MADRID
inter.noise 2019
June 16 - 19
NOISE CONTROL FOR A BETTER ENVIRONMENT

# A parameter mapping technique-based interval perturbation method for predicting the uncertain acoustic field in the enclosure with nonlinear interval parameters

**Wang, Haitao[1], Zeng, Xiangyang[2], Ma, Xiyue[3], Chen, Kean[4]**
**School of Marine Science and Technology, Northwestern Polytechnical University**
**P. B. 58, Youyixi Road, Beilin district, Xi'an, China**

**ABSTRACT**

**Classical wave-based acoustic simulation are usually conducted under the assumption that the parameters are deterministic. While in real engineering, uncertainties on environmental parameters are often unavoidable. Thus, the uncertain acoustical simulation has gradually been a research focus. Because of the effectiveness and simplicity, the interval perturbation method has attracted sustained attentions and been used in many uncertain acoustic problems in recent years. However, the commonly used first order approximation in the perturbation method would no longer be a feasible strategy due to omitting the high orders of the Taylor series when the uncertain parameters are nonlinear. To solve the uncertain problems with nonlinear interval parameters, a parameter mapping technique is proposed in this paper. This technique firstly constructs a function with respect to a nonlinear parameter. Then, by mapping the initial interval of the nonlinear parameter into the newly constructed function space, this function can be considered as a new independent parameter. Finally, the uncertain acoustical problems can be solved by the classical interval perturbation methods in which the first order approximation is capable of giving precise results. The numerical verifications demonstrate that the proposed technique is effective for uncertain acoustic field simulation with nonlinear interval parameters.**

**Keywords:** wave-based simulation, nonlinear interval parameter,
**I-INCE Classification of Subject Number:** 76

## 1. INTRODUCTION

The small enclosure, for example, the cabin of an airplane, is a type of environment in people's daily life that usually requires high sound quality and low noise level. Prediction on the acoustic behaviour inside the small enclosure has been a basic step in the design of such space. The wave-based method are widely for solving low-frequency acoustic field. Classical wave-based acoustic simulation has been conducted under the assumption that the environmental parameters and boundary conditions are

[1] wht@nwpu.edu.cn
[2] zenggxy@nwpu.edu.cn
[3] maxiyue@nwpu.edu.cn
[4] kachen@nwpu.edu.cn

deterministic.[1] However, in the real engineering, due to the environmental changes, physical imperfections and system complexities, uncertainties in environmental parameters, material properties, and boundary conditions are unavoidable, which will lead to the uncertainty of the acoustic field. The uncertain analysis in the acoustic simulation has been a challenging problem and a research focus in recent years.

Typical uncertain acoustic simulations can be divided into three categories, i.e., stochastic analysis, fuzzy analysis and the interval analysis. In the stochastic analysis, probabilistic methods are most commonly used methods. This type of method requires that the uncertain parameters have known probability density functions. Then, the statistical characteristics of the stochastic response can be calculated[2-4]. The fuzzy model is another important type of uncertain analysis method. In this type of method, the uncertain parameters are defined as the fuzzy variables with known fuzzy membership function. The fuzzy response can be obtained based on the fuzzy variables[5-7]. Essentially, the aforementioned two types of methods both are probabilistic methods. Thus, a large amount of information is required to obtain the pre-knowledge of the probability density function and the fuzzy membership function of the uncertain parameters. Unfortunately, it is too difficult or costly to collect sufficient information about the uncertain parameters in the early stage of design. Consequently, the results are often unreliable due to the lack of samples[8].

