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A Support Vector Regression-based Operational Transfer Path Analysis Method for Detecting Vibro-acoustic Sources with Uncertain Path Contributions

Hu, Han[1]
State Key Laboratory of Mechanical System and Vibration, Institute of Vibration, Shock and Noise, Shanghai Jiao Tong University, Shanghai 200240, China
Room B402, ME School, Dongchuan Road 800, Shanghai, China

Mi, Yongzhen[2]
State Key Laboratory of Mechanical System and Vibration, Institute of Vibration, Shock and Noise, Shanghai Jiao Tong University, Shanghai 200240, China
Room B402, ME School, Dongchuan Road 800, Shanghai, China

Zheng, Hui[3]
Collaborative Innovation Center for Advanced Ship and Deep-sea Exploration (CISSE), Shanghai 200240, China
Room A804, ME School, Dongchuan Road 800, Shanghai, China

ABSTRACT

Operational Transfer Path Analysis (OTPA) is a widely-used approach in engineering to detect the dominant source of vibration or noise by comparing the contributions of several pre-defined vibration/noise transfer paths. Because of its dependency on the quality of signals, this approach may identify incorrect path contributions, particularly when random noise is mixed into the collected data. A number of deterministic techniques, e.g. singular value truncation, have been proposed to mitigate this problem. However, the performances of these techniques rely heavily on careful parameter-tuning. To address this issue, a modified OTPA method is presented in this paper which uses support vector regression (SVR) to evaluate the inherent uncertainties associated with the path contributions. Apart from its data-based, automatic way for parameter definition, the main advantage of the proposed method is that it transforms each contribution from a single value to an adjustable interval, so that the reliability of the predictions could be measured and further examined. The effectiveness of the proposed method is verified against traditional OTPA by a simple acoustic emitter-and-receiver numerical example.

Keywords: Transfer Path Analysis, Support Vector Regression, Uncertain Path Contribution
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[1] hhwaiting@sjtu.edu.cn
[2] miyongzhen@sjtu.edu.cn
[3] huizheng@sjtu.edu.cn

1. INTRODUCTION

Operational Transfer Path Analysis (OTPA), a transmissibility function based method, has been proved effective in a range of engineering applications, such as source detecting and vibro-acoustic path contribution evaluation [1]. The method is growing increasingly popular because instead of using real source data, which is difficult to obtain, it only uses data measured in the vicinity of the source and in operational conditions. For many NVH problems, OTPA is a preferable choice, since it eliminates the need to dismantle the source off the system, a time-consuming but mandatory process for the traditional Transfer Path Analysis to get accurate frequency response functions of the system. However, as many authors pointed out, OTPA also suffers from several limitations: 1) errors from neglected important paths, 2) errors from cross-coupling between input signals, 3) errors in the estimation of transmissibility functions [2]. In this paper, we propose a support vector regression (SVR)-based OTPA method in which the transmissibility functions are simultaneously calculated and regularized based on multiple operational data. In particular, the uncertainties of path contributions are quantified through cross validation of multiple models. A 95% prediction interval of the path contributions is observed in the results given by the new method, which proves its higher accuracy than the traditional OTPA method.

2. OTPA Theory

2.1 Traditional OTPA method

In traditional OTPA method, the system under analysis is assumed to be linear time-invariant, and the output of the system is the superposition of various inputs from different transfer paths as

$$Y = XT \tag{1}$$

In the equation Y represents the output (acceleration, force, sound pressure, etc.), X represents the input (acceleration, force, sound pressure, etc.) and T represents the transmissibility function matrix. The equation is frequency depended and for each frequency line, the transmissibility function matrix can be written as follows, where the subscript (opi) represents for the ith experimental operating condition.

