain the optimized piezoelectric reinforcement by imitating the structure of the blade vein. In this paper, we apply SIMP model^[13], the model for a simple theoretical model was established, the related formula deduction, the sensitivity analysis on simple shapes of piezoelectric patches simplified reaction, optimized structure, the optimized mechanism coupling field simulation analysis, contrast before and after the optimization of energy collection efficiency.

2. Model Formula Derivation

Before studying the finite element analysis of piezoelectric materials, it is necessary to deduce the piezoelectric material properties according to the existing theories^[14] and obtain the constitutive equation of piezoelectric materials

$$\begin{aligned}\sigma_{ij} &= C_{ijkl} \varepsilon_{kl} - e_{kij} E_k \\ D_i &= e_{ikl} \varepsilon_{kl} + \epsilon_{ik} E_k\end{aligned}\quad (1)$$

Simplified as a differential equation of displacement degrees of freedom and voltage degrees of freedom in the X, Y and Z directions, and obtained

$$\begin{aligned}T &= c_E S + e^T E \\ D &= eS + \varepsilon_0 \varepsilon_r S E\end{aligned}\quad (2)$$

Where S stands for strain, T for stress, E for electric field and D for electric displacement field. The material parameters S_E , d, and $\varepsilon_r T$ represent the flexibility, coupling properties and the relative dielectric constants of the material under constant stress respectively. ε_0 is the vacuum dielectric constant.

The weak form of electromechanical coupling field equation is shown as follows:

$$\begin{aligned}\int_{\Omega} \delta u^t f_v d\Omega + \int_A \delta u^t f_s dA &= \int_{\Omega} \delta S^t T d\Omega = \int_{\Omega} \delta u^t (C^E \cdot S - e^t \cdot E) d\Omega \\ \int_A \delta \phi \cdot q dA &= \int_{\Omega} \delta E^t D d\Omega = \int_{\Omega} \delta E^t (e \cdot S + \varepsilon^S \cdot E) d\Omega\end{aligned}\quad (3)$$

f_v and f_s are respectively the volume force and the external traction vector acting on the structure, u is the displacement vector, ϕ is the potential vector, q represents the surface charge density on the electrode.

Now interpolate the displacement and potential with N_u and N_{ϕ} into the cell displacement u_e and potential ϕ_e , strain S and electric field strength E can be written as:

$$\begin{aligned}S &= B \cdot u = B \cdot N_u \cdot u_e \\ E &= -B \cdot \phi = -B \cdot N_{\phi} \cdot \phi_e \\ (\int_{\Omega_e} N_u^t B^t C^E B N_u d\Omega) u_e + (\int_{\Omega_e} N_u^t B^t e^T B N_{\phi} d\Omega) \phi_e &= \int_{\Omega_e} N_u^t f_v d\Omega + \int_{A_e} N_u^t f_s dA \\ (\int_{\Omega_e} N_{\phi}^t B^t c B N_u d\Omega) u_e + (-\int_{\Omega_e} N_{\phi}^t B^t \varepsilon^S B N_{\phi} d\Omega) \phi_e &= -\int_{A_e} N_{\phi}^t q dA\end{aligned}\quad (5)$$

The angular symbol e represents the eth finite element. After assembly, for electromechanical coupling of the whole system, the matrix equations of piezoelectric material finite element are as follows:

$$\begin{bmatrix} K_{uu} & K_{u\phi} \\ K_{u\phi}^t & -K_{\phi\phi} \end{bmatrix} \begin{Bmatrix} \hat{u} \\ \hat{\phi} \end{Bmatrix} = \begin{Bmatrix} F \\ Q \end{Bmatrix} \quad (6)$$

Can before to make finite element analysis and topology optimization method together, we will continuous structure design of regional discrete units, and introduce the material attribute variables associated with each unit produced. $\chi(x)$ is the distribution function of the piezoelectric material. If there is a material in the selected region, the value of the function is one. If the material does not exist, the value of its function is zero. Among the topology optimization directions using variable density method, SIMP and RAMP are the most widely used density interpolation methods. In practical application, SIMP is slightly better than RAMP by comparing the expression effect of non-zero and one relative density.