To treat with uncertainties with limited information, an important model, namely the interval model, has been becoming increasingly popular in the uncertain analysis. Compared with the probabilistic methods, the interval technology just requires the variation range of the uncertain parameters as the input information. Thus, it is more appropriate for the numerical analysis of non-probabilistic systems without sufficient information. To solve a interval problem, the Monte Carlo method (MCM) which employs random sampling of the interval parameters to predict the response range is the simplest and most robust approach. However, the computational cost of MCM is usually too high to be acceptable for large-scale engineering systems, and thus its applications are limited. The vertex method is another simple method to solve an interval problem. It achieves the exact response intervals by using all possible combinations of the interval parameters. In such calculations, the non-deterministic problem is transformed into a deterministic one. However, the computational effort of this method increases exponentially with an increase in the number of uncertain parameters. To obtain better performances for both the computational accuracy and efficiency, the interval perturbation method is presented[9]. In this method, the interval system matrices and their inverse matrices are expanded by the Taylor series and the Neumann series, respectively. By retaining the first order and neglecting the high order terms in the Taylor series, the perturbation analysis can be easily implemented by the standard wave-based methods.

Because of the effectiveness and simplicity, the interval perturbation method has attracted sustained attentions and been used in many uncertain acoustic problems in recent years[10-13]. However, it also should be noted that the first order approximation in the perturbation method would no longer be a feasible strategy when the uncertain parameters are non-linear in the system matrices due the omissions of the high orders of the series. To solve the uncertain problems with nonlinear interval parameters, a is proposed in this paper. This technique firstly constructs a function with respect to a nonlinear parameter. Then, by mapping the initial interval of the nonlinear parameter into the newly constructed function space, this function can be considered as a new independent parameter. Finally, the uncertain acoustical problems can be solved by the classical interval perturbation methods in which the first order approximation is capable of giving precise results.

## 2. BASIC WAVE-BASED ACOUSTIC SIMULATION FOR ENCLOSURE

Assuming there is a sound source in the enclosure. The mass of the medium provided by the source in unit time is $\rho_0 q$. The acoustic wave equation with a source is given by

$$\nabla^2 p - \frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} = -\rho_0 \frac{\partial q}{\partial t} \tag{1}$$

where t denotes time, p is the sound pressure, $c_0$ is the speed of sound in air, $\rho_0$ is the equilibrium density of air, and q is the volume velocity of the sound source.

Considering the system in the frequency domain, the Helmholtz equation is expressed as follows,

$$\nabla^2 p_\omega + k^2 p_\omega + \mathrm{j}\rho_0 \omega q_\omega = 0 \tag{2}$$

where $k$ denotes the wave number and is defined by $\omega / c_0$, $\omega$ is the circular frequency, j is the imaginary unit, $p_\omega$ and $q_\omega$ are the sound pressure and the intensity of the sound source in the frequency domain.

The frequently used boundary conditions in the acoustic field simulation usually can be classified into the following two types:

$$\begin{cases} \dfrac{\partial p_\omega}{\partial n} = -\mathrm{j}\rho_0 \omega \dfrac{p_\omega}{Z_s}, \ \Gamma_1 \\ \dfrac{\partial p_\omega}{\partial n} = 0, \ \Gamma_2 \end{cases} \tag{3}$$

where $n$ denotes outward normal direction, $Z_s$ is the specific acoustic impedance of the boundary, $\Gamma_1$ denotes the damping boundary, and $\Gamma_2$ denotes the rigid boundary.

By dividing the problem domain into discretizing form, the sound pressure at any position (for example, $r$) in the problem domain is expressed by

$$p_r = \boldsymbol{N}_r^{\mathrm{T}} \boldsymbol{p} = [N_{r1}, N_{r2}, \cdots, N_{rn}] \begin{Bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{Bmatrix} \tag{4}$$

where $n$ is the number of nodes, $\boldsymbol{N}_r$ is the vector of the shape functions for position $r$, and $\boldsymbol{p}$ is the vector of nodal sound pressures.

