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Study on energy focusing effect of two-dimensional acoustic black hole on flexural waves in composite plate structures

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ABSTRACT

The acoustic black hole (ABH) effect can produce the area with high energy density in plate structure, which is beneficial for setting the damping material to attenuate the vibration and noise of the structure. In order to study the energy focusing effect arising from ABH on flexural waves in composite plate structures, in this paper, different structures with embedded single ABH and ABH array are designed. And the propagation characteristics of energy in composite plate structures with embedded ABH are investigated by using the vibration power flow method at low and mid frequencies. To visualize the power flow of the flexural waves in structures, the data of stress and displacement in plates are extracted from the commercial simulation software, and programs are written to calculate the power flow. The energy focusing effect of two-dimensional ABH on flexural waves at low and mid frequencies in composite plate structures is analysed and compared with the energy focusing effect of ABH on flexural waves in metal plate structures.

Keywords: Acoustic black hole, Energy focalization, Composite
I-INCE Classification of Subject Number: 43

1. INTRODUCTION

Acoustic black hole (ABH) introduces the concept of black hole in physics into the field of wave motion and acoustic vibration, and it is put forward as a new concept. As the thickness of a plate structure decreases smoothly with a power function, the velocity of flexural waves propagating in the plate decreases gradually, the amplitude increases gradually, and its energy is focused [1]. In an ideal case, the flexural wave velocity decreases to zero, which means that the flexural wave never reaches the edge of the structure and therefore never reflects back either. The ABH effect can produce the area with high energy density in plate structure. The characteristic of energy consumption of damping material is that the larger the deformation is, the more energy is consumed. The vibration and noise control of the structure can be realized by attaching less damping

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material, which can reduce the usage of damping material. In addition, the structural characteristics of ABH itself further reduce the weight of the plate. Therefore, the ABH effect is beneficial to attenuate the vibration and noise of structures and the lightweight effect of structures can be achieved at the same time.

The geometrical acoustics approach was used to investigate the flexural wave propagation in a wedge structure, and to compute the corresponding coefficient of reflection [1,2]. Boundary Element model and Finite Element model were utilized to investigate the radiated sound power performance and vibration response of several plate structures, and embedded periodic acoustic black hole designs are compared with baseline uniform plates [3]. Based on a numerical model, the natural frequencies and mode shapes of vibration of a cylindrical plate with embedded ABH were analysed [4]. Current researches mainly focus on metal plate structures with embedded ABH at high frequency.

In this paper, different structures with embedded single ABH and ABH array are designed. And the propagation characteristics of energy in composite plate structures with embedded ABH are investigated by using the vibrational power flow method at low and mid frequencies. The energy focusing effect of two-dimensional ABH on flexural waves at low and mid frequencies in composite plate structures is analysed and compared with the energy focusing effect of ABH on flexural waves in metal plate structures.

2. ACOUSTIC BLACK HOLE THEORY

The wedge-shaped edge of a typical one-dimensional ABH structure is shown in Fig.1 and it satisfies a thickness profile following the power law equation as $h(x) = \varepsilon x^m (m \geq 2)$ [5]. When the power-law exponent m is equal to or larger than two, the velocity of flexural waves propagating in the plate decreases gradually and its energy is focused [6]. In an ideal case, the flexural wave velocity decreases to zero, which means that flexural waves never reach the edge of the structure and therefore never reflect back either. This phenomenon is called the "acoustic black hole" effect.

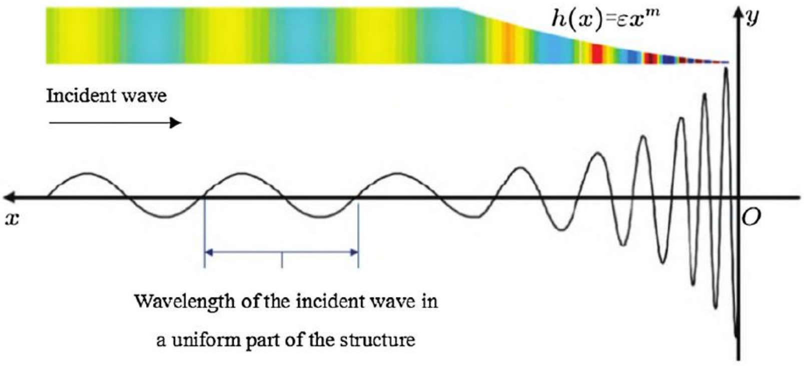


Fig. 1. Propagation of flexural wave in the one-dimensional ABH [7].