In the piezoelectric material interpolation model of SIMP with penalty coefficient and polarization direction, the relative density of any finite element can be expressed as:

$$\varphi(x_i) = x_i^P, x_i \in [x_{\min}, 1], i = 1, 2, 3, \dots, n \quad (7)$$

The strain energy of the structure is:

$$W^S = u^t K_{uu} u \quad (8)$$

The electric energy generated by the structure is:

$$W^E = \varphi^t K_{\phi\phi} \phi \quad (9)$$

Then the energy conversion efficiency is:

$$\zeta = \frac{\varphi^t K_{\phi\phi} \phi}{u^t K_{uu} u + \varphi^t K_{\phi\phi} \phi} \quad (10)$$

$$u^t K_{uu} u = \sum_{i=1}^n x_i^P u_i^t k_{uu} u_i \quad (11)$$

$$\varphi^t K_{\phi\phi} \phi = \sum_{i=1}^n x_i^P \varphi_i^t k_{\phi\phi} \varphi_i$$

Based on SIMP piezoelectric material interpolation model with penalty coefficient and polarization direction, the piezoelectric material topological optimization model with maximum energy conversion efficiency under the volume constraint can be written as:

$$\text{Maximize: } \zeta_1 = \frac{\varphi^t K_{\phi\phi} \phi}{u^t K_{uu} u} \quad (12)$$

$$\text{or Minimize: } \zeta_2 = \frac{1}{\zeta} = 1 + \frac{u^t K_{uu} u}{\varphi^t K_{\phi\phi} \phi} \quad (13)$$

$$\text{Subject to: } \frac{V(x)}{V_0} = f \quad (14)$$

$$\begin{bmatrix} K_{uu} & K_{u\phi} \\ K_{u\phi}^t & -K_{\phi\phi} \end{bmatrix} \begin{Bmatrix} u \\ \phi \end{Bmatrix} = \begin{Bmatrix} F \\ Q \end{Bmatrix} \quad (15)$$

$$0 < x_{\min} \leq x \leq 1$$

Where $V(x)$ and V_0 respectively represent the volume size of the piezoelectric material and the volume size of the total design area, and f is the predetermined volume fraction.

Strain energy sensitivity analysis:

When the structure is in an equilibrium state, the finite element equilibrium equation must satisfy. Therefore, we define a new equation to describe the finite element equilibrium equation in the form of strain energy:

$$\Pi^{S^*} = \Pi^S = \frac{1}{2} u^t K_{uu} u + \lambda_1^t (K_{uu} u + K_{u\phi} \phi - F) + \mu_1^t (K_{\phi u} u - K_{\phi\phi} \phi) \quad (16)$$

Where lambda λ_1 and u_1 are any adjoint displacement or adjoint potential vectors, the first derivative of the above expression with respect to the design variable is:

$$\begin{aligned} \frac{\partial \Pi^{S^*}}{\partial x_e} &= \frac{\partial \Pi^S}{\partial x_e} = (u^t K_{uu} + \lambda_1^t K_{uu} + \mu_1^t K_{\phi u}) \frac{\partial u}{\partial x_e} + (\lambda_1^t K_{u\phi} - \mu_1^t K_{\phi\phi}) \frac{\partial \phi}{\partial x_e} \\ &+ \frac{1}{2} u^t \frac{\partial K_{uu}}{\partial x_e} u + \lambda_1^t \frac{\partial K_{uu}}{\partial x_e} u + \lambda_1^t \frac{\partial K_{u\phi}}{\partial x_e} \phi + \mu_1^t \frac{\partial K_{\phi u}}{\partial x_e} u - \mu_1^t \frac{\partial K_{\phi\phi}}{\partial x_e} \phi - \lambda_1^t \frac{\partial F}{\partial x_e} \end{aligned} \quad (17)$$

