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Screening Criterion for Small Bore Attachments on Thin-Walled Pipes

Fahmy, Salah, Dr.[1]
AXI Group
28, Hamilton Crescent,
Bishopton, Renfrewshire, Scotland, United Kingdom

Gerges, Rafael[2]
LAEPI – Laboratory of Personal Protective Equipment
Joe Collaço Street, n. 491, Santa Mônica, Florianópolis, Santa Catarina, Brazil

ABSTRACT

This analysis provides a fatigue screening criterion for Small-Bore-Attachment on thin-wall pipes. The investigation was instigated by lack of industry standard of guidance on the issue. The analysis proposes a criterion that requires one-channel measurement - as opposed to the two-channels method usually applied. As well as specifying the level of excitation in terms of the SBA natural frequency, the analysis provides:
a) An estimate of fatigue life for a broadband random excitation of limited duration.
b) An estimate of the number of impacts a Small-Bore-Attachment would sustain from a transient type excitation.
A distinction is made between a Small-Bore Attachment (SBA) and a Small-Bore Branch (SBB). This analysis is concerned with SBA only.
Mathematical analysis is provided only to demonstrate how it is useful to have Math as a tool, in spirit of previous publications', though most conclusions can be intuitively obtained.

Keywords: Fatigue Failure, Environment, Vibration
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1. INTRODUCTION

Current industry standard approach for Small-Bore Attachments screening relies on EI Guidelines[3] whereby a combination of 10 generic arbitrarily sized small-bore attachments to header connections were modelled yielding a general acceptance criterion (that is not specific to a certain Small-Bore Attachment.

[1] sfahmy@talk21.com
[2] rafael@laepi.com.br

The screening rule is derived from a static analysis that gives the allowable vibration velocity in terms of the SBA natural frequency. The criterion is known to conservative. However, could be non-conservative when applied to thin-walled pipes (e.g. to 10S schedule pipes and below)[4]. In addition, the Guideline may not be applicable to thin wall header that are internally excited.

In general, thin-walled pipes are more prone to fatigue failure, in comparison to thick ones due to the following reasons:

- They are more susceptible to metallurgical change and material weakening during welding.
- They carry higher stress by virtue of their thin walls.
- They allow higher vibration loads to be transmitted from the relatively rigid SBA to the thin-walled pipe.
- Thin walled pipe tends to reduce the amount of material damping which has a large effect on fatigue life.
- A thin walled pipes exhibit deformation patterns that thick pipes do not.

A factor that increases their survival, is perhaps they are usually manufactured from high strength stainless, and therefore have higher endurance limit.

In deriving a screening criterion difficulty are encountered which prohibit the derivation of a single general criterion.

The first of these difficulties lies in the multiplicity of variables affecting the stress at a weld, namely[5]:

- Variability in geometrical properties.
- Variability in inertial properties.
- Variability in small bore branch - header combinations.
- Variability in excitation, direction, intermittency and spectra.

A further difficulty is the variability in the fatigue strength which is affected by factors such as weld profile, residual stresses, environmental degradation, heat affected zone and weld location or profile.

During a screening strategy (e.g. VRA), it is not known if loading may not change becoming random or intermittent. Accumulated damage that a particular SBC is also not known. There is also the question of variability in the fatigue strength due to the occurrence of micro cracks, leading to what may appear abrupt failure, for instance a single large transient could initiate a crack that subsequent smaller excitations could cause it to propagate or indeed vice versa[6].

Above factors may explain the absence of a generic design guideline for a small-bore attachment and the need to address the problem on a case-by-case basis, which, in some way is the objective of the present investigation[7].

2. APPROACH

The proposed strategy for screening of SBA's, considering the factors discussed above, is therefore as follows:

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[4] Investigation of Bruce Flare Header Small More Branch Failure, 01/06/2006.

[5] JMD publications on Small Bore Branch.

[6] There is a scenario in the nuclear industry for fault beyond design basis which postulates that the main shock of an earthquake would initiate a crack while aftershocks, which usually continues for longer period of time result in the propagation of crack resulting in a catastrophic Loss-of-Coolant Accident (LOCA). A similar situation could arise in a petrochemical installation in a variety of situation (e.g. Large Transient, Wave slamming, supply boat impact, etc.)