The flexural wave propagation in such structure can be investigated by using the geometrical acoustics approach [1,2]. The integrated wave phase $\Phi(x)$ can be written as the integral expression from an arbitrary point x of the wedge to the wedge edge

$$\Phi = \int_0^x \kappa(x)dx \tag{1}$$

where $\kappa(x)$ is the wavenumber and it can be written by

$$\kappa(x) = \left(\frac{12\kappa_p^2}{\left(\varepsilon x^m\right)^2} \right)^{1/4} \tag{2}$$

When $m \geq 2$, we can easily get that the integral in Equation (1) diverges. This means that the phase Φ becomes infinite. Therefore, flexural waves never reach the edge.

3. POWER FLOW THEORY

The vibration power flow represents the ability of external force to work or structure to dissipate energy per unit time, and is an important physical quantity that describes the transmission of vibration energy in the structure. The instantaneous power flow can be written by

$$P = F(t) \cdot V(t) \tag{3}$$

where P represents power flow; $F(t)$ is external force and $V(t)$ indicates response velocity.

For vibration analysis, it is of little significance to study the instantaneous power flow at a certain moment, and the average power flow in a certain period of time can better reflect the energy intensity of the external excitation input structure. Thus, the vibration power flow is expressed as

$$P = \frac{1}{T} \lim_{T \to \infty} \int_0^T F(t) \cdot V(t) dt \tag{4}$$

The stress state at any point in the elastic body can be expressed by six stress components, namely normal stress σ_x, σ_y, σ_z, and shear stress τ_{xy}, τ_{yz}, τ_{zx}. Taking a micro-element in the elastic body, the stress form is as Fig. 2.

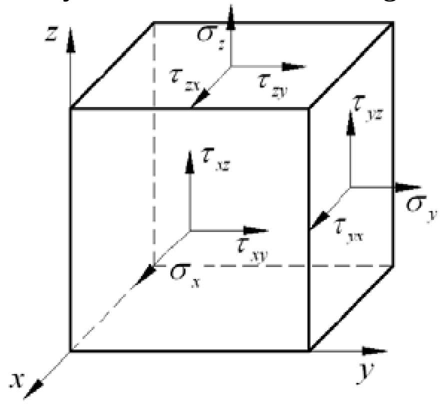


Fig. 2. Micro-element stress form

The power flow per unit area is [8]

$$p_n = -\frac{1}{2} \mathrm{Re}\left(\sigma_n v_n^* + \tau_{n1} v_1^* + \tau_{n2} v_2^*\right) \tag{5}$$

where σ_n represents normal stress in normal n direction; τ_{n1} and τ_{n2} are shear stress in the direction of 1 and 2; v_n^*, v_1^*, v_2^* indicate complex conjugate of velocities in the normal n, 1 and 2 direction respectively.

For solid elements, the power flow expressed by the stress and displacement parameters is

$$p_x = -\frac{\omega}{2} \mathrm{Im}\left(\sigma_x u^* + \tau_{xy} v^* + \tau_{xz} w^*\right) \tag{6}$$

$$p_y = -\frac{\omega}{2} \mathrm{Im}\left(\tau_{yx} u^* + \sigma_y v^* + \tau_{yz} w^*\right) \tag{7}$$

$$p_z = -\frac{\omega}{2} \mathrm{Im}\left(\tau_{zx} u^* + \tau_{zy} v^* + \sigma_z w^*\right) \tag{8}$$

where P_x, P_y, P_z indicates power flow in the x, y and z direction.

4. TWO-DIMENSIONAL ACOUSTIC BLACK HOLE

The schematic of the two-dimensional ABH structure studied in this paper is shown in Fig. 3. The ABH is embedded in a plate structure, and the thickness of the uniform portion outside the ABH structure is h_2. Since the thickness of the actual structure cannot be gradually reduced to zero according to the power function form, there is always a truncation at the center, so the two-dimensional ABH structure in this paper has a circular platform with uniform thickness. The power-law exponent of thickness variation of the non-uniform portion is $m = 2.2$, the truncation thickness is h_1, and the truncation radius is r_1, so $h_1 = \varepsilon x_1^{2.2}$. That is, the relationship between the thickness and position of the ABH portion is as shown in Equation (9). Geometrical parameters of the two-dimensional ABH used in this paper are tabulated in Table 1.

$$h(x) = \begin{cases} h_1, (x \le x_1) \\ \varepsilon x^{2.2}, (x_1 \le x \le x_2) \end{cases} \tag{9}$$

