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Loss Factors Identification from E-SEA Techniques

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ABSTRACT

When designing a Statistical Energy Model (SEA) both the extension of the system with respect to the real structure/fluid and the connections between its parts are key issues in large complex systems. A previous analysis of its practical extent may allow simplifying the model to a smaller one while still taking into account all the significant energy from the paths of higher orders.

On the other hand, the analysis of the connections between the parts of the system, will help provide an adequate and accurate SEA matrix where all the connexions (even the non-resonant ones) are considered, and all the non-connected subsystems are identified. In this work, a methodology to identify and classify the coupling loss factors of a given system is proposed. This methodology is based on the application of pattern recognition techniques (such a clustering) to experimental results obtained through an experimental SEA. A description of the methodology is presented, and its advantages and performances are highlighted by an application case.

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1. INTRODUCTION

One of the most widespread numerical methods to model vibroacoustic systems at high frequency is the Statistical Energy Analysis (SEA) [1]. This method relates groups of modes of different nature from different parts of the system between them. At the end of the process a system of linear equations is obtained that describes the whole system and is useful to determine the response of the system. There are three main quantities involved in this method: input power into each of the subsystems, energy contained in them and the loss factors of the system.

Within a SEA model the loss factors quantify the relative power dissipated within a subsystem (internal loss factors, ILF) or the power transmission from one subsystem to another (coupling loss factors, CLF). The total power leaving a given subsystem is called total loss factor of this subsystem, TLF [1].

All the loss factors (LFs) of a system form a matrix that relates the energy in the system to the input power and that can be considered the system definition for an SEA model. This way, determining this SEA matrix becomes the key issue of SEA modelling.

Essentially, there is three different ways to obtain SEA LFs: analytical equations, numerical methods and experimental tests. Analytical equations exist only for simple subsytems so that the number of vibroacoustic systems that can be studied with them is small and likely not the most realistic.

An alternative to the theoretical equations is obtain the LFs by test. This set of methods is called Experimental SEA, E-SEA. Most of the current E-SEA methods are based on the Power Injection Method (PIM) [2, 3] in which a given power is driven into each of the subsystems consecutively. In this way a sufficient set of equations is obtained and all the LFs of the system can be determined. The drawbacks of this method are that it needs a whole set of equations/excitations of all the existing subsystems and the difficulty that can be found to excite some of them, for instance in-plane ones.

There are some numerical methods that can be used as well to obtain the LFs. Among them, the so-called Virtual SEA stands out. This is a numerical method based on a Finite Element Analysis, FEA, to simulate a PIM test and, from it, estimate the LFs of the SEA model.

Thereby, the PIM is a key method to be able to characterize a SEA model. Apart from the practical problems that arise when setting up a test based on the PIM (like the ones stated above) numerical difficulties can be involved as well. Two of the most common are the occurrence of results inconsistent with the properties a LF should fulfil, and the difficulty to identify non-connected subsystems due to the orders of magnitude involved.

In this work, a feasible methodology able to single out zeros in the SEA matrix, and consequently, to identify non-connected subsystems is presented. To that end, the use of a Monte Carlo Model is proposed together with clustering techniques. The Monte Carlo model provides a statistical sample whose properties may be used as input into clustering models that allow classifying the LFs of the system and identifying those pertaining to non-connected subsystems.

2. E-SEA METHOD

Essentially, an SEA model of a vibroacoustic system made up of n subsystems consists of a linear system of equations in which the input power into the different subsystems is related to the energy in these same subsystems (both in a given frequency band centred in

ω) by a coefficient matrix made up of LFs:

$$P_i = \omega \sum_{k=1}^{n} l_{ik} E_k \tag{1}$$

where P_i and E_k are the input power and energy, respectively, and $\mathbf{L} = (l_{ij})$ is the SEA matrix defined as:

$$l_{ij} = \begin{cases} \sum_{k=1}^{n} \eta_{jk} & i = j \quad \text{(2a)} \\ -\eta_{ji} & i \neq j \quad \text{(2b)} \end{cases}$$

being η_{ij} the loss factor from subsystem i to subsystem j.