[7] InterNoise Aug 2016, Hamburg, Germany

- Identifying a database of SBA's–header combinations and carry out a finite element stress analysis to derive a numerical screening criterion.
- Estimating fatigue life to intermitted broadband random excitation.
- Establishing the number of such transients that result in fatigue failure.

For a combination that is vastly unique, a separate FEA will need be carried out.

3. THEORY

3.1 Small Bore Attachment Response to Constant Amplitude Excitation

The force acting on the SBA is equal to its modal mass times the absolute acceleration of the small bore. Denoting the hot spot stress per unit force at the weld by S_0, the stress S can be written as

$$S = S_0 . m . a_b \tag{1}$$

Equating the stress S in Equation to the allowable fatigue strength, S_a and solving for a_b, gives

$$a_b = \frac{S_a}{m . S_0} \tag{2}$$

The acceleration, therefore, is inversely proportional to the equipment mass, such that a small-bore attachment carrying a lighter mass would benefit more from a small reduction in the mass in comparison to a heavier equipment. Substituting m for $(k/\omega n2)$, where k is the SBA stiffness, one obtains,

$$a_b = \frac{\omega_n^2 . S_a}{k . S_0} = \frac{(2.\pi.f_n)^2 . S_a}{k . S_0} \tag{3}$$

The benefit of using Equation (3) is that it does not require a prior knowledge of the equipment's mass[8], only the frequency, (which is measurable), and the branch stiffness which could be easily obtained. Casting the dynamic equation in terms of k and S_0 is a key factor in this approach. In terms of velocity

$$v_{br} = \frac{\omega_n . S_a}{k . S_0} = \frac{2.\pi.f_n . S_a}{k . S_0} \tag{4}$$

If the logarithm of Equation (4) is taken, the resulting equation is similar to the familiar Wachel & Bates criteria, with the exception that while Wachel & Bates describes the velocity as a function of the square root of the branch frequency, Equation (4) gives a linear relationship. It should be noted that while Wachel & Bates is empirically derived, this formulation is derived analytically.

3.2 Fatigue Life Prediction for Random Excitation

For broadband random header excitation of limited duration, the time to fail is calculated from the following expression (See Appendix A),

$$T . f_n . \left(\frac{S_0 . k . a_b}{(2.\pi.f_n)^2}\right)^b = \frac{c/2^{b/2}}{\Gamma(1 + b/2)} \tag{5}$$

[8] The mass **m**, is the SBA modal mass, ~ 1.10 equipment mass.

or

$$T = \frac{c/2^{b/2}}{\left[\Gamma(1+b/2).f_n^{(1-2b)}.\left(\frac{S_0.k.a_b}{(2.\pi)^2}\right)^b\right]} \tag{6}$$

where

T	is the time to failure	[s]
f_n	is the branch frequency	[Hz]
S_0	is the stress per unit force	[(N/mm^2)/N]
a_b	is the SBA acceleration (rms)	[m/s^2
b and c	are constants associated with fatigue equation	
Γ	is the gamma function of argument (1+b/2)	

In this derivation above, the Palmgren-Miner fatigue damage summation rule is assumed.

Below the SBA acceleration level given by Equation 3, life time T, is infinite. Endurance limit of protected joints, the $(m + 2)$ rule should apply ($b_m = b + 2$), while for unprotected joints, the slope should remain at b and the life time is calculated from Equation 6.

The total number of cycles to failure $N = T.f_n$ is equal to the total number of cycles for crack initiation N_1 plus the total number of cycles for crack propagation to failure N_2,

$$i.e. \quad N = N_1 + N_2 \tag{7}$$

Gamma function for $b = 3$, i.e. $\Gamma(2.5)$, is $0.75(\pi)^{0.5} = 1.33$.

3.3 Fatigue Assessment due to Intermittent Pipework Excitation

Fatigue damage resulting from intermittent transient header excitation is derived from first principles. It should be noted that EI Guidelines do not address fatigue damage resulting from such events which is practically the case for non-vibrating SBA (i.e. located on dry lines).