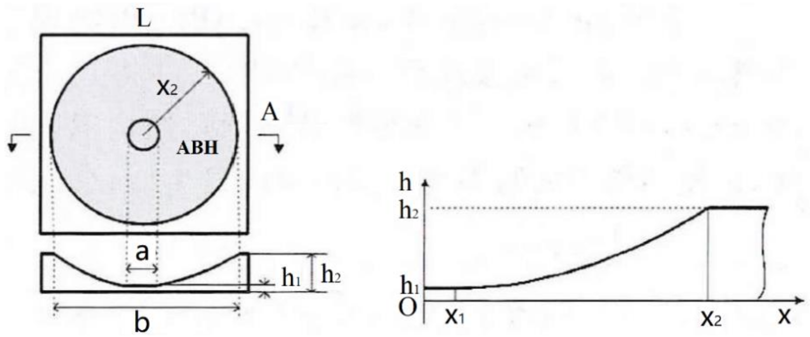


Fig. 3. Schematic of two-dimensional ABH

Table 1 Geometrical parameters of the two-dimensional ABH

Parameters	h_1(m)	h_2(m)	x_2(m)	m
Value	0.0001	0.004	0.06	2.2

5. SIMULATION ANALYSIS OF THE VIBRATION POWER FLOW OF THE PLATE STRUCTURE WITH EMBEDDED SINGLE ABH

5.1 Simulation Analysis of Composite Plate Structure

In order to study the propagation characteristics of flexural waves in the composite plate structure with embedded single ABH, the model (0.39m × 0.26m × 0.004m) is established in the finite element software ABAQUS, as shown in Fig. 4.

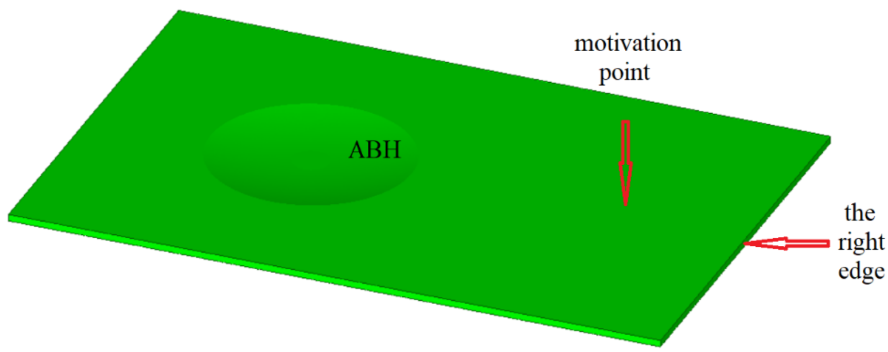


Fig. 4. The finite element model of plate with embedded single ABH

Material of the plate structure in this section is carbon fiber epoxy resin composite. The ply orientation of composite laminate is $[0/45/90/-45/0]_8$ and the thickness of each ply is 0.0001m. The ply stack plot of composite laminate is shown in Fig. 5. For the variable thickness region of the plate structure, the number of plies gradually decreases as the thickness decreases. Material properties used in the finite element model are tabulated in Table 2.

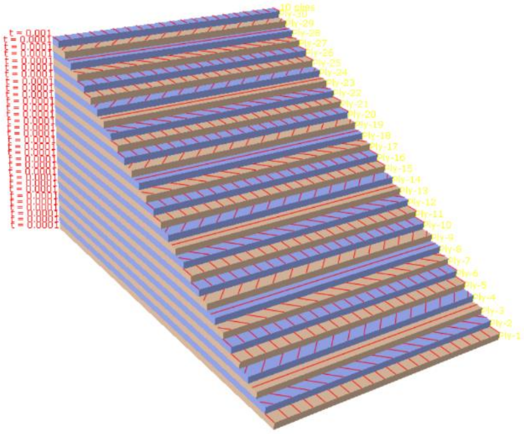
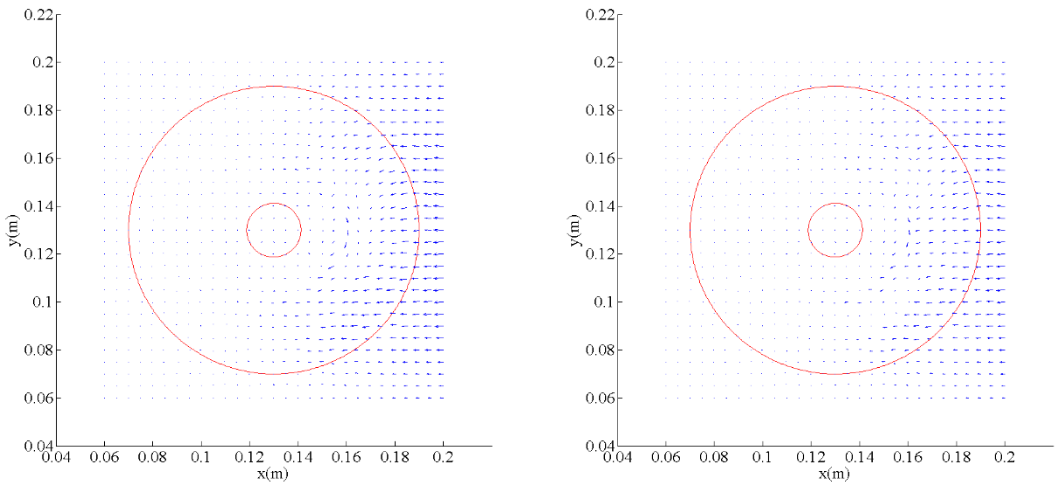


