MADRID
inter.noise 2019
June 16 - 19
NOISE CONTROL FOR A BETTER ENVIRONMENT

Modeling and experimental investigation on the vibration of main drive chain in escalator

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Abstract

In this paper, a coupled nonlinear string model was built to analyze the dynamic behavior of the main drive chain in escalator. Following, Wavelet-Galerk in method was applied to solve the coupled non linear equation. Meanwhile, a testing rig was designed to experimentally investigate the vibration of drive chain in escalator. The transverse vibration of drive chain was captured with high-speed camera under different traction conditions. And then it was quantitative ly obtained following an images processing which was put forward according to binary and morphological algorithm. Finally, compared with experimental results, the validity of the string model and the effects of pre-tension were discussed.

Keywords: Escalator; Drive chain; String vibrations
I-INCE Classification of Subject Number: 14

1. Introduction

Chain drives are widely used to transmit power between rotational machine elements, where positive driving, high efficiency, and low maintenance cost are required[1].Compared to belt drives, chain drives are characterized by the discrete nature of the chain links and sprocket teeth, which lead to noise and vibration[2]. Therefore, vibration of chain is an important problem that needs to be resolved in design and the operation of the chain drives[3]. There are two factors resulting in vibration of the chain, which are internal and external stimuli. Periodic torsional loading and imbalance in the drive are external sources. Polygonal action and roller-tooth impacts are external sources[4].Because the chain, lying on the sprockets, forms a polygon rather than a circle, the ends of the chain experience periodic fluctuation of velocities, which is known as polygonal action [5].

Many experiments have been carried out to investigate the dynamic behavior of roller chain drives. In 1995, James [6] used a strain gage mounted on a link side plate to determine chain tension during normal operation over a wide range of linear chain speeds and preloads. What's more, Roberto and Yan [7] built a novel mechatronic test rig to study the chain lubrication and its effect on the temperature, efficiency and vibrations of the motorbike chain transmission. Lenkov et al. [8] have designed the experimental installation of a chain drive for investigation of the influence of chain tension on the amplitude of forced oscillations. Also there are many experiments to measure the impact force of the chain rollers on the sprocket teeth [9-11].

Under the assumption that the chain span motion decouples from the dynamics of sprockets and attached machinery, the studies of roller chain drive dynamics are found to border the analysis of axially moving strings [12]. Early in 1957, Mahalingam [13] treated the chain as a uniform string to determine the natural frequencies. In 1987, Ariartnam and Asokanthan [14] studied the periodic fluctuation of power transmitting chains brought about by factors such as polygonal action and eccentricity of sprockets by treating the chain as a traveling uniform heavy string. Nowadays, Destyl et al. [15] derived and studied a novel model of the coupled $\mathcal{PT}$-symmetric discrete nonlinear Schrödinger equations which described parametrically driven chains of the coupled pendula pairs connected to the nearest neighbors in the longitudinal and transverse directions. In studies of string and roller chain drive dynamics, it is often assumed that polygonal action leads to a parametric excitation described by time harmonic variation of span tension or velocity [16].

Attempting to solve the coupled vibration problems of axially moving string, many researchers have investigated the applications of various mathematical methods on it. Generally, employing the Galerkin discretization, the equation of motion is reduced to a set of nonlinear ordinary differential equations (ODE) with coupled terms. [17-19]. For example, Chen [20] applied the method of multiple scales based on the Galerkin discretization to solve the governing equation of transverse nonlinear vibration in the parametric resonances. Nowadays, approximation techniques play an indispensable role in complementing exact solutions. Wavelets technique represents a newly developed powerful mathematical tool, which has been broadly applied to solve differential equations. Pernot [21] introduced a wavelet Galerkin method in order to obtain transient and periodic solutions of multi-degree-of-freedom dynamical systems with time-periodic coefficients. Liu et al. [22] presented an efficient wavelet-based algorithm for solving a class of fractional vibration, diffusion and wave equations with strong nonlinearities. Likewise, a new wavelet approximation scheme of bounded functions based on techniques of boundary extension and Coiflet-type wavelet expansion has been developed by Wang [23].

In this study, considering transverse and longitudinal vibration, a coupled nonlinear string model was built to analyze the dynamic behavior of the main drive chain in escalator. The coupled dynamics vibration equation was derived by The Hamilton's principle and solved with the Wavelet-Galerkin method. In next section, a new experimental method for detecting dynamic vibration of main drive chain in escalator by utilizing image processing technology was developed. Finally, compared with experimental results, the validity of the string model, the effects of pre-tension and excitation of chain were discussed, which in principle are conductive to the study and engineering design of the main drive chain system in escalator.