Then, an integral equation to solve the discretizing nodal pressure can be simultaneously deduced by Equations 2-4 as follows,

$$\int_\Omega \left[ (\nabla \boldsymbol{N})(\nabla \boldsymbol{N})^{\mathrm{T}} \right] \mathrm{d}v \cdot \boldsymbol{p}_\omega - \int_\Omega \left( k^2 \boldsymbol{N}\boldsymbol{N}^{\mathrm{T}} \right) \mathrm{d}v \cdot \boldsymbol{p}_\omega - \int_\Omega \left( \mathrm{j}\rho_0 \omega \boldsymbol{N} q_\omega \right) \mathrm{d}v + \int_{\Gamma_1} \frac{\mathrm{j}\rho_0 \omega}{Z_s} \left( \boldsymbol{N}\boldsymbol{N}^{\mathrm{T}} \right) \mathrm{d}s \cdot \boldsymbol{p}_\omega = 0 \tag{5}$$

In the above equation, $\boldsymbol{K} = \int_\Omega (\nabla \boldsymbol{N})(\nabla \boldsymbol{N})^{\mathrm{T}} \mathrm{d}v$ is defined as the stiffness matrix, $\boldsymbol{M} = \frac{1}{c^2}\int_\Omega \boldsymbol{N}\boldsymbol{N}^{\mathrm{T}} \mathrm{d}v$ as the mass matrix, $\boldsymbol{C} = \frac{\rho_0}{Z_s}\int_{\Gamma_1} \boldsymbol{N}\boldsymbol{N}^{\mathrm{T}} \mathrm{d}s$ as the damping matrix, and $\boldsymbol{F} = \int_\Omega \mathrm{j}\rho_0 \omega \boldsymbol{N} q_\omega \mathrm{d}v$ as the load matrix. If the sound source is a point source at $r_0$, the intensity of the source can be described as $q_\omega = q_\omega(r_0)\delta(r - r_0)$. By defining $Q = q_\omega(r_0)\mathrm{d}v$ as the volume velocity, the load matrix can be written as $\boldsymbol{F} = \int_\Omega \mathrm{j}\rho_0 \omega \boldsymbol{N} q_\omega(r_0)\delta(r - r_0)\mathrm{d}v = \mathrm{j}\rho_0 \omega \boldsymbol{N} q_\omega(r_0)\mathrm{d}v = \mathrm{j}\rho_0 \omega \boldsymbol{N} Q$, and finally, the system equation can be obtained,

$$\left(\boldsymbol{K}+\mathrm{j}\omega\boldsymbol{C}-\omega^2\boldsymbol{M}\right)\boldsymbol{p}_\omega=\boldsymbol{F} \tag{6}$$

Finally, the nodal sound pressure and the pressure at the receiving point can be obtained.

## 3. PERTURBATION METHOD BASED ON THE INTERVAL MAPPING TECHNIQUE

In engineering practice, because of the manufacturing tolerances, environment changes or some other unpredictable factors, uncertainties are inevitable. If the objective information of uncertain parameter is insufficient to construct probabilistic models, the interval model is an alternative non-probabilistic way to describe the uncertain parameters when the lower and upper bounds of uncertainties are well defined. Assuming there are l uncertain parameters in the problem, an interval vector $\boldsymbol{b}^I=\left[b_1^I,b_2^I,\cdots,b_l^I\right]^{\mathrm{T}}$ is introduced to represent the interval uncertainties in this paper, and it can be expressed as

$$\begin{cases}\boldsymbol{b}^I\in\left[\underline{\boldsymbol{b}},\overline{\boldsymbol{b}}\right]=\boldsymbol{b}^m+\Delta\boldsymbol{b}^I\\ \Delta\boldsymbol{b}^I\in\left[-\Delta\boldsymbol{b},\Delta\boldsymbol{b}\right]\\ \boldsymbol{b}^m=\dfrac{\underline{\boldsymbol{b}}+\overline{\boldsymbol{b}}}{2}\\ \Delta\boldsymbol{b}=\dfrac{\overline{\boldsymbol{b}}-\underline{\boldsymbol{b}}}{2}\end{cases} \tag{7}$$

where $\underline{\boldsymbol{b}}$ and $\overline{\boldsymbol{b}}$ denote the lower and upper bounds of the uncertain parameters, respectively, $\boldsymbol{b}^m$ is the mean value vector, $\Delta\boldsymbol{b}^I$ is the deviation interval vector, $\Delta\boldsymbol{b}$ is the deviation amplitude vector.