Fig. 5. The ply stack plot of the composite laminate

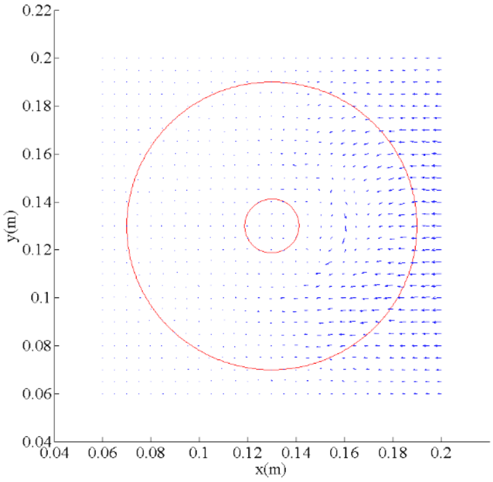
Table 2 Material properties

Parameters	ρ $(\mathrm{kg/m^3})$	E_1 (GPa)	E_2 (GPa)	E_3 (GPa)	G_{12} (GPa)	G_{13} (GPa)	G_{23} (GPa)	ν_{12}	ν_{13}	ν_{23}
Value	1600	181	10.3	10.3	7.17	7.17	4.27	0.308	0.308	0.456

The structure is excited by harmonics with excitation frequencies of 1000 Hz, 1050 Hz and 1100 Hz respectively, and the excitation force size set in the established finite element simulation model is 1N. As shown in Fig. 4, the motivation point is 0.170m from the center of the ABH. The right edge of the structure is fixed. The data required to calculate the power flow of each node include the real and imaginary parts of the stress and displacement, and then MATLAB programs are written to calculate the power flow. The power flow distribution diagrams are shown in Fig. 6. The length of the arrow in the diagrams indicates the magnitude of the power flow, and the direction indicates the direction of propagation of the power flow.



a) 1000 Hz *b) 1050 Hz*



c) 1100 Hz

Fig. 6. The power flow distribution diagram

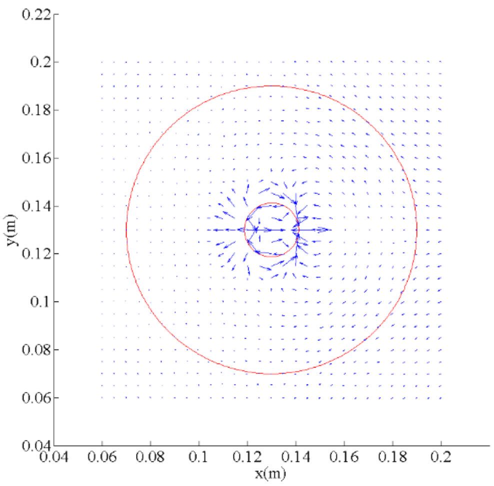
As can be seen from Fig. 6, The propagation direction of the power flow does not deflect significantly in the region of ABH structure, and energy is not focused in the center of the ABH structure.