Since the design variables can be associated with the stiffness matrix in the SIMP model, it is very easy to obtain the derivatives of all the stiffness matrices. The last term in the above equation is the number of external loads on the structure, but the external load has nothing to do with the design variable, so its derivative is always zero. In order to eliminate the first derivative $\frac{\partial u}{\partial x_e}$ and $\frac{\partial \phi}{\partial x_e}$ of displacement and potential in the above equation, we set the adjoint vectors lambda λ_1 and u_1 as the solution to the following coupled equation.

$$\begin{aligned} K_{uu} \lambda_1 + K_{u\phi} \mu_1 &= -K_{uu} u \\ K_{\phi u} \lambda_1 - K_{\phi\phi} \mu_1 &= 0 \end{aligned} \quad (18)$$

The above equation is actually equivalent to solving the equilibrium equation when the external load F is $-K_{uu} u$. The sensitivity of the strain energy is obtained by solving for lambda λ_1 and u_1 and plugging it into the first order partial derivative equation of the strain energy.

3. Feasibility analysis of bionics

Structural similarity; In this paper, PVDF is used as the piezoelectric material. The piezoelectric material has the characteristics of high flexibility, small mass and high shape reinvention, which is similar to the leaf properties. Meanwhile, the PVDF sheet after reinforcement is like the leaf vein. Figure 2 shows the distribution of veins of the three types of leaves.

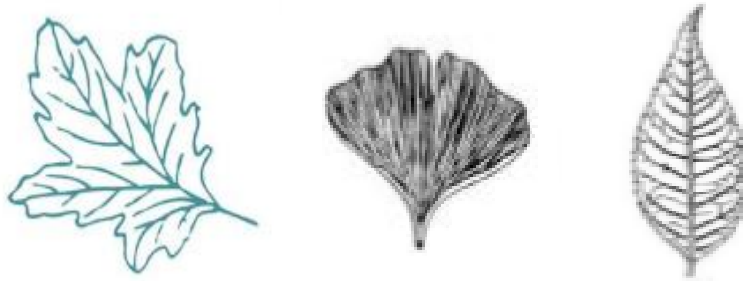


Figure 2: Veins distribution of the three types of leaves

Similarity of vein structure after optimization:

Due to the existence of the flow around the cylinder, there will be staggered distribution of vortices behind the cylinder, which will transfer the continuity

backward, and finally act on the piezoelectric plate, causing the pressure difference between the two sides of the piezoelectric plate and resulting in deformation. The potential difference between the two sides of the piezoelectric plate completes the process of energy collection. ^[15]When the flow field reaches a stable value and the flow rate reaches a stable value, the force exerted on the piezoelectric element can be simplified as a periodic force. Since the optimization studied in this paper can further improve the energy collection efficiency, a certain instantaneous moment is selected, that is, the force is a constant value when applied.

The palmate-shaped blade was used for example analysis, and the geometric parameters (size, thickness) and physical parameters (elastic modulus, poisson's ratio) of the model were set. At the same time, the external force was applied to simplify the load, and the volume fraction constraint, removal rate and convergence accuracy of the reinforcement were set. The results are shown in Figure 3 below:

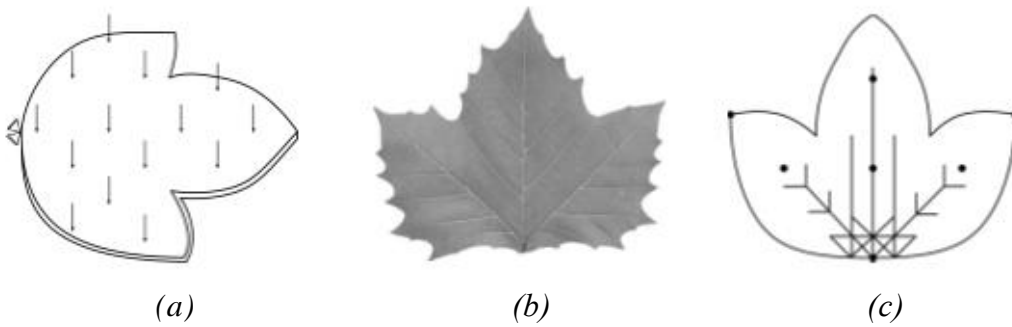


Figure 3: (a) simple diagram of leaf stress
(b) leaf original vein distribution diagram