The fatigue damage D resulting from of a single header excitation event is derived in Appendix B. The fatigue damage is shown to depend on the initial velocity of impact to the power b, given by

$$D = \left(\frac{f_n.T}{b.c}\right).(S_0.\omega_n.m)^b.V_0^b \tag{8}$$

where

D	is the fatigue damage	
f_n	is the branch out-of-plane natural frequency	[Hz]
T	is the response time	[s]
b and c	are constants associated with fatigue equation	
S_0	is the hot spot stress per unit force	[(N/mm2)/N]
ω_n	$= 2.\pi.f_n$	[rad/s]
m	is the flange and equipment mass	[kg]
V_0	is the initial velocity of small-bore branch due to header acceleration impulse	[mm/s]

Equation (8) gives an estimate of the fatigue damage occurring as a result for one transient. For a number of transients N_T, (e.g. flare blowdown) the total fatigue damag

$$D_{Total} = N_T.D = \left(\frac{N_T.f_n.T}{b.c}\right).(S_0.\omega_n.m)^b.V_0^b \tag{9}$$

Failure occurs when the total accumulative damage, D_{Total} approaches unity. Substituting $D_{Total} = 1.00$ in Equation (9), the following functional relationship between the SBA impact velocity and the number of transients, N_T is obtained

$$N_T = \frac{b.c}{[V_0^b.(S_0.\omega_n.m)^b.f_n.T]} \tag{10}$$

It can be easily shown that the product $f_n.T$ is equal to $\frac{1}{2}.\pi.\zeta$ while the product $S_0.\omega_n.m$ is equal to $k.S_0/\omega_n$. Substituting in Equation (10), the following equation is obtained

$$N_T = \frac{2.\pi.b.c.\zeta}{\left[V_0^b.\left(\frac{k.S_0}{2.\pi.f_n}\right)^b\right]} \tag{11}$$

Some numerical values are given below

$b = 3$		
$c = 1.726.10^{12}$	for weld Class F at mean level	[(N/mm^2)^3]
$c = 0.630.10^{12}$	for weld Class F at (mean-2σ) level	[(N/mm^2)^3]
$c = 0.380.10^{12}$	for weld Class F at (mean-3σ) level	[(N/mm^2)^3]

It follows that if the damping is halved, fatigue life reduces by a factor of 8 while if it is doubled, fatigue life increases by a factor of 8. Damping results mainly from hysteresis cycles and at high frequency, from air resistance and can be easily measured during a site visit.

4. FINDINGS AND APPLICATIONS TO ON-SITE PROBLEM

A database of small-bore attachments on thin wall-pipe is created and a generic conservative criterion is developed. This criterion applies to out-of-plane vibration. For a combination that does not exist in database, it will be necessary to carry out an FEA on a case-by-case basis. Based on experience, it is estimated that this will take two hours per SBA. Analysis of results gives the following empirical correlation of the product $k.S_0$,

$$k.S_0 = (300 - 0.9D/t).(d/90)^{0.5}.(200/h) \tag{12}$$

where

k	is the SBA stiffness	[N/m]
S_0	is the hot spot stress per unit force	[(N/mm^2)/N]
D	is the pipe outer diameter	[mm]
t	is the main pipe thickness	[mm]
d	is the diameter of the SBA at base	[mm]
h	is the small-bore branch height	[mm]

5. CASE OF STUDY

The application of the method is illustrated in below. It is worth noting that this specific SBA has failed, despite being screened by MTD, resulting in plant shutdown.

The 10S Sch10 pipe that failed has $\frac{D}{t} = \frac{273.05}{4.191} = 65.15$, $d = 90\ mm$ and $h = 280\ mm$. Substituting in Equation (12) gives

$$k.S_0 = 172.04\ (N/mm^3) \tag{13}$$

The frequency of the SBA was calculated using FEA at 49 Hz. During screening, the frequency of the SBA is also measured at 43 Hz.

Substituting in Equation (8), assuming S_a is equal to 20 (0-PK) N/mm2 for a Class F weld, the screening velocity above is calculated at 25.26 mm/s rms.

The value 20 (0-PK) N/mm2 is

$$S_a = 16.8\ N/mm^2 \tag{14}$$

The screening velocity therefore for this SBA was reduced to (16.8/20) x 25.26 = 21.22 mm/s (rms) which is below the EI Guidelines.

Equation (11) allows determining the number of transients to induce failure as a function of impact velocity (see Appendix B)

The prediction of fatigue life under random variable-amplitude loading of limited duration can also be determined from above (Appendix A).