For the SEA model to be completely defined the n^2 η_{ij} values must be known. An E-SEA model gets them by defining a set of n independent equations from Equation 1. To do that, each subsystem is excited individually in a sequential way. The result is written in matrix form as

$$\mathbf{P} = \omega \mathbf{L} \mathbf{E} \tag{3}$$

where $\mathbf{P}$ is a $n \times n$ matrix made of n input power combinations (by columns) and $\mathbf{E}$ is a $n \times n$ matrix whose columns are the energy of the system under each one of the input power configurations. $\mathbf{L}$ can be isolated from Equation 3 and the E-SEA model solved.

3. NON-CONNECTED SUBSYSTEMS IDENTIFICATION

E-SEA matrix systems usually involve matrices whose entries range several orders of magnitude. As a consequence their condition number is high and numerical problems arise. Another consequence is that as some low LFs with low value (between very weakly coupled subsystems) are expected, these ones are not distinguished from the ones corresponding to non-coupled subsystems.

The aim of this work is to identify these non-connected subsystems. To this end, it is necessary to select a set of characteristics of the different LFs that allows extracting the information needed. The set of parameters chosen is statistically obtained by a Monte Carlo model [4].

In the Monte Carlo model used the input parameters are the energy states for each of the input powers driven into the system. A Gaussian distribution is assumed for which the mean and variance are determined from the E-SEA tests. Once the Monte Carlo results sample is obtained, the average, variance, skewness, kurtosis or any other statistical parameter of each one of the loss factor η_{ij} can be calculated.

Each of them will have a different statistical distribution parameters depending on their nature. These parameters (or a subset of them) can be used to group the different η_{ij} according to this nature. An efficient way to do it is by using clustering techniques as the *K-means* method [5–7].

This method obtains a partition of a set of d-dimensional points such that the squared error between the calculated mean of a cluster and the points in the cluster is minimized. In the case of interest each point represents a given η_{ij} and its dimension d is the number of statistical parameters used. If μ_k is the mean of cluster c_k the squared error between μ_k and the points in c_k is

$$J(c_k) = \sum_{\eta_{ij} \in c_k} \left\| \eta_{ij} - \mu_k \right\|^2 \tag{4}$$

The aim of *K-means* is to minimize the sum of the squared error over all K clusters, that is:

$$J(C) = \sum_{k=1}^{K} \sum_{\eta_{ij} \in c_k} \left\| \eta_{ij} - \mu_k \right\|^2 \tag{5}$$

Since the squared error always decreases with an increase in the number of clusters K, this number must be stated in advance. The algorithm to solve this problem is iterative according to these four steps:

1. Chose an initial partition with K clusters of your set
2. Make a new partition by assigning each element to its closest cluster center
3. Compute new cluster centres
4. repeat until cluster membership is stable enough

In this work, the *K-means* method is applied to a box described in the next section. In this application case a set with $d = 2$ is good enough to identify the set of non-connected LFs.

4. APPLICATION CASE

The system under study is a rectangular box made up of six panels (see Fig. 1) whose dimensions are shown in Table 1. The faces are substructures line-connected through the edges. The internal loss factors of all the plates are assumed to be 0.05, and the effect of the air within the box is neglected (that is, no acoustic subsystem is considered). Only the bending field is considered, therefore the system has only six different subsystems.

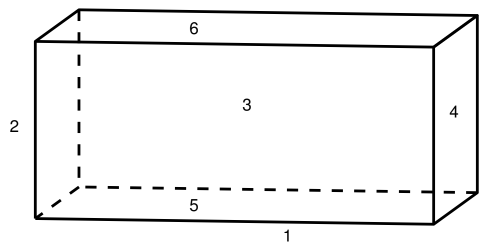


Figure 1: System sketch

To define the base case, coupling loss factors are defined for all the joints, in both directions. Then, the energy state of the system in third octave bands, $\mathbf{E_0}$, is determined for a power matrix $\mathbf{P_0}$.