By introducing the interval parameters into the wave-based method, the system equation as shown in Equation 6 can be written as

$$\boldsymbol{D}\left(\boldsymbol{b}^I\right)\boldsymbol{p}_\omega^I=\boldsymbol{F}\left(\boldsymbol{b}^I\right) \tag{8}$$

where $\boldsymbol{D}\left(\boldsymbol{b}^I\right)=\boldsymbol{K}\left(\boldsymbol{b}^I\right)+\mathrm{j}\omega\boldsymbol{C}\left(\boldsymbol{b}^I\right)-\omega^2\boldsymbol{M}\left(\boldsymbol{b}^I\right)$ is the interval dynamic stiffness matrix, $\boldsymbol{F}\left(\boldsymbol{b}^I\right)$ is the interval load vector and $\boldsymbol{p}_\omega^I$ is the interval nodal response vector.

By employing the Taylor series, the interval dynamic stiffness matrix $\boldsymbol{D}\left(\boldsymbol{b}^I\right)$ and the interval load vector $\boldsymbol{F}\left(\boldsymbol{b}^I\right)$ can be expanded at the mean value of the random variables, and they can be expressed as

$$\begin{gathered}\boldsymbol{D}\left(\boldsymbol{b}^I\right)=\boldsymbol{D}\left(\boldsymbol{b}^m\right)+\sum_{i=1}^{l}\frac{\partial\boldsymbol{D}\left(\boldsymbol{b}^m\right)}{\partial b_i^I}\left(\Delta b_i^I\right)+\frac{1}{2!}\sum_{i=1}^{l}\sum_{t=1}^{l}\frac{\partial\boldsymbol{D}\left(\boldsymbol{b}^m\right)}{\partial b_i^I}\frac{\partial\boldsymbol{D}\left(\boldsymbol{b}^m\right)}{\partial b_t^I}\left(\Delta b_i^I\right)\left(\Delta b_t^I\right)+\cdots\\ \boldsymbol{F}\left(\boldsymbol{b}^I\right)=\boldsymbol{F}\left(\boldsymbol{b}^m\right)+\sum_{i=1}^{l}\frac{\partial\boldsymbol{F}\left(\boldsymbol{b}^m\right)}{\partial b_i^I}\left(\Delta b_i^I\right)+\frac{1}{2!}\sum_{i=1}^{l}\sum_{t=1}^{l}\frac{\partial\boldsymbol{F}\left(\boldsymbol{b}^m\right)}{\partial b_i^I}\frac{\partial\boldsymbol{F}\left(\boldsymbol{b}^m\right)}{\partial b_t^I}\left(\Delta b_i^I\right)\left(\Delta b_t^I\right)+\cdots\end{gathered} \tag{9}$$

Equation 9 shows that the Taylor expansions of the dynamic stiffness matrix and the interval load have complicated forms. To precisely and efficiently obtain the calculating results, conventional wisdom is neglecting the high orders of the Taylor expansions and only retaining the first order one. Then, the interval system equation for calculating the interval response can be expressed as,

$$\boldsymbol{p}_{\omega m}+\Delta\boldsymbol{p}_\omega=\left(\boldsymbol{D}_m+\Delta\boldsymbol{D}\right)^{-1}\left(\boldsymbol{F}_m+\Delta\boldsymbol{F}\right) \tag{10}$$