6. ACKNOWLEDGEMENTS

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7. REFERENCES

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APPENDIX A

DYNAMICS OF SMALL BORE ATTACHMENT

Consider $\boldsymbol{x_H}$ to be the motion of the header and $\boldsymbol{x_B}$ to be the motion of the branch line *relative* to the header. Because of the offset of the branch mass, this would induce a bending moment which would in turn result in the header shell to deform an angle θ. The small-bore branch therefore has two components, one due to flexing of fitting, assuming infinitely rigid header while the other is due to rotation at the header connection, assuming infinitely rigid fitting. The equation of motion of mass of the connection is given by:

$$m.a_t + k_B.x_B = 0 \tag{15}$$

where

a_t	is the total acceleration of the connection centre of mass, which is the second derivative of the total displacement, x_t given by

$$x_t = x_B + x_H \tag{16}$$

$$x_B = x_{B0} + L.\theta \tag{17}$$

m	is the modal mass of the small-bore fitting
k_B	is the modal stiffness of the branch fitting
L	is the length between the centre of mass and header
x_{B0}	is the movement of the branch for infinitely stiff header

Equilibrium of the branch fitting alone, neglecting its inertia, yields

$$k_{B0}.x_{B0}.L = k_{B\theta}.\theta \tag{18}$$

where

k_θ	is the header shell stiffness	[N.m/rad]

Substituting Equations (17) and (18) into equation (15) results in the following equation of motion of branch:

$$a_B + \omega_n^2.x_B = -a_H \tag{19}$$

where

$$\omega_n^2 = \frac{k_B}{m} \tag{20}$$

$$k_B = \frac{k_{B0}}{(1 + k_{B0}.L^2/k_\theta)} \tag{21}$$

a_H	is the header acceleration

It should be noted that the dynamic force exerted on the header due to branch line vibration is given by $k_B.x_B$ which, from Equation (1), is also equal to $m.x_t$[9].The solution of Equation (19) for harmonic header acceleration is given by:

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[9] See Section 6 for a clarification as to how this force is incorporated in the calculation of dynamic stresses.

$$(-\omega^2 + 2.i.\zeta.\omega.\omega_n + \omega_n^2).x_B(i.\omega) = -a_H(i.\omega) \quad (22)$$

where a damping term, $(2.i.\zeta.\omega.\omega_n)$ is introduced.

In order to solve Equation (22), it is necessary to define the header excitation. For the purpose of this investigation, it will be assumed that the branch line is wholly excited by the header and that the vibration is random. This assumption does not limit the application of the findings to any other type of excitation. For example, for harmonic excitation, the SBC response is given by,

$$a_B = \frac{a_H}{[(1-\beta^2) + 2.i.\zeta.\beta]} \quad (23)$$

If it is assumed that the excitation spectrum has a broad band in the vicinity of the resonance frequency of the SBA and that the branch fitting is lightly damped such that it peaks steeply at resonance, the mean square response of the branch fitting $k_B < x_B >$, can be obtained in terms of the power spectral density of the header acceleration, $G_H(f)$[10], as follows

$$k_B^2 < x_B^2 >= \pi^2.m^2.f_n.G_H(f)/\zeta \quad (24)$$

Where $f_n = \omega_n/2\pi$

$$G_H(f) =< a_H^2 >/\Delta f_H \quad (25)$$

Δf_H is the band width of excitation, with the symbol $< x >$ signifies temporal average of the variable x

The expression derived above gives the mean square force on the header, $< F^2 > = k_B^2 < x_B^2 >$ for random excitation. To allow for the time dependent aspect of this random excitation, a time response function enveloping the spectral acceleration can be included if required. Peak values of response can also be estimated. In the calculation, a pseudo peak value of 1.414 times the RMS value of F, $(< F^2 >)^{1/2}$, is assumed.

It is important to note that for this type of excitation, the branch line fitting vibrates at its “natural frequency”- with random but slowly varying amplitude [2].

The forgoing analysis quantifies the effect of the different parameters on the dynamic force and gives some insight to the problem. For instance, the analysis shows tha the dynamic force per unit header acceleration is inversely proportional to the square root of the damping ratio, $\boldsymbol{\zeta}$ and directly proportional to the equipment mass, $\boldsymbol{m}$ and the square root of the SBA natural frequency, $\boldsymbol{f_n}$

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[10] For the derivation of this expression, see for example Ref. [2].

The stiffening of the fitting therefore would lead to an increase in the force transmitted to the header. In practice however, this *does* not happen as the introduction, for example of bracing, would 'split' the path of force leading to a reduction in the force transmitted from the connection to the header and therefore the stress. Further, the addition of any reinforcement would increase the effective wall thickness and this would offset the increase in the dynamic force.