(c) leaf vein distribution diagram was obtained after optimization

The optimized results showed that the optimized reinforcement pattern was very similar to the vein distribution pattern of the prototype plant leaves, so the bionic structure was feasible in this paper.

4. Topology Optimization Analysis

In this paper, a simple trapezoidal shape piezoelectric element is taken as an example. The Figure 4 below is a schematic diagram of a simple trapezoidal shape.

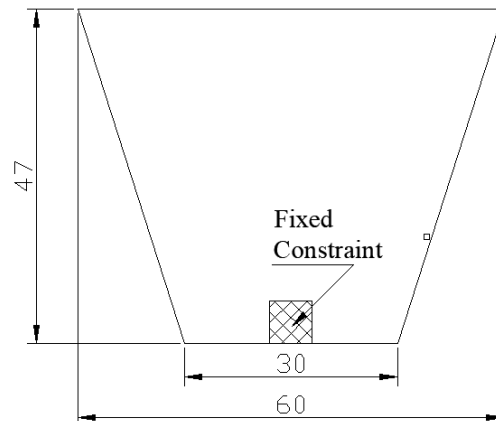


Figure 4: Schematic diagram of a simple trapezoidal shape piezoelectric wafer

Under the steady wind speed of 11m/s, the displacement and strain of each point are shown in Figure 5 and Figure 6 below:

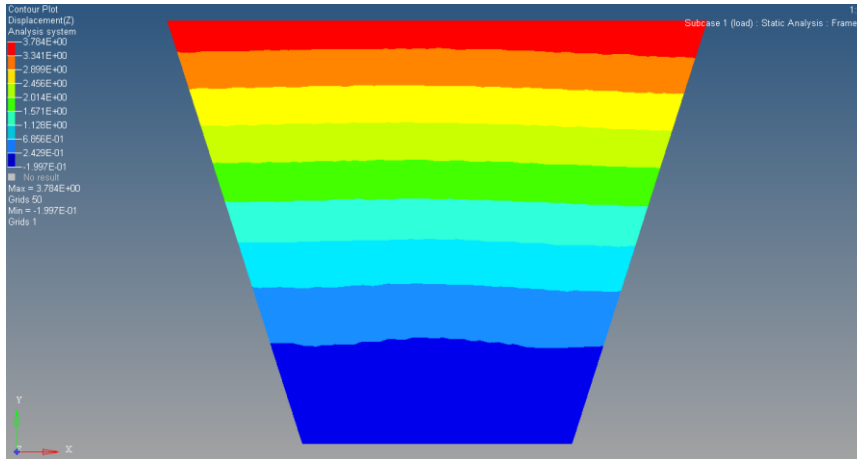


Figure 5: Displacement diagram of piezoelectric element

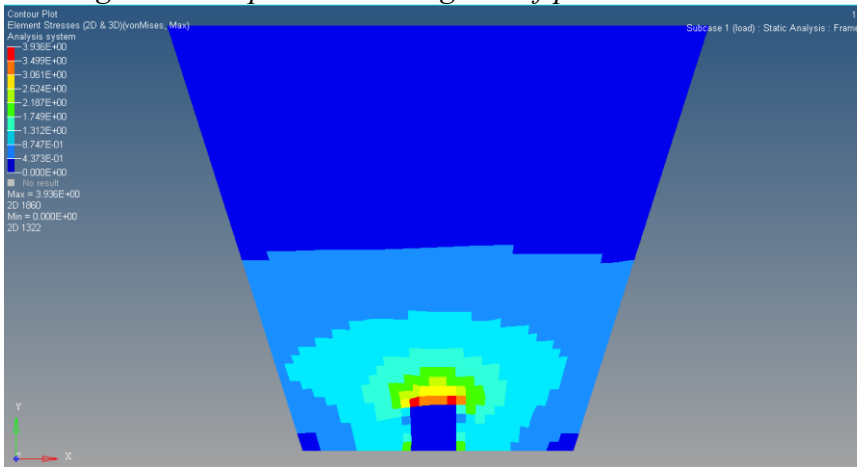


Figure 6: Strain diagram of piezoelectric element

Then, through COMSOL software and wind tunnel experiment, the time-domain analysis curve of the piezoelectric element is shown in Figure 7 below under the wind speed of 11m/s.

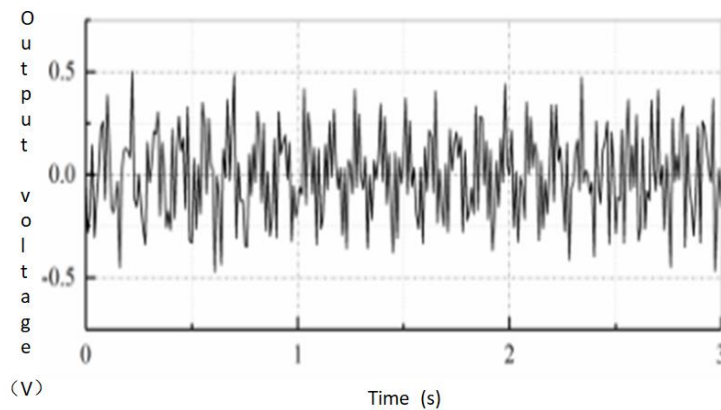


Figure 7: COMSOL simulation 11m/s piezoelectric element output open-circuit voltage time-domain curve

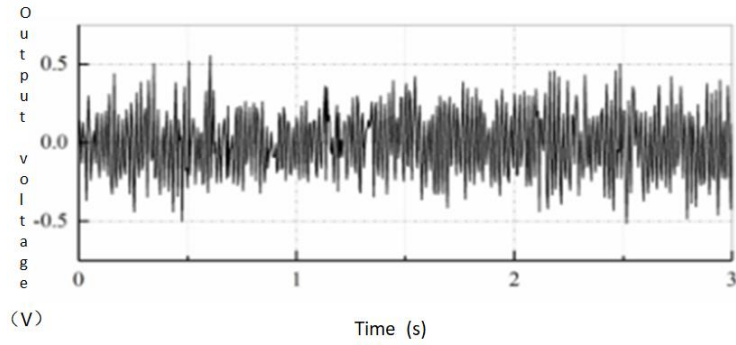


Figure 8: Time domain curve of output open circuit voltage of 11m/s piezoelectric element in wind tunnel test

Through comparison, it can be seen that the amplitude of the curve obtained by the two methods is about 0.5v, and the waveform is also similar. When compared to the experimental curve is more messy, the crest is not obvious. The simulation voltage is 0.22v and the test voltage is 0.212v by using the two methods of root-mean-square method, so the estimated results of the vortex street on the piezoelectric plate can be applied to the actual analysis.