where $\boldsymbol{p}_{\omega m}$ is the response at the mean values of the uncertain parameters, $\Delta\boldsymbol{p}_{\omega}$ is the perturbation response caused by the interval parameters, $\boldsymbol{D}_m$ and $\boldsymbol{F}_m$ are the system matrices at the mean values, $\Delta\boldsymbol{D}$ and $\Delta\boldsymbol{F}$ are the perturbation matrices, and they can be expressed as,

$$\begin{cases} \boldsymbol{D}_m = \boldsymbol{D}\left(\boldsymbol{b}^m\right), \Delta\boldsymbol{D} = \sum_{i=1}^{l} \dfrac{\partial \boldsymbol{D}\left(\boldsymbol{b}^m\right)}{\partial b_i^I} \Delta b_i^I \\ \boldsymbol{F}_m = \boldsymbol{F}\left(\boldsymbol{b}^m\right), \Delta\boldsymbol{F} = \sum_{i=1}^{l} \dfrac{\partial \boldsymbol{F}\left(\boldsymbol{b}^m\right)}{\partial b_i^I} \Delta b_i^I \end{cases} \tag{11}$$

The first order Taylor expansion in the classical perturbation method is a feasible strategy when the uncertain parameters are linear. However, it may suffer a large relative error for solving non-linear parameter problem due to the high order information loss. In such condition, the interval system matrices in Eq. (10) should be expanded into high order Taylor series. Theoretically, if the system equation has $n$+1 order derivatives with respect to the uncertain parameter $b_i^I$, it needs to be expanded into at least a $n$ order Taylor series to ensure the results have high accuracy. However, calculating with the complete expressions of the Taylor expansions will cause a very heavy computational burden. To avoid this problem, a novel parameter mapping technique is proposed in this paper.

Assuming the system function contains a set of non-linear uncertain parameters $\boldsymbol{b}^I$ which can be expressed by,

$$\boldsymbol{Y}^I = \boldsymbol{f}\left(\boldsymbol{b}^I\right), b_i^I \in \left[\underline{b_i}, \overline{b_i}\right] \tag{12}$$

where $\boldsymbol{Y}^I$ is a set of linear variables introduced here to describe the uncertain parameters and $\boldsymbol{f}(\cdot)$ denotes the functionship between $\boldsymbol{Y}^I$ and $\boldsymbol{b}^I$. By considering $\boldsymbol{Y}^I$ as the new uncertain parameters, the system equation is transformed into

$$\boldsymbol{D}\left(\boldsymbol{Y}^I\right)\boldsymbol{p}_{\omega}^I = \boldsymbol{F}\left(\boldsymbol{Y}^I\right) \tag{13}$$

where the uncertain parameter $\boldsymbol{Y}^I$ and its interval can be written as

$$\begin{cases} \boldsymbol{Y}^I \in \left[\min \boldsymbol{f}\left(\boldsymbol{b}^I\right), \max \boldsymbol{f}\left(\boldsymbol{b}^I\right)\right] = \boldsymbol{Y}^m + \Delta\boldsymbol{Y}^I \\ \Delta\boldsymbol{Y}^I \in \left[-\Delta\boldsymbol{Y}, \Delta\boldsymbol{Y}\right] \\ \boldsymbol{Y}^m = \dfrac{\underline{\boldsymbol{Y}} + \overline{\boldsymbol{Y}}}{2} = \dfrac{\min \boldsymbol{f}\left(\boldsymbol{b}^I\right) + \max \boldsymbol{f}\left(\boldsymbol{b}^I\right)}{2} \\ \Delta\boldsymbol{Y} = \dfrac{\overline{\boldsymbol{Y}} - \underline{\boldsymbol{Y}}}{2} = \dfrac{\max \boldsymbol{f}\left(\boldsymbol{b}^I\right) - \min \boldsymbol{f}\left(\boldsymbol{b}^I\right)}{2} \end{cases} \tag{14}$$