2. Equation of motion for drive chain

It is proposed here the chain can be modeled as a uniform traveling string for the purpose of this analysis under the assumption that the chain span motion decouples from the dynamics of sprockets and attached machinery.

2.1 Governing equation of drive chain

The schematic of main drive chain system of escalator is shown in Fig.1. The main drive chain transmits the power at speed *v* between driving and driven sprocket.

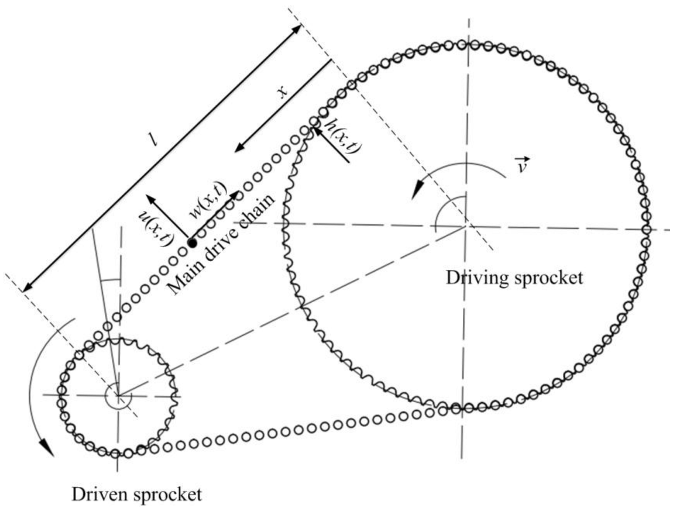


Fig. 1 schematic of main drive chain system of escalator

It is proposed here the chain can be modeled as an axially moving string with both ends hinged, which is shown in Fig.2. ρ c T E A are, respectively, density per unit length, damping coefficient, tension, Young's modulus and the cross-section area. $w(x,t)$ and $u(x,t)$ are longitudinal displacement and transverse displacement at time t and axial coordinate x related to coordinates translating at speed v. $h(x,t)$ is the excitation. Besides, the origin of the coordinate is set at one end of the string and the instantaneous length of the string is l.

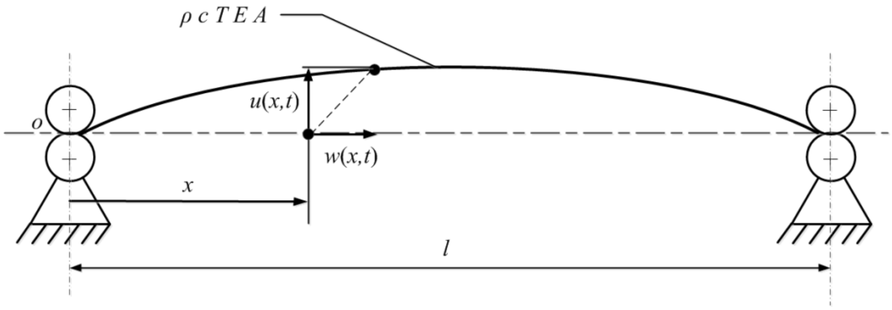


Fig. 2 A theoretical model of the string with the longitudinal and transverse vibration

The kinetic energy of the axially moving string is given by

$$K = \frac{1}{2}\rho\int_0^l \vec{v}\bullet\vec{v}dx \tag{1}$$

where

$$\vec{v} = \left(v + \frac{\partial w}{\partial t} + v\frac{\partial w}{\partial x}\right)\vec{i} + \left(\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial x}\right)\vec{j}$$

The potential energy is obtained as

$$U = \int_0^l \left(\frac{1}{2}EA\varepsilon^2 + T\varepsilon\right)dx \tag{2}$$

The displacement strain relation ε is expressed as

$$\varepsilon = \frac{\partial w}{\partial x} + \frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2 \tag{3}$$