The forgoing analysis addresses the dynamic force acting on the header rather that the resulting stresses. Dynamic stress would depend upon other geometrical parameter of the branch-header combination such as header wall thickness, branch fitting to header diameter, weld contour etc. Therefore, in comparing two different fitting-header combinations, the one with higher natural frequency or higher mass should not necessarily result in higher dynamic stress.

In the Finite Element Analysis, the rms dynamic stress (per unit header rms acceleration) is obtained by calculating the stress due to a unit force (applied at centre of mass of assembly) and multiplying the stress values obtained by the quantity,

$$[(\pi/2).\eta.m^2.f_n/(\Delta f_H.\zeta)]^2 \tag{26}$$

Where Δf_H from Equation (25) is set equal to $1/G_H(f)$ for unit rms header acceleration. The factor $\boldsymbol{\eta}$ is introduced to allow for the finite bandwidth of excitation and is a function of system parameters and excitation bandwidth [2].

APPENDIX B

Mathematical Derivation of Fatigue Damage Resulting from Intermittent Impact Loads

The equation of motion of a secondary mass spring system resulting from a primary system excitation can be written as:

$$m.\frac{d^2x}{dt^2} + c.\frac{dx}{dt} + k.x = -m.a_H \tag{27}$$

where

m	is the secondary system modal mass	[kg]
c	is the secondary system damping coefficient	[N/mm/s]
k	is the secondary system stiffness	[N/mm]
x	is the relative displacement	[mm]
a_H	is the primary structure acceleration	[mm/s^2]

The solution of the Equation of motion (27) due to a transient excitation, assuming small damping, is given by[11]

[11] W.T. Thomson, Theory of Vibration with Application, George Allen & Unwin, 1981.

$$x = \left(\frac{V_0}{\omega_n}\right). e^{-\zeta\omega_n t}. \sin(\omega_n t) \quad (28)$$

where

V_0	is the initial velocity due to impact	[mm/s]
ω_n	is the system circular frequency $= 2.\pi.f_n$	[rad/s]
ζ	is the damping ratio	

The force F is given by the product of the stiffness k and the displacement x, i.e. $F = k\ x$, which from Equation (27), can be written as

$$F = k.\left(\frac{V_0}{\omega_n}\right). e^{-\zeta\omega_n t}. \sin(\omega_n t) \quad (29)$$

Noting that $k = m.\omega_n^2$, Equation (15) can be written as

$$F = (m.\omega_n.V_0). e^{-\zeta\omega_n t}. \sin(\omega_n t) \quad (30)$$

If the stress per unit force is S_0 , the stress resulting from the primary system excitation, S can be written as,

$$S = S_0.F \quad (31)$$

which from Equation (15) is

$$S = S_0.k.x = S_0.k.\left(\frac{V_0}{\omega_n}\right). e^{-\zeta\omega_n t}. \sin(\omega_n t) \quad (32)$$

or from Equation (16) is,

$$S = S_0.(m.\omega_n.V_0). e^{-\zeta\omega_n t}. \sin(\omega_n t) \quad (33)$$

It should be noted that in Equation (31), the definition of the stress S as amplitude, range or rms value follows the definition of the force F with no effect on the value of the value S_0.

Equations (32) or (33) are equivalent and can be used interchangeably depending on the ease of obtaining the value of the stiffness or of the modal mass. Usually the mass m, is easier to determine than the stiffness k. In subsequent development therefore, Equation (33) will be used in favour of Equation (32).

If we let

$$S_1 = S_0.(m.\omega_n.V_0) \quad (34)$$

it follows from Equation (33) that

$$S = S_1. e^{-\zeta\omega_n t}. \sin(\omega_n t) \quad (35)$$

The maximum stress excursion due to an acceleration of short duration occurs at time = $\pi/2\omega_n$. Substituting into Equation (35), it follows that,

$$S_{max} = S_1. e^{-\zeta\omega_n \pi/2\omega_n}. \sin(\omega_n \pi/2\omega_n) = S_1. e^{-\zeta\pi/2} \quad (36)$$

In practice, the value $\zeta\pi/2$ is in the range 0.02 -0.05. Substituting in Equation (36), the value $e^{-\zeta\pi/2}$ is in the range 0.9 to 0.95. It follows that a conservative estimate of the maximum value of stress during any excursion, SD_{max} is equal to S_1 and the following simplified equation will be used in subsequent analysis:

$$S_{max} = S_0.(m.\omega_n.V_0) = S_0.(2.\pi.m.f_n.V_0) \tag{37}$$

In what follows, an attempt is made to estimate the fatigue damage occurring as a result of such excitation.