$$\begin{bmatrix} t_{11} & t_{12} & \cdots & t_{1l} \\ t_{21} & t_{22} & \cdots & t_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ t_{n1} & t_{n2} & \cdots & t_{nl} \end{bmatrix} = \begin{bmatrix} x_{1(op1)} & x_{2(op1)} & \cdots & x_{n(op1)} \\ x_{1(op2)} & x_{2(op2)} & \cdots & x_{n(op2)} \\ \vdots & \vdots & \ddots & \vdots \\ x_{1(ops)} & x_{2(ops)} & \cdots & x_{n(ops)} \end{bmatrix}^{+} \begin{bmatrix} y_{1(op1)} & y_{2(op1)} & \cdots & y_{l(op1)} \\ y_{1(op2)} & y_{2(op2)} & \cdots & y_{l(op2)} \\ \vdots & \vdots & \ddots & \vdots \\ y_{1(ops)} & y_{2(ops)} & \cdots & y_{l(ops)} \end{bmatrix} \tag{2}$$

It should be noted that the number of experimental operating conditions s should be no less than the number of inputs to ensure reversibility of the input matrix X . The paths contribution to the ith output under the jth experimental operating condition can be calculated as

$$y_{i(opj)} = x_{1(opj)}t_{1i} + x_{2(opj)}t_{2i} + \cdots + x_{n(opj)}t_{ni} \tag{3}$$

However, as the input signals are measurements from reference points around the real source of the system, some measurement noise and cross-coupling from other sources are also inevitably considered into the input signals. That, to a certain degree, causes the uncertainty of OTPA. In traditional OTPA method, the input distortion due to noise and cross-coupling is solved by cross talk cancellation (CTC), namely using truncated singular value decomposition (TSVD) to prevent poor estimates of the transmissibility function. The singular value decomposition of the input matrix X can be expressed as

$$X = U\Lambda V^{T} \tag{4}$$

U is an $s \times s$ unitary matrix, V is a $n \times n$ unitary matrix and V^T denotes its conjugate transpose. Λ is an $s \times n$ matrix ($s \geq n$) having the form as

$$\Lambda = \begin{bmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \ddots & \\ & & & \sigma_n \\ & & & \vdots \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{5}$$

where the elements on the diagonal are singular values and satisfy $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_n \geq 0$. Generally, larger singular value contains more information of input matrix and small singular value contributes little and can be seen as noise. The truncation can be done according to the contribution rate (CR) as $CR = (\sigma_i / \sum_{i=1}^{n} \sigma_i) \times 100\%$, $(0 \leq i \leq n)$. By setting a threshold τ, those singular value whose $CR \leq \tau$ will be rejected. The new transmissibility function matrix thus becomes

$$\tilde{T} = \tilde{X}^{-1} Y = V \tilde{\Lambda}^{-1} U^T Y \tag{6}$$

Therefore the paths contribution of input signals under another k experimental operational condition can be expressed as

$$y_{i(opj)} = x_{1(opj)} \tilde{t}_{1i} + x_{2(opj)} \tilde{t}_{2i} + \cdots + x_{n(opj)} \tilde{t}_{ni} \tag{7}$$

2.2 SVR based OTPA method

Support vector regression is a machine learning algorithm originally invented by Vapnik and his colleagues [3] and has been largely developed during the past few decades. It is useful to recognize subtle patterns in complicated data sets and generalize well to unseen data in both linear and non-linear task. In this paper we assume the system is of linear property in most time, so we focus on using SVR to solve linear regression problem. And the function we estimate does not contain any bias term. For instance, suppose we have s operational data and n input signals in every frequency line $\{(x_1,y_1),(x_2,y_2),...,(x_s,y_s)\} \subset \mathbb{R}^n \times \mathbb{R}$. In $\varepsilon - SVR$ our objective is to find a function $f(x) = \langle \omega, x \rangle$ with $\omega \in \mathbb{R}^n$ that has an up limit of ε deviation of the target point y_i for all training data, and at the same time as flat as possible, which can be expressed as