Using Hamilton's principle, the governing equations of motion for the axially moving string system with longitudinal and transverse motions are derived as

$$\begin{aligned} &\rho\left(\frac{\partial^2 w}{\partial t^2} + 2V\frac{\partial^2 w}{\partial x\partial t} + V^2\frac{\partial^2 w}{\partial x^2}\right) - (EA+T)\frac{\partial^2 w}{\partial x^2} - EA\frac{\partial}{\partial x}\left[\frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2\right] = 0 \\ &\rho\left(\frac{\partial^2 u}{\partial t^2} + 2V\frac{\partial^2 u}{\partial x\partial t} + V^2\frac{\partial^2 u}{\partial x^2}\right) - EA\frac{\partial}{\partial x}\left[\frac{\partial w}{\partial x}\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^3\right] - T\frac{\partial^2 u}{\partial x^2} = h(x,t) \end{aligned} \tag{4}$$

The initial conditions and boundary conditions are given

$$w(0,t)=w(l,t)=0,\ w(x,0)=0$$
$$u(0,t)=u(l,t)=0,\quad u_{xx}(0,t)=u_{xx}(l,t)=0,\quad u(x,0)=0 \tag{5}$$

Equation (4) is a strong nonlinear differential equation. Following, we should to truncate the infinite dimensional partial differential equation (PDE) into a finite-dimensional ordinary differential equation.

2.2 Discretization of the governing equation in Wavelet Galerkin method

Firstly, some dimensionless variables are introduced as follows:

$$\bar{x} = \frac{x}{l}\ , \quad \bar{u} = \frac{u}{l}\ , \quad \bar{w} = \frac{w}{l} \tag{6}$$

We introduce $u_1 = \frac{\partial u}{\partial x}$ and $w_1 = \frac{\partial w}{\partial x}$, and according to the wavelet theory [22], the dimensionless functions can be approximated through the scaling function series as, respectively,

$$u \approx \sum_{k=1}^{2^j-1} u(x_k,t)\Phi_{j,k}(x)\ , \qquad w \approx \sum_{k=1}^{2^j-1} w(x_k,t)\tilde{\Phi}_{j,k}(x)\ ,$$

$$u_1 \approx \sum_{k=0}^{2^j} u_1(x_k,t)\varphi_{j,k}(x)\ , \qquad w_1 \approx \sum_{k=0}^{2^j} w_1(x_k,t)\varphi_{j,k}(x)\ , \tag{7}$$

To transform Equation (4) into a matrix equation,, by applying the wavelet Galekin method, Equation (4) is expressed as:

$$\begin{aligned}
&\rho l \sum_{k=1}^{2^j} \frac{\partial^2 w(x_k,t)}{\partial t^2} a_{lk} + \rho V \sum_{k=0}^{2^j-1} \frac{\partial w(x_k,t)}{\partial t} b_{lk} - \frac{T-\rho V^2}{l} \sum_{k=1}^{2^j} w(x_k,t) c_{lk} \\
&-\frac{1}{l}\sum_{k=1}^{2^j} f[u_1(x_k,t), w_1(x_k,t)] m_{lk} \approx 0 \\
&\rho l \sum_{k=1}^{2^j-1} \frac{\partial^2 u(x_k,t)}{\partial t^2} d_{lk} + \rho V \sum_{k=0}^{2^j-1} \frac{\partial u(x_k,t)}{\partial t} e_{lk} - \frac{(-\rho V^2 + T)}{l} \sum_{k=1}^{2^j-1} u(x_k,t) f_{lk} \\
&-\frac{1}{l}\sum_{k=0}^{2^j} g[u_1(x_k,t), w_1(x_k,t)] p_{lk} - \sum_{k=0}^{2^j} h(x,t) q_{lk} \approx 0
\end{aligned} \tag{8}$$

where

$$a_{lk} = \int_0^1 \tilde{\Phi}_{j,k}(x)\tilde{\Phi}_{j,l}(x)dx \qquad b_{lk} = \int_0^1 \tilde{\Phi}'_{j,k}(x)\tilde{\Phi}_{j,l}(x)dx \qquad c_{lk} = \int_0^1 \tilde{\Phi}''_{j,k}(x)\tilde{\Phi}_{j,l}(x)dx$$

$$m_{lk} = \int_0^1 \varphi'_{j,k}(x)\tilde{\Phi}_{j,l}(x)dx \qquad d_{lk} = \int_0^1 \Phi_{j,k}(x)\Phi_{j,l}(x)dx \qquad e_{lk} = \int_0^1 \Phi'_{j,k}(x)\Phi_{j,l}(x)dx$$

$$f_{lk} = \int_0^1 \Phi''_{j,k}(x)\Phi_{j,l}(x)dx \qquad p_{lk} = \int_0^1 \varphi'_{j,k}(x)\Phi_{j,l}(x)dx \qquad q_{lk} = \int_0^1 \varphi_{j,k}(x)\Phi_{j,l}(x)dx$$

$$f[u_1(x_k,t), w_1(x_k,t)] = \frac{1}{2} EA u_1^2(x_k,t)$$

$$g[u_1(x_k,t), w_1(x_k,t)] = EA[w_1(x_k,t) \bullet u_1(x_k,t) + \frac{1}{2} u_1^3(x_k,t)]$$