To solve the uncertain response, both the dynamic stiff matrix and the interval load vector need to be expanded into Taylor series with respect to the new uncertain parameter $\boldsymbol{Y}^I$. Since $\boldsymbol{Y}^I$ is a set of linear variables, the system only has first order derivatives with respect to these parameters. Therefore, the first order Taylor expansions have been capable of giving sufficient accurate approximations of the system matrices. The new expansions of the system matrices can be written as

$$\begin{aligned} \boldsymbol{D}\left(\boldsymbol{Y}^I\right) &= \boldsymbol{D}\left(\boldsymbol{Y}^m\right) + \sum_{i=1}^{l} \frac{\partial \boldsymbol{D}\left(\boldsymbol{Y}^m\right)}{\partial Y_i^I}\left(\Delta Y_i^I\right) \\ \boldsymbol{F}\left(\boldsymbol{Y}^I\right) &= \boldsymbol{F}\left(\boldsymbol{Y}^m\right) + \sum_{i=1}^{l} \frac{\partial \boldsymbol{F}\left(\boldsymbol{Y}^m\right)}{\partial Y_i^I}\left(\Delta Y_i^I\right) \end{aligned} \tag{15}$$

To solve the response in case of Equation 15, the dynamic stiff matrix $\left(\boldsymbol{D}_m + \Delta\boldsymbol{D}\right)^{-1}$ in Equation 10 can be expanded by employing the Neumann series

$$\left(\boldsymbol{D}_m + \Delta\boldsymbol{D}\right)^{-1} = \left(\boldsymbol{D}_m\right)^{-1} + \sum_{\gamma=1}^{\infty}\left(\boldsymbol{D}_m\right)^{-1}\left(-\Delta\boldsymbol{D}\left(\boldsymbol{D}_m\right)^{-1}\right)^{\gamma} \tag{16}$$

Thus, Equation 10 can be expressed by

$$\begin{aligned} \boldsymbol{p}_{\omega m} + \Delta\boldsymbol{p}_{\omega} = {} & \left(\boldsymbol{D}_m\right)^{-1}\boldsymbol{F}_m + \left(\boldsymbol{D}_m\right)^{-1}\Delta\boldsymbol{F} + \sum_{\gamma=1}^{\infty}\left(\boldsymbol{D}_m\right)^{-1}\left(-\Delta\boldsymbol{D}\left(\boldsymbol{D}_m\right)^{-1}\right)^{\gamma}\boldsymbol{F}_m \\ & + \sum_{\gamma=1}^{\infty}\left(\boldsymbol{D}_m\right)^{-1}\left(-\Delta\boldsymbol{D}\left(\boldsymbol{D}_m\right)^{-1}\right)^{\gamma}\Delta\boldsymbol{F} \end{aligned} \tag{17}$$

By neglecting the higher-order cross terms, Equation 17 can be rewritten as

$$\boldsymbol{p}_{\omega m} + \Delta\boldsymbol{p}_{\omega} = \left(\boldsymbol{D}_m\right)^{-1}\boldsymbol{F}_m + \left(\boldsymbol{D}_m\right)^{-1}\Delta\boldsymbol{F} - \left(\boldsymbol{D}_m\right)^{-1}\Delta\boldsymbol{D}\left(\boldsymbol{D}_m\right)^{-1}\boldsymbol{F}_m \tag{18}$$

where $\boldsymbol{p}_{\omega m} = \left(\boldsymbol{D}_m\right)^{-1}\boldsymbol{F}_m$ and $\Delta\boldsymbol{p}_{\omega} = \left(\boldsymbol{D}_m\right)^{-1}\Delta\boldsymbol{F} - \left(\boldsymbol{D}_m\right)^{-1}\Delta\boldsymbol{D}\left(\boldsymbol{D}_m\right)^{-1}\boldsymbol{F}_m$.

Finally, the interval response can be obtained.