5.2 Simulation Analysis of Metal Plate Structure

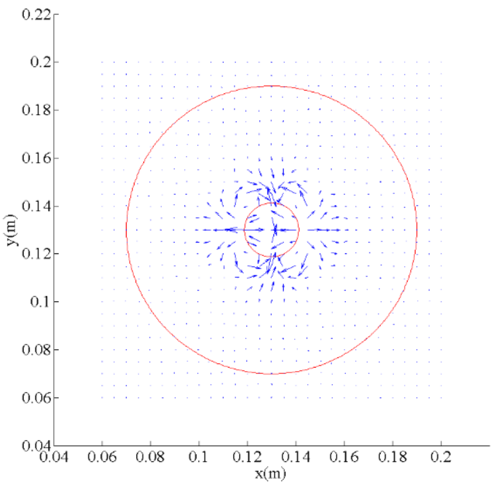
Material of the plate structure in this section is metal. Material properties used in the finite element model are tabulated in Table 3. The dimensions of the metal plate structure and the parameters of simulation analysis are the same as those of the composite plate structure in section 5.1. The power flow distribution diagrams are shown in Fig. 7.

Table 3 Material properties

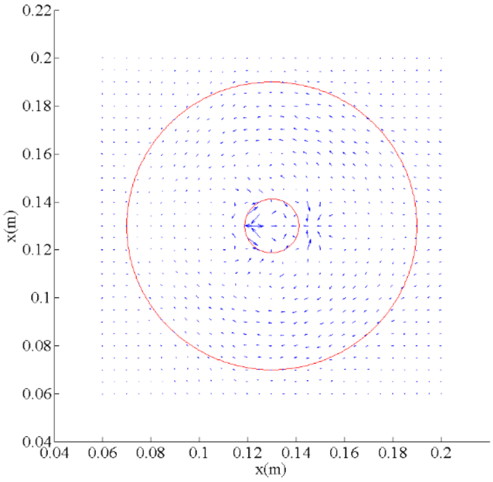
Parameters	$\rho\left(\text{kg/m}^3\right)$	$E\,(\text{GPa})$	ν
Value	7800	209	0.3



a) 1000 Hz



b) 1050 Hz



c) 1100 Hz

Fig. 7. The power flow distribution diagram

As can be seen from Fig. 7, the power flow in the central portion of the ABH is significantly larger than the power flow elsewhere, that is, energy is focused in the center of the ABH structure.

6. SIMULATION ANALYSIS OF THE VIBRATION POWER FLOW OF THE STRUCTURE WITH EMBEDDED ABH ARRAY

6.1 Simulation Analysis of Composite Plate Structure

In order to study the propagation characteristics of flexural waves in the composite plate structure with embedded ABH array, the model ($0.5\text{m} \times 0.5\text{m} \times 0.004\text{m}$) is established in the finite element software ABAQUS, as shown in Fig. 8.

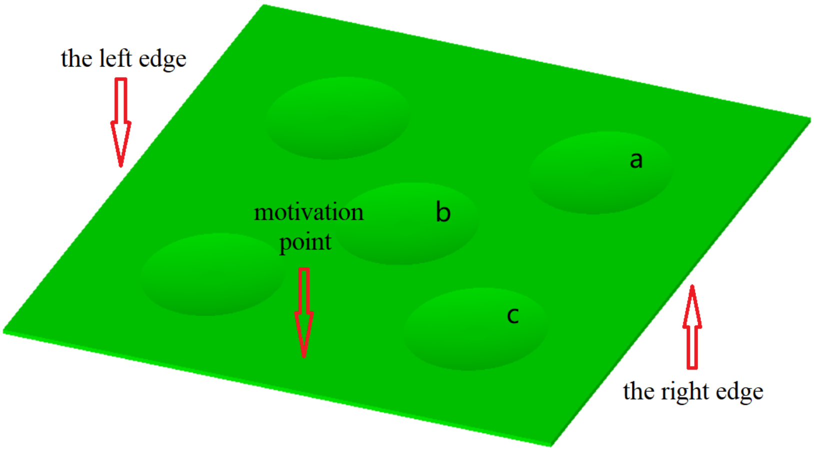
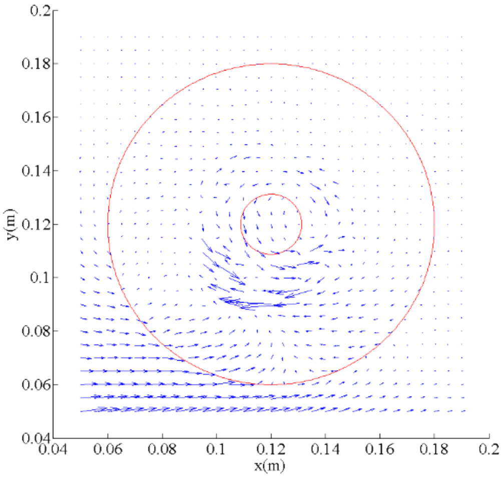
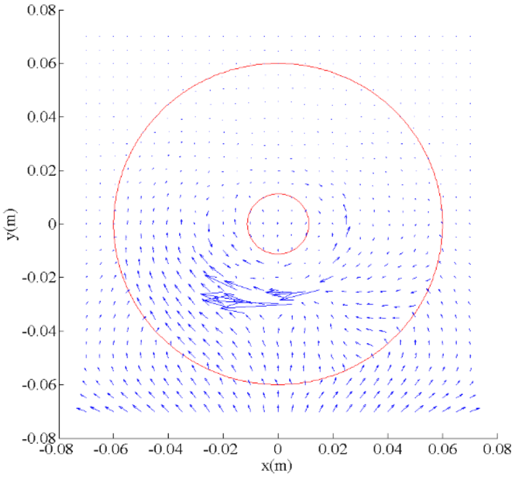