As above, by applying the Wavelet-Galerkin method, without inversion of matrix and special solution technique to deal with the nonlinear spatial and temporal operators, we can obtain the discretized nonlinear algebraic equations:

$$\begin{aligned}
&\frac{d\mathbf{U}}{dt}=\mathbf{U}_t \\
&\frac{d\mathbf{W}}{dt}=\mathbf{W}_t \\
&\frac{d\mathbf{U}_t}{dt}=-\frac{V}{l}\mathbf{D}^{-1}\mathbf{E}\mathbf{U}_t+\frac{(T-\rho V^2)}{\rho l}\mathbf{D}^{-1}\mathbf{F}\mathbf{U}+\frac{1}{\rho l^2}\mathbf{D}^{-1}\mathbf{P}\mathbf{G}_t+\frac{1}{\rho l}\mathbf{D}^{-1}\mathbf{Q}\mathbf{H} \\
&\frac{d\mathbf{W}_t}{dt}=-\frac{V}{l}\mathbf{A}^{-1}\mathbf{B}\mathbf{W}_t+\frac{T-\rho V^2}{\rho l}\mathbf{A}^{-1}\mathbf{C}\mathbf{W}+\frac{1}{\rho l^2}\mathbf{A}^{-1}\mathbf{M}\mathbf{F}_t
\end{aligned} \tag{9}$$

Where the matrices and vectors are A={a_{lk}}, B={b_k}, C={c_{lk}}, D={d_{lk}}, E={e_{lk}}, M={m_{lk}}, F={f_{lk}},P={p_{lk}}, Q={q_{lk}}, U=[$u(x_1,t)$, $u(x_2,t)$, ⋯, $u(x_{2^{m-1}},t)$,]^T, H=[$h(x_0,t)$, $h(x_1,t)$, ⋯, $h(x_{2^{m-1}},t)$, $h(x_{2^m},t)$]^T.

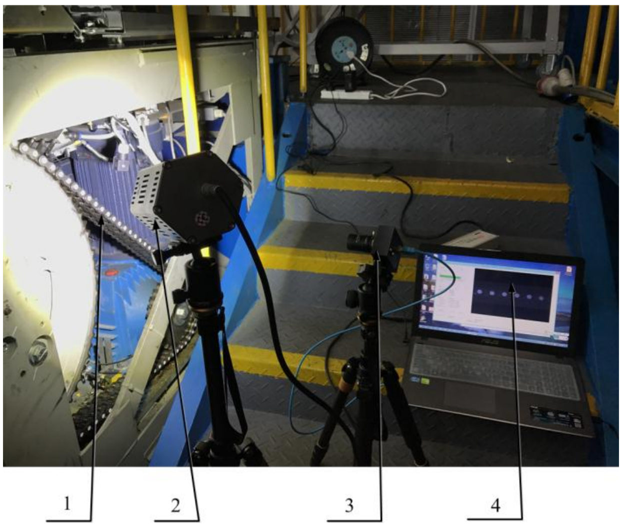
So far, the discretization in space of the governing equation in Equation (4) has been fulfilled. It can be seen the wavelet-Galerkin procedure uncoupled the complicated spatial and temporal dependences in the original PDEs, results in a set of optimally minimized ODEs. Then based on the parameters given by Table 1, numerical solutions can be computed by applying the wavelet Galekin method.

Table 1. The values of the parameters

l	ρ	V	E	A	T
0.66m	5.4kg/m	0.449m/s	2.3Gpa	$2\times10^{-4}m^2$	-

3. **Experimental design and configuration**

The experiment was carried out by the camera measurement method for the transverse vibration of chain. Figure 3 depicts the chain vibration measurement system, which consists of a LED light source, a high speed industrial camera, the marked object and a computer for image processing.