## 4. NUMERICAL VERIFICATION

The accuracy of the proposed method is explored by predicting an uncertain field in a cubic cavity as shown in Figure 1. The length, width and height of the cavity are 1.1m, 1.2m and 1m, respectively. The specific acoustic impedance of the inner surfaces of the cavity is set to be 16600. A bottom corner is considered to be the origin of the coordinate system. There is a point source at coordinate of (0.1, 0.2, 0.3)m. The receiver locates at (0.8, 0.6, 0.9)m.

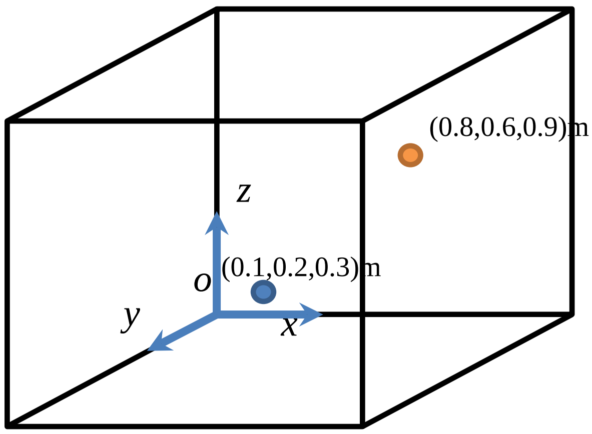


Figure 1 The cubic cavity

By considering that the environmental temperature changes in an interval, the speed of sound can be treated as interval parameters. Here we assume that the speed of sound changes from 309m/s to 359m/s.

The bounds of the frequency response at the receiver from 100Hz to 300Hz are calculated using the proposed method and the classical perturbation method. Reference results are obtained using the vertex method. All the methods involved in this paper are implemented under the framework of meshless method[14, 15]. The fluid domain in the cavity is divided into 729 nodes. The comparisons of the upper bound and lower bound are illustrated in Figure 2 and Figure 3.

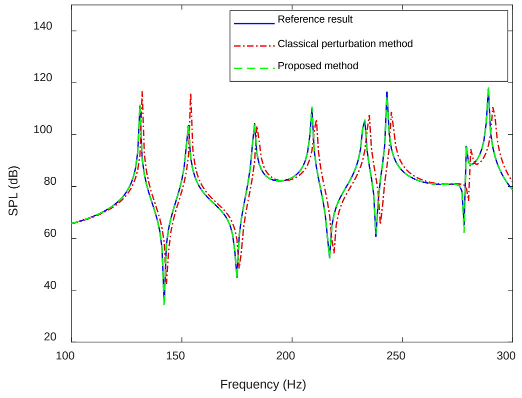


Figure 2 Results of different methods for upper bound of the freqeuncy response

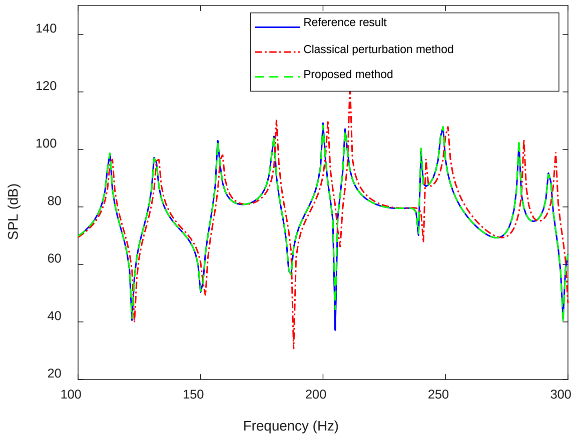


Figure 3 Results of different methods for lower bound of the freqeuncy response

Figure 2 and Figure 3 illustrate that the classical perturbation method fail to exactly predict the upper bound and the lower bound of the frequency responses when the speed of sound are uncertain. However, the proposed method gives very close results with the reference results. The comparisons demonstrate that the classical perturbation method cannot give correct results for problems with nonlinear interval parameters, while the proposed method is capable of giving correct results with very simple processing to the uncertain parameters.