Fig. 8. The finite element model of plate with embedded ABH array

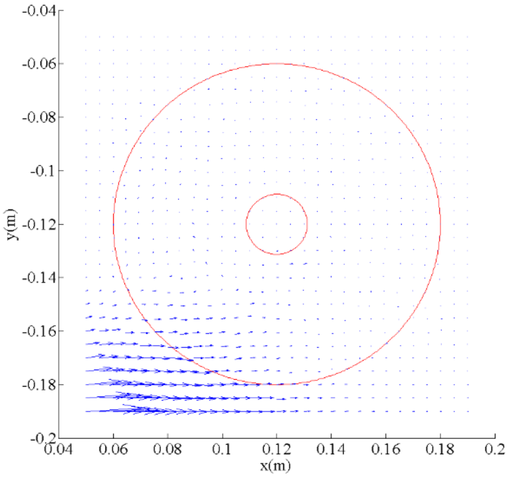
Material properties are the same as those of the structure in section 5.1. Since the plate structure is bilaterally symmetrical, only the power flow of three ABHs in the structure, namely the ABH(a), (b) and (c) in Figure 4, is investigated. The structure is excited by harmonics with excitation frequencies of 1000 Hz and 1100 Hz respectively, and the excitation force size set in the established finite element simulation model is 1N. As shown in Fig. 8, the motivation point is 0.21m from the center of the ABH(b) in the middle of the plate structure. The right and left edge of the structure are fixed.The power flow distribution diagrams are shown in Figs. 9-10.



a) The ABH(a)

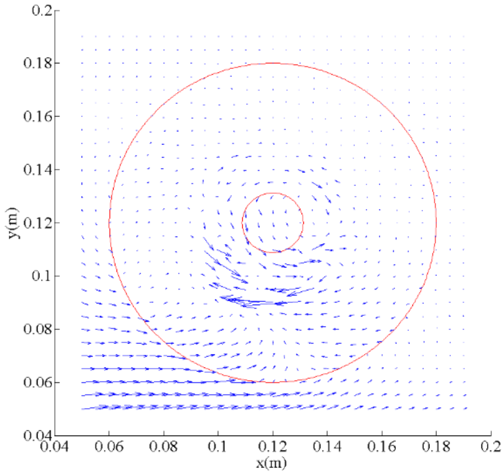


b) The ABH(b)

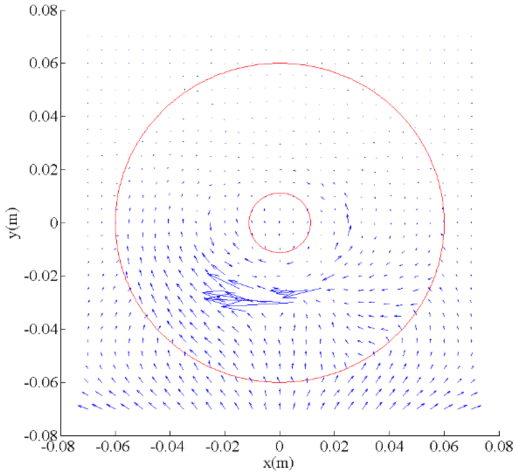


c) The ABH(c)

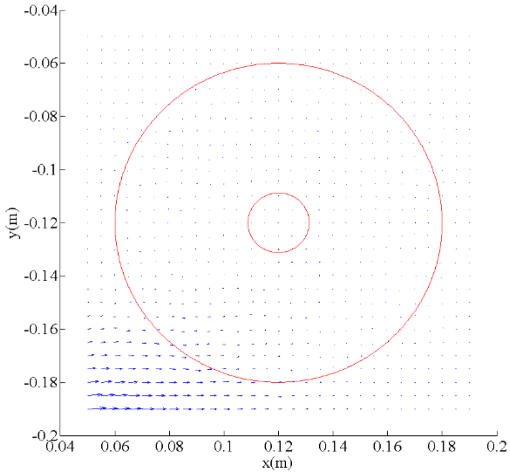
Fig. 9. The power flow distribution diagram when excitation frequency is 1000 Hz



a) The ABH(a)



b) The ABH(b)