$$\begin{aligned} &\text{minimize} \quad \frac{1}{2}\|\omega\|^2 + C\sum_{i=1}^{l}(\xi_i + \xi_i^*) \\ &\text{subject to} \begin{cases} y_i - \langle \omega, x_i \rangle \leq \varepsilon + \xi_i \\ \langle \omega, x_i \rangle - y_i \leq \varepsilon + \xi_i^* \\ \xi_i, \xi_i^* \qquad \geq 0 \end{cases} \end{aligned} \tag{8}$$

where $\langle \cdot, \cdot \rangle$ represents for dot product in $\mathbb{R}^n$, C is a positive constant that determines the trade-off between the flatness of f and the amount up to which deviations larger than ε are tolerated. ξ_i, ξ_i^* are slack variables introduced to make the margin “soft” to cope with infeasible constraints. The ε-insensitive loss function $|\xi|_\varepsilon$ has been described by

$$|\xi|_{\varepsilon} = \begin{cases} 0 & if \ |\xi| < \varepsilon \\ |\xi| - \varepsilon & otherwise \end{cases} \tag{9}$$

In most cases, it turns out that optimization problem (8) can be solved easily in its dual formulation (and also provides the key for extending SVR to some nonlinear functions) as

$$\begin{aligned} L &:= \frac{1}{2}\|\omega\|^2 + C\sum_{i=1}^{l}(\xi_i + \xi_i^*) - \sum_{i=1}^{l}(\eta_i \xi_i + \eta_i^* \xi_i^*) \\ &- \sum_{i=1}^{l}\alpha_i(\varepsilon + \xi_i - y_i + \langle \omega, x_i \rangle) \\ &- \sum_{i=1}^{l}\alpha_i^*(\varepsilon + \xi_i^* + y_i - \langle \omega, x_i \rangle) \end{aligned} \tag{10}$$

L denotes the Lagrangian and $\alpha_i, \alpha_i^{(*)}, \eta_i, \eta_i^{(*)}$ are non-negative Lagrangian multipliers (also called dual variables). It follows from saddle point condition that the partial derivatives of L with respect to the primal variables (ω, ξ, ξ^*) have to vanish for optimality.

$$\frac{\partial L}{\partial \omega} = \omega - \sum_{i=1}^{l}(\alpha_i^* - \alpha_i)x_i = 0 \tag{11}$$

$$\frac{\partial L}{\partial \xi^{(*)}} = C - \alpha^{(*)} - \eta^{(*)} = 0 \tag{12}$$

$\alpha^{(*)}$ refers to α and α^*. Substituting Equation (11) and (12) into (10) yields the dual optimization problem.

$$\text{maximize} \quad \begin{cases} -\frac{1}{2}\sum_{i,j=1}^{l}(\alpha_i - \alpha_i^*)(\alpha_j - \alpha_j^*)\langle x_i, x_j \rangle \\ -\varepsilon\sum_{i=1}^{l}(\alpha_i + \alpha_i^*) + \sum_{i=1}^{l} y_i(\alpha_i - \alpha_i^*) \end{cases} \tag{13}$$

$$\text{subject to} \quad 0 \le \alpha_i^{(*)} \le C$$

Equation (11) can be rewritten as $\omega = \sum_{i=1}^{l}(\alpha_i^* - \alpha_i)x_i$ and therefore we can get the so-called support vector expansion $f(x) = \sum_{i=1}^{l}(\alpha_i - \alpha_i^*)\langle x_i, x \rangle$.

Hyper-parameters C and ε play the role of regularization in SVR and after being carefully picked, they can largely cancel the distortion of measurement noise as well as cross-coupling effect. For the selection of those parameters, detailed investigation has been done by Vladimir Cherkassky and Yunqian Ma [4], who proposed to choose regularization parameter as

$$C = \max(|\bar{y} + 3\sigma_y|, |\bar{y} - 3\sigma_y|) \tag{14}$$

$$\varepsilon = 3\sigma\sqrt{\frac{\ln s}{s}} \tag{15}$$

where $\bar{y}$ and σ_y are the mean and standard deviation of the output, respectively. s is the number of training samples and σ is the estimate of noise according to the following formula

$$\sigma = \sqrt{\frac{1}{s-n}\sum_{i=1}^{s}\left(y_i - \hat{y}_i\right)^2} \tag{16}$$

where y_i is the ith response output and $\hat{y}_i$ is the least-square fitting of the training data.