1.Marks 2. LED 3. High speed camera 4. PC control and record

Fig.3 Structure of chain vibration measuring equipment

In the experiment, a series of marks were purposely made at each junction center of chain links. During the traction, the positions of those marks were recorded with camera and the raw images were processed first with five detailed processes: image reading, threshold calculation and binarization, dilation, erosion and opening operation. Vibrations of chain could be calculated after image processing. Besides, the experiment for camera calibration was performed and the results showed that the measurement relative error of displacement was roughly in the range of -2.7%~2.8%.

4. Results and discussion

4.1Analysis of the validity of the string model

There are two traction conditions for escalator: running down and running up. Notably, it will need larger traction force for running up than running down under the same load. In this section, experiment and numerical simulation have been done to investigate the vibration of descending and ascending escalator chain under empty load.

Firstly, the dynamic analysis of ascending escalator chain under empty load is presented. Figure 4 depicts the comparison of transverse vibrations for one-third position of the ascending chain between experiment and numerical simulation. Notably, the tension of the chain is about 3300N. It can be seen that both of behaviors including amplitude and waveform for experiment and numerical simulation are comparable.

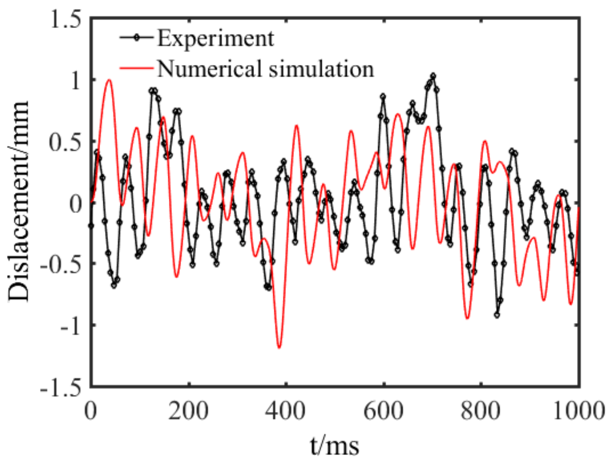


Fig.4 Comparison of transverse vibrations for one-third position of the ascending chain between experiment and numerical simulation

Meanwhile, amplitude-frequency characteristics for one-third position of the chain between numerical and experimental data have been shown in Figure 5. It can be seen the natural frequency is identified as 18.8Hz from experiment while it is 18.52Hz from numerical simulation. Obviously, the numerical result is in good line with the experimental result and the relative deviation is about 1.5%. This confirms that the proposed string model performs well in simulating the vibration of ascending escalator under empty load. It should be noted that some inconsistencies between experimental and numerical results also exist in other frequencies, which may be on account of some practical factors, such as environment noise and inexact excitation in simulation.

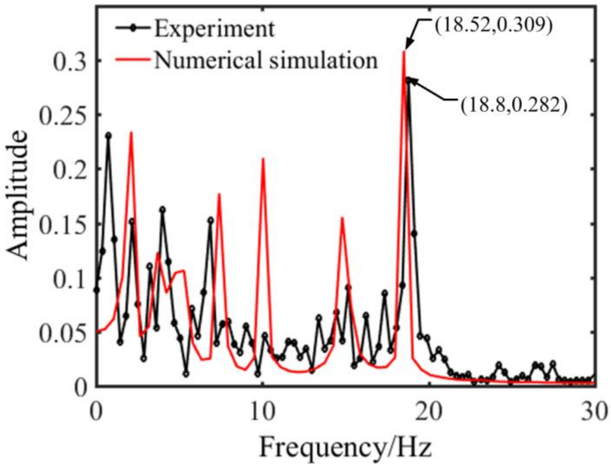


Fig.5 Comparison of frequency characteristics for one-third position of the ascending chain between experiment and numerical simulation

Following, the dynamic analysis of descending escalator chain under empty load is provided. Figure 6 shows the comparison for one-third position of the descending chain between experiment and numerical simulation. Notably, the tension of the chain is about 10N. Obviously, dynamic behaviors including amplitude, waveform and frequency for experiment and numerical simulation are comparable. Namely, the proposed string model is not valid for the chain of descending escalator under empty load.