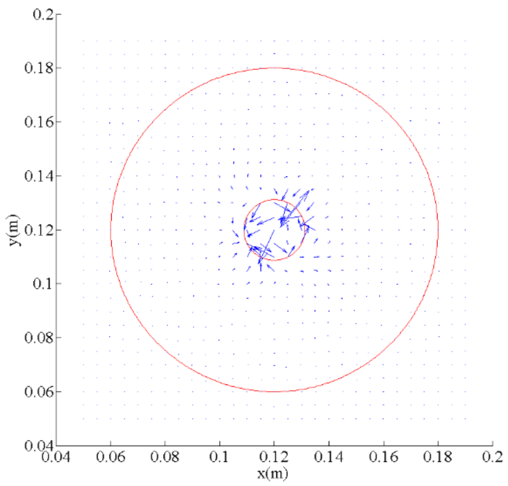
c) The ABH(c)

Fig. 10. The power flow distribution diagram when excitation frequency is 1100 Hz

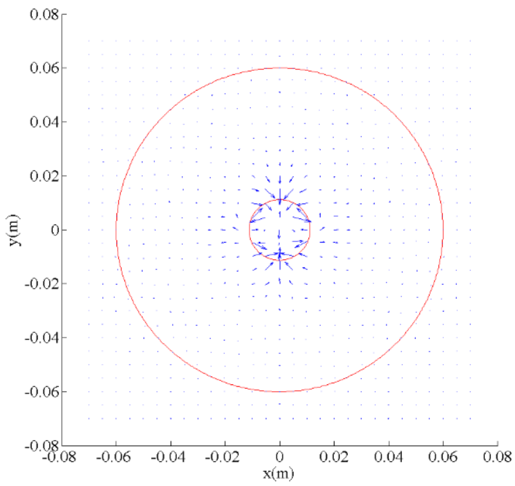
As can be seen from Figs. 9-10, the propagation direction of the power flow is deflected somewhat at the ABH(a) and the ABH(b) at 1000 Hz and 1100 Hz, and its energy is focused but not fully focused in the central portion of the ABH(a) and the ABH(b). It means that the ABH(a) and the ABH(b) have the effect of energy focalization, but the phenomenon is not obvious. The propagation direction of the power flow does not deflect significantly in the ABH(c) at 1000hz and 1100hz, and energy is not focused in the center of the ABH(c).

6.2 Simulation Analysis of Metal Plate Structure

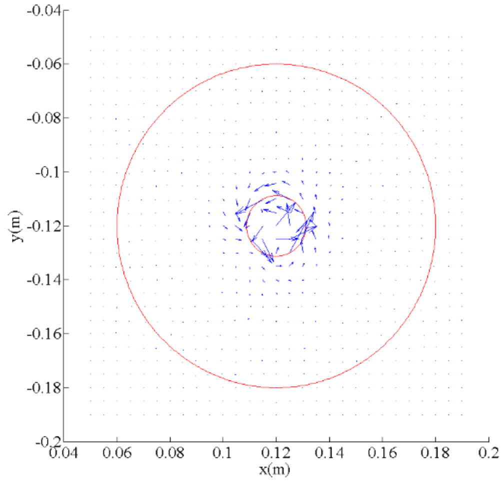
Material of the plate structure in this section is metal and the material properties are the same as those of the structure in scction 5.2. The dimensions of the metal plate structure and the parameters of simulation analysis are the same as those of the composite plate structure in section 6.1. The power flow distribution diagrams are shown in Figs. 11-12.



a) The ABH(a)

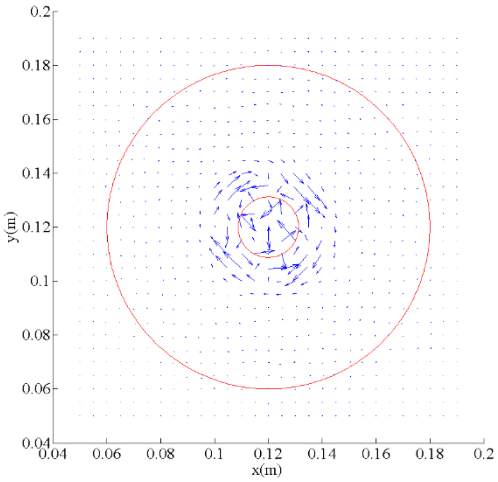


b) The ABH(b)