Root mean square error (RMSE) is usually used to measure the errors for accuracy assessment, which is expressed as

$$RMSE = \sqrt{\frac{\sum_{i=1}^{s}(y_i - \hat{y}_i)^2}{s}} \tag{17}$$

3. A simple numerical case

3.1 Acoustical field with spherically radiating sources

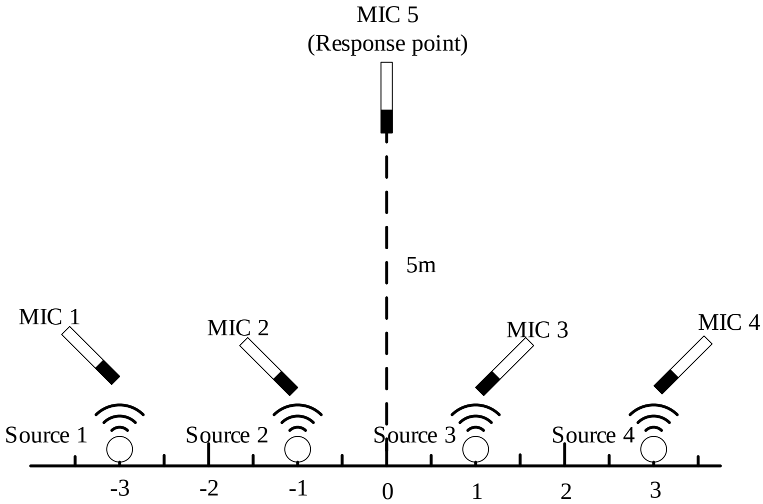

Fig. 1 Acoustical field of 4 sources

As shown in Fig. 1, an acoustical problem with 4 spherically radiating sources is designed to evaluate the performances of the new SVR-based OTPA method in comparison with the traditional OTPA method. The positions of sound sources and response point (MIC 5) are marked in the figure and the vertical distance between each sound source and its nearby microphone is 0.2m while the horizontal distance is 0. The radius of the 4 sources is 0.05m ($a = 0.05m$). The sound velocity of the sources are $v_1 = v_2 = v_3 = 30m/s$, $v_4 = 15m/s$. Assume all the sound sources and response point are in a free sound field, which the sound pressure $p_i(r,\omega_i)$ can be expressed as

$$p_i(r,\omega) = v_i \frac{j\omega\rho_0}{1+jka} \cdot \frac{e^{-jk(r_i-a)}}{r_i}, \; i = 1 \sim 4 \tag{18}$$

where $j\omega$ denotes the complex angular frequency, $\rho_0 = 1.29kg/m^3$ is the density of the air, $k = \frac{\omega}{c}$ is the wave number. $c = 340m/s$ is the sound velocity in the air, and r_i is the distance between source i and other points in the sound field.

The reference point (MIC 1~4) receive not only the sound of its nearby sound source but also from other sources even though far away from it, and moreover with random noise of the sound field. So the sound pressure for the ith microphone get is

$$\hat{p}_{itot} = \sum_{j=1}^{4} v_j \frac{j\omega_j \rho_0 a^2}{1 + jk_j a} \cdot \frac{e^{-jk_j(r_{ji}-a)}}{r_{ji}} + noise(\omega),\ i = 1 \sim 5 \tag{19}$$

in which $noise(\omega)$ is random noise whose amplitude is 0.2 of the surface velocities of sound sources. The frequency resolution is $\Delta f = 1Hz$ and the analytical frequency band is 0~100 Hz. Furthermore, the characteristic sound pressure frequencies of the 4 sources are $f_1 = 15Hz$, $f_2 = 25Hz$, $f_3 = 35Hz$ and $f_4 = 15Hz$.