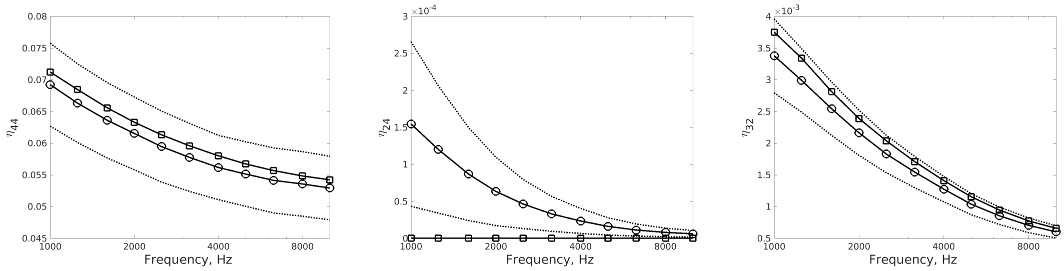
For the Monte Carlo model, gaussian distributions are assumed for both $\mathbf{P}$ and $\mathbf{E}$ with a relative standard deviation of 10%. The Monte Carlo sample size is set to 10^5. From the

Table 1: Geometry of the system.

Panel	Height, (*m*)	Length, (*m*)	Thickness, (*mm*)
1, 3	0.7	1.5	10
2, 4	0.7	0.78	10
5, 6	0.78	1.5	20

resulting sample, the set $\{\mathbf{L_i}\}$ is obtained by means of Equation 3 and those instances that contain non-positive η_{ij} are rejected. As a result, only about the 3% of the whole sample gives valid SEA matrices.

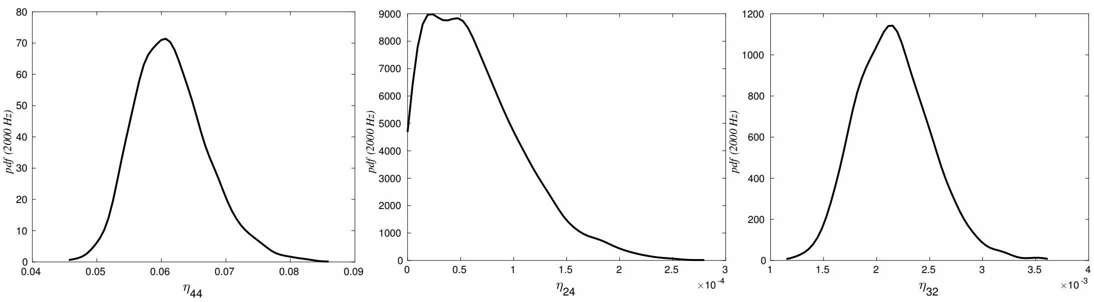
Fig. 2 shows the results of the η_{ij} for some of the loss factors: one ILF, the CLF between two non-coupled subsystems and the CLF between two adjacent faces. In general the relative standard deviation is about 10 to 15 %. An outstanding exception are those results corresponding to non-coupled subsystems that have spreads over 60 %.



(a) *(b)* *(c)*

Figure 2: η_{ij} obtained from the Monte Carlo model. (−○−) mean value; (−□−) reference; (·······) upper and lower 1-σ limits

Results obtained for the LFs can be analysed also in terms of their probability density function, *pdf* for each frequency band. Fig. 3 shows the probability density function of the same loss factors presented in Fig. 2 obtained by a kernel density estimation for the 2 kHz frequency band. It can be observed that the *pdf* of the η_{ij} corresponding to two non-connected subsystems have a great skewness as, for instance, η_{24}.



(a) *(b)* *(c)*

Figure 3: Probablity density function of the η_{ij} of (a) subsystem 4, (b) two non-connected subsystems, and (c) two connected subsystems.

The difference between the *pdf* of connected and non-connected subsystems provides a way to identify these latter subsystems by the use of techniques as the *K-means* clustering that allows partitioning the set of results in K clusters. In this case, the parameters that define our axes are the mean value and skewness of the η_{ij} distributions. The algorithm groups the η_{ij} values as a function of the distance among all of them. Each group is defined by its centroid and the points associated. The results of a cluster analysis based on the mean value and skewness of η_{ij} for the 2 kHz band are shown in Fig. 4. Three clusters can be observed where connected and non-connected subsystems are isolated.