## 4. CONCLUSIONS

To solve the uncertain problems with nonlinear interval parameters, a parameter mapping technique is proposed in this paper. This technique firstly constructs a function with respect to a nonlinear parameter. Then, by mapping the initial interval of the nonlinear parameter into the newly constructed function space, this function can be considered as a new independent parameter. Finally, the uncertain acoustical problems can be solved by the classical interval perturbation methods in which the first order approximation is capable of giving precise results. The numerical verifications demonstrate that the proposed technique is effective for uncertain acoustic field simulation with nonlinear interval parameters.

## 5. ACKNOWLEDGEMENTS

This study was supported in part by the National Natural Science Foundation of China (Grant No.11604266) and the Research Funds of the Central Finance (Grant No.MJ-2015-F-044) .

## 6. REFERENCES

1. L. L. Thompson, “A review of finite-element methods for time-harmonic acoustics”, *J Acoust Soc Am*, **119** (3), 1315-1330, (2006).
2. K. R. James, D. R. Dowling, “A probability density function method for acoustic field uncertainty analysis”, *J Acoust Soc Am*, **118** (5), 2802-2810, (2005).
3. K. R. James, D. R. Dowling, “A method for approximating acoustic-field-amplitude uncertainty caused by environmental uncertainties”, *J Acoust Soc Am*, **124** (3), 1465-1476, (2008).
4. K. R. James, D. R. Dowling, “Pekeris waveguide comparisons of methods for predicting acoustic field amplitude uncertainty caused by a spatially uniform environmental uncertainty (L)”, *J Acoust Soc Am*, **129** (2), 589-592, (2011).
5. D. Moens, D. Vandepitte, “A fuzzy finite element procedure for the calculation of uncertain frequency-response functions of damped structures: Part 1-Procedure”, *J Sound Vib*, **288**, 431-462, (2005).
6. H. D. Gersem, D. Moens, W. Desmet, D. Vandepitte, “A fuzzy finite element procedure for the calculation of uncertain frequency response functions of damped structures: Part 2-Numerical case studies”, *J Sound Vib*, **288**, 463-486, (2005).
7. H. Yin, D. Yu, S. Yin, B. Xia, “Possibility-based robust design optimization for the structural-acoustic system with fuzzy parameters”, *Mech Syst Signal Process*, **102**, 329-345, (2018).
8. D. Moens, D. Vandepitte, “A survey of non-probabilistic uncertainty treatment in finite element analysis”, *Comput Methods Appl Mech Engrg*, **194** (12-16), 1527-1555, (2005).
9. S. H. Chen, Z. P. Qiu, D. T. Song, “New method for computing the upper and lower bounds on frequencies of structures with interval parameters”, *Mechanics Research Communications*, **21**, 583-592, (1994).
10. B. Xia, D. Yu, “Modified sub-interval perturbation finite element method for 2D acoustic field prediction with large uncertain-but-bounded parameters”, *J Sound Vib*, **331**, 3774-3790, (2012).
11. B. Xia, D. Yu, J. Liu, “Hybrid uncertain analysis of acoustic field with interval random parameters”, *Comput Methods Appl Mech Engrg*, **256**, 56-69, (2013).
12. C. Wang, Z. Qiu, “An interval perturbation method for exterior acoustic field prediction with uncertain-but-bounded parameters”, *J Fluid Struct*, **49**, 441-449, (2014).
13. J. Feng, D. Wu, W. Gao, G. Li, “Hybrid uncertain natural frequency analysis for structures with random and interval fields”, *Comput Methods Appl Mech Engrg* **328**, 365-389, (2018).
14. T. Belytschko, Y. Y. Lu, L. Gu, “Element-free Galerkin methods”, *Int J Num Meth Eng*, **37**, 229-256, (1994).
15. H. T. Wang, X. Y. Zeng, “Calculation of sound fields in small enclosures using a meshless model”, *Appl Acoust*, **74** (3), 459-466, (2013).