3.2 Numerical simulation

In the simulation, we define the number of experimental operating conditions to be 11. The first 10 conditions are used as a known dataset (training set), and the last condition is used as the data to be predicted (test set). Note that for each condition the operating data spans the whole frequency band (0~100 Hz). For traditional OTPA method, we use all training data to build the OTPA model with $\tau = 3\%$ and test the performance of the model by test data. For SVR-based OTPA method, we use every 8 out of 10 operating conditions in the training set to build $C_{10}^{8} = 45$ models, and evaluate each model by the test data. The mean of all 45 outputs are used as the final result, and the deviation of the output are selected as an indicator of the uncertainty of total or each transfer path caused by measurement noise. The 95% predicted uncertainty (95 PPU) is also calculated.

As shown in Fig. 2, both the SVR-based OTPA method and traditional OTPA method show good generalization ability and their performances in fitting the test result is similar, especially for such characteristic frequencies as 25Hz and 35Hz. However, the response value at 15Hz is obviously underestimated by both methods. These errors are resulted from the unwanted coherence between Source 1 and Source 4, which adds numerical difficulties in inversing the transmissibility function matrix. In fact, the decomposition of highly coherent sources is known as one of the most challenging obstacles in OTPA [5].

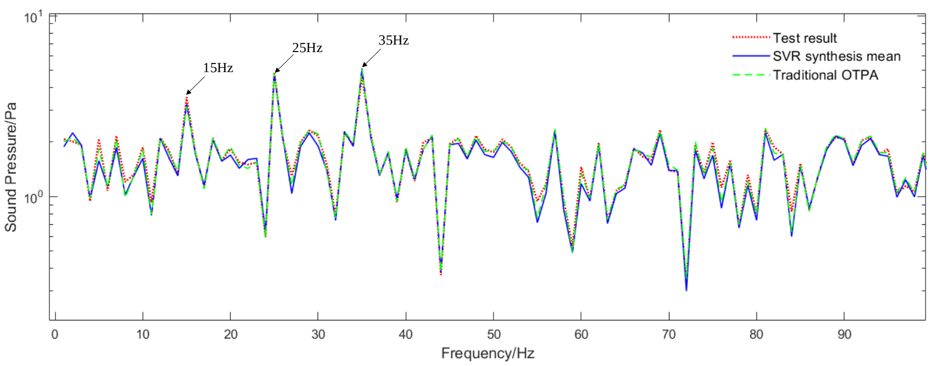

Fig. 2 Test and predicted data of response point

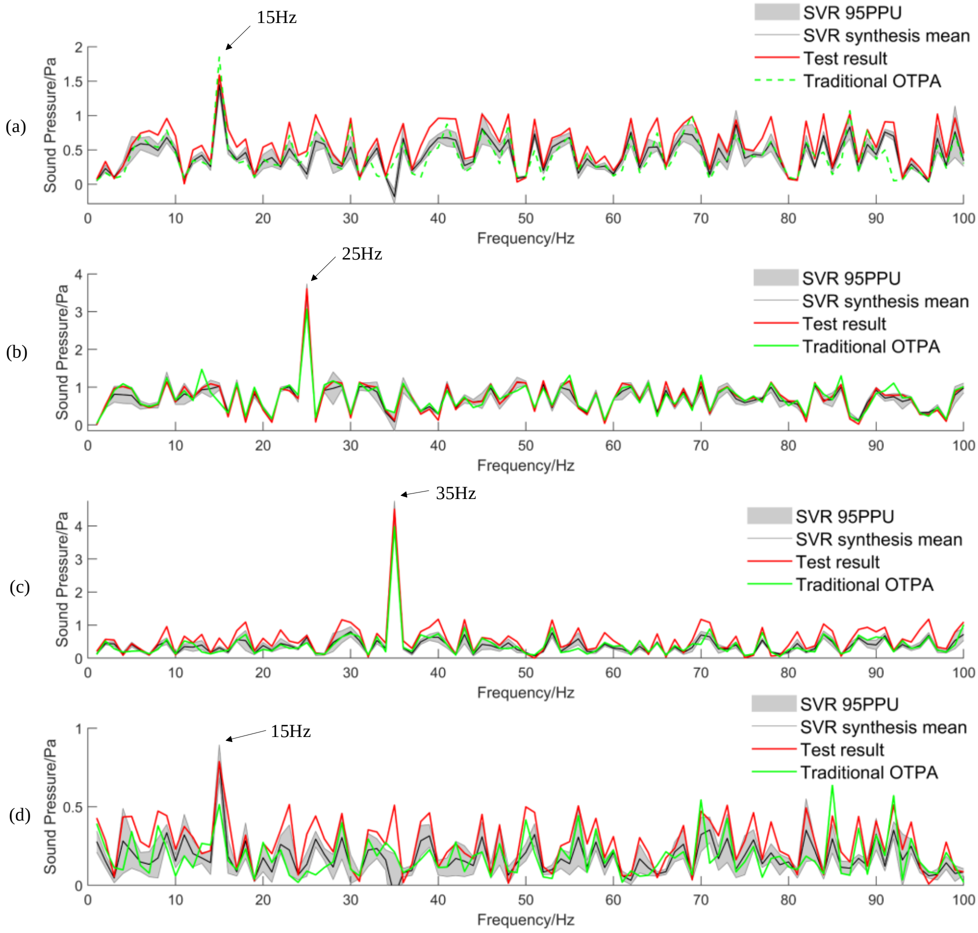

Fig. 3 Path contribution of (a) Path 1 (b) Path 2 (c) Path 3 (d) Path 4 of test result, SVR synthesis mean and 95PPU and traditional OTPA method