At each event, the secondary system will accumulate a certain amount of fatigue damage which can be reasonably estimated according the Palmgren-Miner rule. This rule is known to be conservative.

According to the Palmgren-Miner rule, the incremental damage δD due to an incremental number of cycles $\delta n(S)$ occurring between stress level S and $S + \delta S$, is given by

$$\delta D = \omega_n.(S)/N(S) \tag{38}$$

where $N(S)$ is the number of cycles at stress level S enough to induce fatigue failure in a constant-amplitude fatigue test with stress amplitude S, which can be estimated from a typical $S - N$ empirical relationship,

$$NS^b = C \qquad (C = 3) \tag{39}$$

The total damage resulting from one event of the primary system excitation is given by integrating the incremental damage δD resulting from an elemental number of cycles $\omega_n.(S)$, over the number of cycles at all stress levels, i.e.

$$D = \int dD = \int dn(S)/N(S) \tag{40}$$

To evaluate this integral, the differential $dn(S)$ is written as $(dn(S)/dS)\ dS$ and the distribution of the number of cycles n for different stress levels S is sought.

Substituting S^b/C for N from Equation (39), the evaluation of Equation (40) therefore reduces to

$$D = \int dD = \int S^b(dn(S)/dS)dS/C \tag{41}$$

The distribution of the number of cycle's n of the secondary system at any stress level S due to transient motion is worked out as follows:

We know that there is zero number of cycles over and above the maximum value determined by Equation (37), namely

$$(S)_{max} = S_0.(2.\pi.m.f_n.V_0) \tag{42}$$

There are also an infinite number of cycles at zero stress level. These two conditions permit constructing a function that represents the variation of the number of cycles with stress level during the small-bore vibration. A function that satisfies these two conditions is

$$n(S) = -f_n.T.\ln[S/(S)_{max}] \quad (43)$$

for which

$$(dn(S)/dS) = -f_n.T/S \quad (44)$$

where T is the response duration and is proportional to $1/f_n\zeta$.

Substituting Equation (44) into Equation (42) and integrating between $S = 0$ and $S = (S_D)_{max}$, the following expression is obtained,

$$D = \int dD = -\left(f_n.\frac{T}{C}\right).\int S^{b-1}.dS = \left(f_n.\frac{T}{bC}\right).(S)_{max}^b \quad (45)$$

Therefore, the fatigue damage that the secondary system would suffer as a result of a single impact is given by

$$D = (f_n.T/b.C).S_0^b.(2.\pi.m.f_n.V_0) \quad (46)$$

which can be written as

$$D = (f_n.T/b.C).(S_0.\omega_n.m)^b.V_0^b \quad (47)$$

Equation (46) gives an estimate of the fatigue damage occurring as a result of one impact. For a number of such impacts N_T, the total fatigue damage D_{total}, is

$$D_{Total} = N_T.D = (N_T.f_n.T/b.C).(S_0.\omega_n.m)^b.V_0^b \quad (48)$$

According to the Palmgren-Miner hypothesis, failure will occur when D_{total} approaches the value 'one'. Substituting $D_{total} = 1.00$ in Equation (48), the following functional relationship is obtained between the SBC impact velocity and the number of transients, N_T

$$N_T = b.C/[V_0^b.(S_0.\omega_n.m)^b.f_n.T] \quad (49)$$

Or, if one denotes the stress per unit velocity of the SBC as S_0' (N/mm^2/m/s), Equation (49) becomes

$$N_T = b.C/\left[V_0^b.\left(S_0'\right)^b.f_n.T\right] \quad (50)$$

It should be noted that the product f_nT is equal to $1/2\pi\zeta$, thus Equation (50) simplifies to:

$$N_T = 2.\pi.b.C.\zeta/\left(V_0^b.S_0'\right)^b \quad (51)$$

If the value of the stress value S_0 is known, for example from a computer modelling of the system, Equation (51) can be written as

$$N_T = 2.\pi.b.C.\zeta/(S_0)^b \quad (52)$$