The simulation results obtained through topology optimization simulation are shown in figure 9. The structure is very similar to the vein structure of the leaf above. After the optimized structure is cut out with similar shape after processing, the experimental analysis shows the output open-circuit voltage diagram as follows:

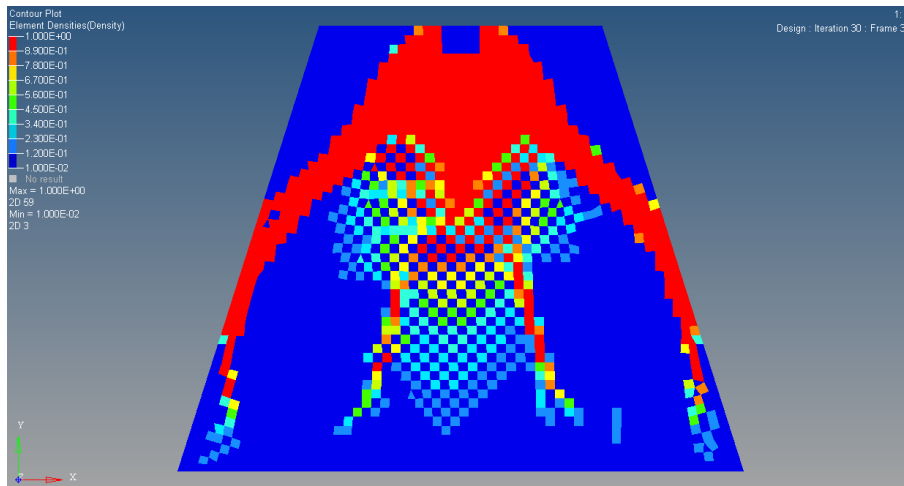


Figure 9: Topology optimization diagram of piezoelectric wafer

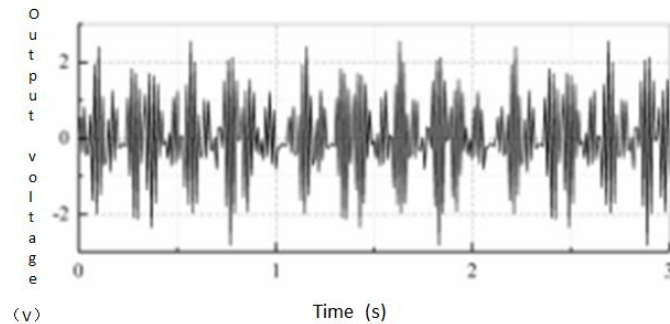


Figure 10: COMSOL simulation 11m/s topology optimization piezoelectric sheet output open circuit voltage

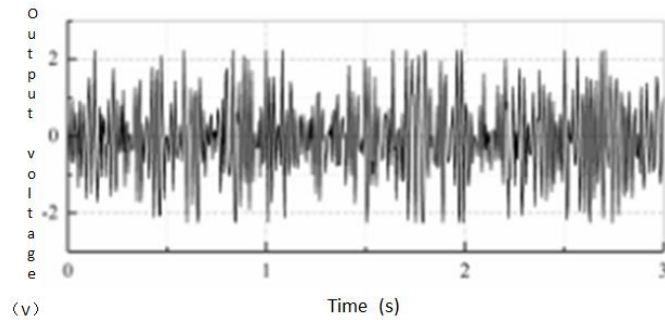


Figure 11: Wind tunnel test 11m/s topology optimization piezoelectric output open circuit voltage

The above figure shows that COMSOL simulation results are similar to the experimental results, with similar waveforms. The peak voltage is around 2.1v, and the open circuit voltage obtained by root mean square method is 0.85v, which is four times higher than the energy collection efficiency of the unoptimized piezoelectric element.

5. CONCLUSIONS

In this paper, there will be some errors in the process of analysis, which mainly occur in two aspects: wind speed fluctuation in the wind tunnel and noise in the signal test and analysis system. But the general trend of the results of the experiment error does not have any influence, simple trapezoidal shape before optimization of piezoelectric patches under the constant wind speed of 11 m/s around the peak output open circuit voltage of 0.5 V, open circuit voltage at around 0.2V, is obtained by topology optimization of piezoelectric patches after reinforcement in the same 11 m/s wind tunnel conditions to obtain the output of the open circuit voltage peak value is about 2.1 V, the open circuit voltage at around 0.85 V, energy collection efficiency obviously had more than four times, this method is simple and effective, has the significance of further study.

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