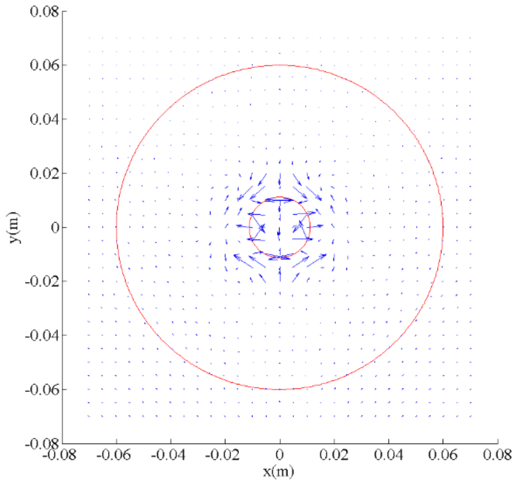


c) The ABH(c)

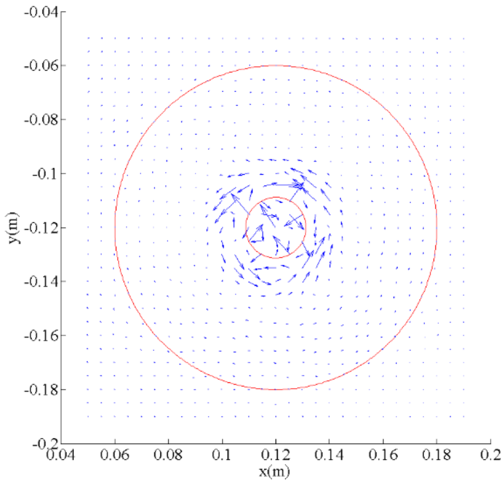
Fig. 11. The power flow distribution diagram when excitation frequency is 1000hz



a) The ABH(a)



b) The ABH(b)



c) The ABH(c)

Fig. 12. The power flow distribution diagram when excitation frequency is 1100 Hz

As can be seen from Figs. 11-12, the power flow in the central portion of the ABH is significantly larger than the power flow elsewhere, that is, energy is focused in the center of the ABH structure.

7. CONCLUSIONS

In this paper, the finite element models of different structures with embedded single ABH and ABH array are established, and the propagation characteristics of energy in composite plate structures with embedded ABH are investigated by using the vibration power flow method at low and mid frequencies. The data of stress and displacement in plates are extracted from the commercial simulation software, and programs are written to calculate the power flow. The energy focusing effect of two-dimensional ABH on flexural waves at low and mid frequencies in composite plate structures is analysed and compared with the energy focusing effect of ABH on flexural waves in metal plate structures. The following conclusions can be reached:

(1) It can be seen from the propagation characteristics of energy in the composite plate structure with embedded single ABH designed in this paper at 1000 Hz, 1050 Hz and 1100 Hz that the propagation direction of the power flow does not deflect significantly in the region of ABH structure, and energy is not focused in the center of the ABH structure. However, the propagation direction of the power flow in the composite plate with embedded ABH array designed in this paper is deflected somewhat in the ABH(a) and the ABH(b) at 1000 Hz and 1100Hz, and its energy is focused but not fully focused in the central portion of the ABH(a) and the ABH(b). It means that the ABH(a) and the ABH(b) have the effect of energy focalization, but the phenomenon is not obvious. The propagation direction of the power flow in the composite plate with embedded ABH array does not deflect significantly in the ABH(c) at 1000 Hz and 1100 Hz, and the energy is not focused in the center of the ABH(c). Therefore, compared with the composite plate structure with embedded single ABH, the composite plate structure with embedded ABH array designed in this paper can change the propagation characteristics of the power flow and the effect of energy focalization.

(2) From the propagation characteristics of energy in the metal plate structures with embedded single ABH and ABH array designed in this paper, it can be seen that the power flow in the central portion of the ABH is significantly larger than the power flow elsewhere, that is, the energy is focused in the center of the ABH structure.

(3) Compared with the metal plate structures with embedded single ABH and ABH array, the energy focusing effect of two-dimensional ABH on flexural waves in composite plate structures with embedded single ABH and ABH array is not significant.

8. ACKNOWLEDGEMENTS

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