For each transfer path (*Path i* meaning the transfer path from *Source i* to the response point) the path contributions of the test result, the output of proposed method and traditional OTPA are compared in Fig. 3. In addition, 95PPU of the output of the proposed method is also shown in the figure. It is evident that the performance of the SVR method is much better than traditional OTPA method in predicting the path contribution. The same conclusion can also be drawn in terms of root mean square error (Table. *1*): the RMSE values of proposed method prediction are smaller than that of traditional OTPA method prediction, neither the total response nor the path contribution.

The provided 95PPU of proposed method shows how far prediction changes in different models built upon existing experimental operating data, and how severe the noise affects the result. If higher noise at some frequency is involved in the data, the result at that point will surely show a larger uncertainty interval. However, as traditional OTPA method only calculates synthesis once, the result can be any inside that interval, which may mislead to identify the wrong dominant path.

The uncertainty interval in Fig. 3 (d) seems to be more notable than others, while it is so because the scale of the Y-axis of (d) is evidently smaller and the uncertainty level is actually the same as others. Furthermore, it can be referred that the uncertainty for path contribution is related to the variance of the noise while not in relation to the value at characteristic frequency (15Hz, 25Hz, and 35Hz). Therefore, an accurate prediction will be made if a high signal to noise rate is guaranteed.

Table. 1 RMSE of predicted result using SVR method and Traditional OTPA method

RMSE	Total Response	Path1	Path2	Path3	Path4
SVR synthesis mean	0.1179	0.1978	0.0910	0.2403	0.1311
Traditional OTPA	0.2170	0.6087	0.8206	0.6919	0.1996