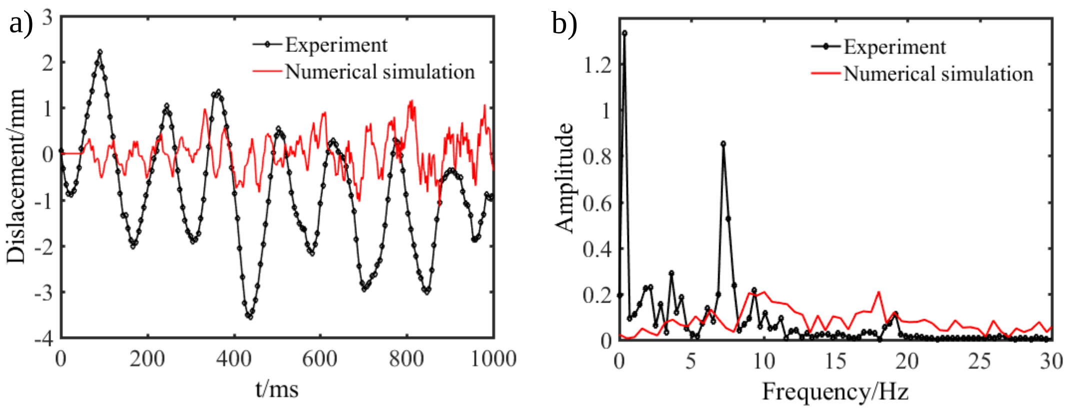


Fig.6 Comparison for one-third position of the descending chain between experiment and numerical simulation (a) transverse vibrations (b) frequency characteristics

In particular, the chain is characterized by the nature of the chain links and sprocket teeth. When the tension of the chain is so small, we can't neglect the different mechanical mechanism between the chain links and junctions. Namely, continuous model can't be applied to study chain. That is the reason the proposed string model is valid for the ascending chain but failed for the descending chain.

4.2 The effects of pre-tension

In this section, the relationship between amplitude, frequency and tension has been investigated. As referred above, the proposed string model is valid when the tension is large enough, so we studied the transverse vibrations of the string model when the tension is larger than 2000N. Figure 7 shows the relationship between amplitude, frequency and tension. It can be seen that with the increase of tension, the amplitude decreases while the frequency increases.

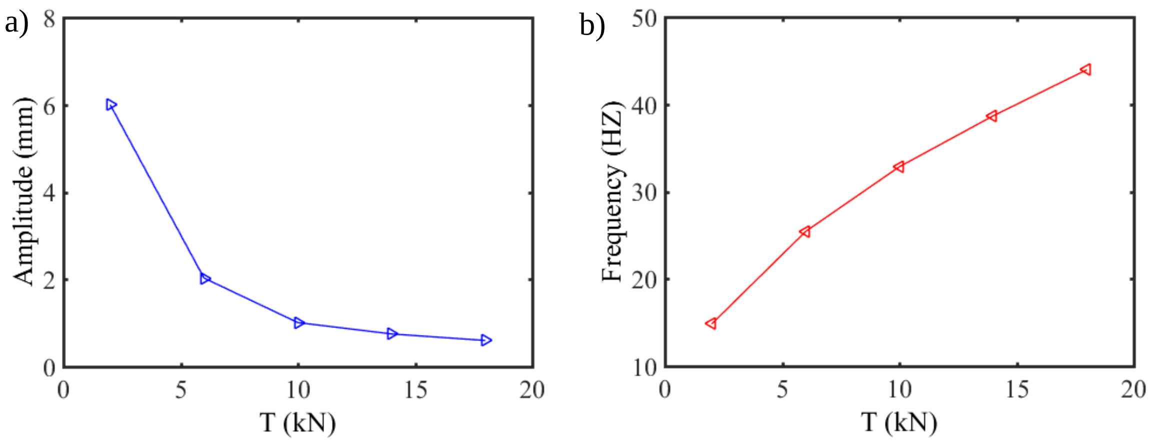


Fig.7 The relationship between tension and (a) amplitude (b) frequency

5. Conclusion

In this paper, a Wavelet-Galerkin method is presented for solving the dynamic behavior of the main drive chain in escalator. It can be found the Wavelet-Galerkin method makes it possible to avoid numerically calculating integral of multifold-products of scaling functions and their derivatives in the string vibration equations. Furthermore, the experiment was carried out by the camera measurement method for the transverse vibration of chain, whose results were compared with numerical results. Based upon these results, the following conclusions are obtained:

(1) The Wavelet-Galerkin method presented in this paper can be applied to solve the coupled nonlinear string vibration equations.

(2) The proposed string model is valid for the ascending chain but failed for the descending chain under low load.

(3) For the transverse vibrations of the string model in this paper when the tension is large, with the increase of tension, the amplitude decreases while the frequency increases.

6. Acknowledgment

This research was supported by the National Key Research and Development Program of China under Grant No. 2017YFC0805700.

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