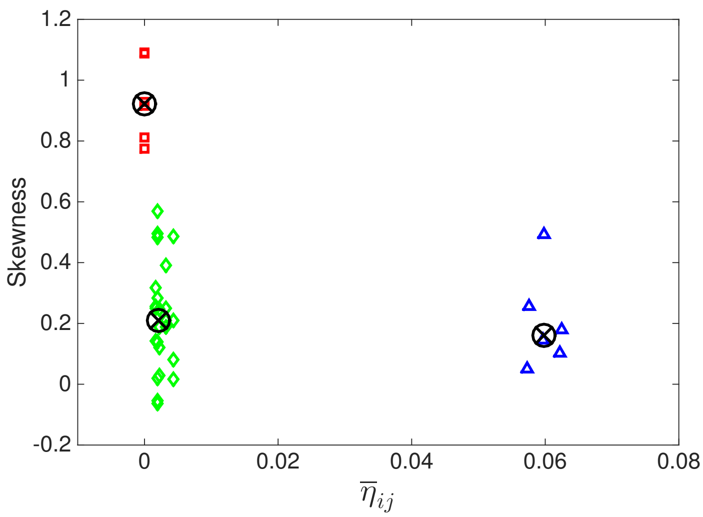


Figure 4: Cluster analysis for the 2 kHz frequency band. (−△−) TLFs; (−□−) non-connected CLFs; (− ◇ −) connected CLFs.

5. CONCLUSIONS

Although the E-SEA approach seems to be simple –a linear system of equations–, it gives rise to a set of numerical problems that make it difficult to deal with. One of these problems is the difficulty to identify non-connected subsystems that should have $\eta_{ij} = 0$. On one hand obtaining a zero from a numerical method is almost impossible and on the other hand, the typical values of the CLFs are so small that distinguishing weakly-coupled from non-coupled subsystems is very difficult.

In this work, a useful method to identify non-connected subsystems has been presented. It uses a set of statistical characteristics obtained from a Monte Carlo simulation based on the result of an E-SEA model.

Once the statistical characteristics of the different η_{ij} are obtained, a widespread clustering method, the K-means, is used to classify them in such a way that all the non-connected subsystems are recovered.

This method is applied to a numerical case consisting of a structural box. The E-SEA model is posed, then the Monte Carlo simulation is carried out and once the non-consistent results are neglected, the statistical parameters are obtained. From this set it has been seen that only two of them –mean and skewness– are sufficient to single out the required CLFs.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

[1] A. Le Bot. *Foundation of Statistical Energy Analysis in Vibroacoustics*. OUP Oxford, 2015.

[2] Brandon C. Bloss and Mohan D. Rao. Estimation of frequency-averaged loss factors by the power injection and the impulse response decay methods. *The Journal of the Acoustical Society of America*, 117(1):240–249, 2005.

[3] M.J. Fernández, F. Simón, and M. Chimeno. Uncertainty in the measurement of coupling loss factors using the power injection method. In *internoise*, 2010.

[4] D.P. Kroese, T. Taimre, and Z.I. Botev. *Handbook of Monte Carlo Methods*. Wiley, 2011.

[5] H. Steinhaus. Sur la division des corp materiels en parties. *Bull. Acad. Polon. Sci.*, 4(12):801–804, 1956. cited By 339.

[6] J. MacQueen. Some methods for classification and analysis of multivariate observations. In *Proceedings of the Fifth Berkeley Symposium on Mathematical Statistics and Probability, Volume 1: Statistics*, pages 281–297, Berkeley, Calif., 1967. University of California Press.

[7] Anil K. Jain. Data clustering: 50 years beyond k-means. *Pattern Recognition Letters*, 31(8):651 – 666, 2010. Award winning papers from the 19th International Conference on Pattern Recognition (ICPR).