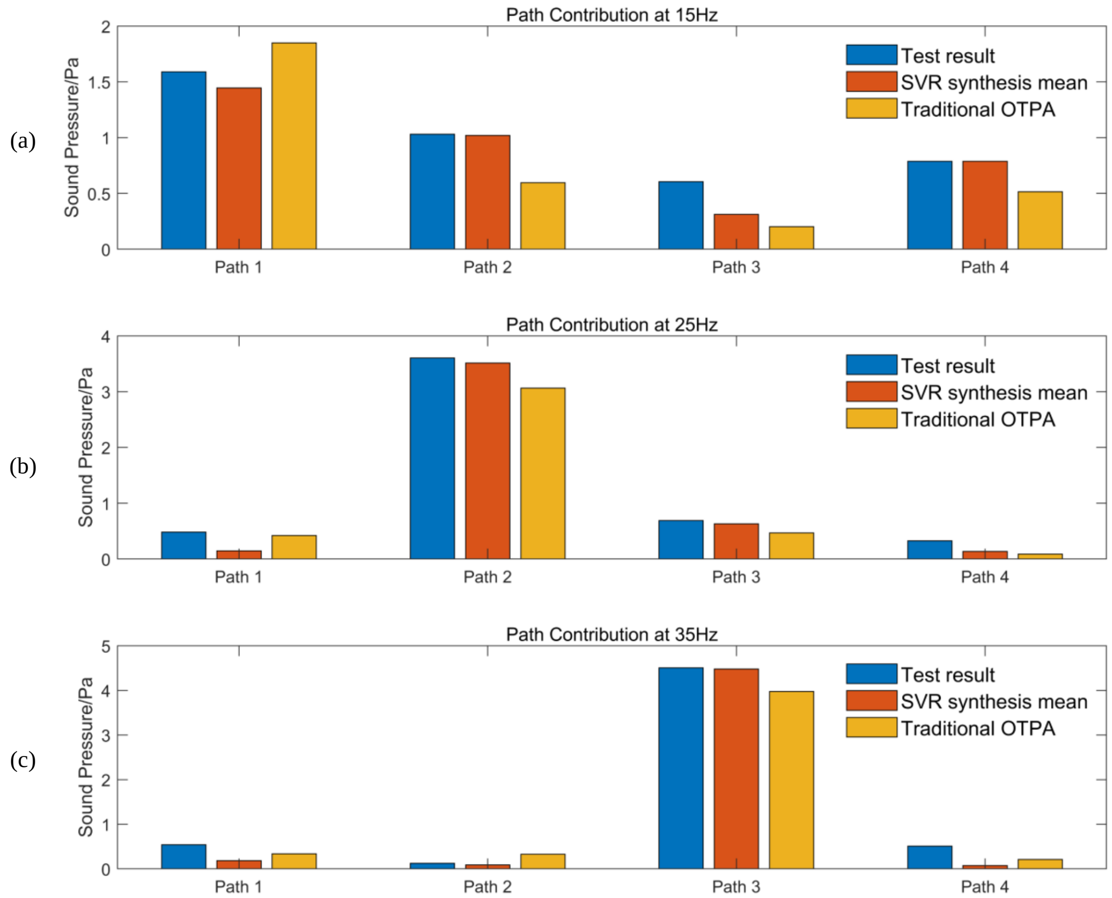

Fig. 4 Path contribution at characteristic frequency of (a) 15Hz (b) 25Hz (c) 35Hz

Detecting the dominant path at each characteristic frequency is the final and most important task for OTPA. In this case, as shown in Fig. 4, path contributions predicted by two OTPA methods are compared with the test result. Intuitively, both methods can identify the right dominant path, but SVR-based OTPA method outperforms the other one in terms of prediction accuracy. The traditional OTPA method fails to give precise predictions at certain frequencies: The dominant path prediction in Fig. 4 (a) is overestimated and underestimated in Fig. 4 (b) and (c).

However, the ranking of path contributions is not as reliable as identification of the dominant one. Even though we get the right ranking of two OTPA methods in Fig. 4 (a), their predicted ranking in Fig. 4 (c) is not predicted as expected. That reminds us that OTPA methods should not be extended to give contribution ranking to paths.

4. CONCLUSIONS

A new SVR-based OTPA method is proposed in this paper, with emphasis on its ability to quantify the uncertainties of transfer path contributions. A simple acoustic emitter-and-receiver case is designed and the performance of the new method is examined by comparing with the traditional SVD truncated OTPA method. The results show that the SVR-based OTPA method outperforms the traditional one in terms of prediction accuracy and confidence. Therefore, the proposed method can be used as a better alternative for transfer path analysis in NVH applications.

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