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## **Investigation on the Improved Adaptive Algorithm in Vibration Reduction with Narrow-band Gaussian Disturbance**

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### **ABSTRACT**

**Effects of active vibration control, making use of the adaptive algorithm, can be significantly affected by property of the secondary path, which relates to the transfer function of controlled structures. The secondary path is too difficult to be identified**

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**for constructing under narrow-band Gaussian excitation, due to multiple natural frequencies and complex mode shapes. Therefore, it is hard to achieve satisfactory active vibration attenuation in narrow-band frequency range. In this paper, an improved adaptive algorithm is presented for the purpose of vibration suppression under narrow-band Gaussian disturbance. Several adaptive algorithms with the secondary path identification are thoroughly demonstrated to pave the way to the proposed adaptive algorithm. In the meantime, experiment of active vibration control is designed to verify the better performance compared with that of the original algorithm. Results show that large attenuation of the response under the narrow-band Gaussian disturbance is achieved with the improved algorithm. Moreover, the secondary path identification of the improved algorithm can be artificially directed, which has potential in realize active vibration control with limited bandwidth vibration disturbance.**

**Keywords:** Active vibration control, Adaptive algorithm, Secondary path identification  
**I-INCE Classification of Subject Number:** 46

## **1. INTRODUCTION**

Active vibration control system is widely used to reduce structural vibration and noise. The vibration attenuation effect is mainly determined by the pre-estimating signal, which is calculated firstly through a control algorithm and then implemented by the secondary actuator. As a result, the adaption of algorithm is colselly related to the effect of vibration response. Due to the high robustness, fast convergence rate and good stability, most intelligent control algorithms are effective for periodic, impact or random excitation. As a result, they are mainly used to attenuate the resonance phenomenon of structure excited by various excitations.

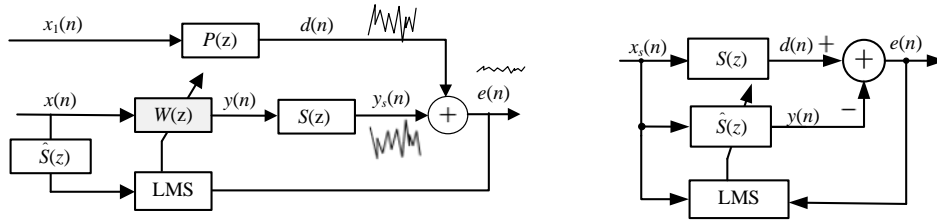
Among those intelligent control algorithms, the adaptive algorithm has been paid much attention to achieve the vibration attenuation under periodic or quasi-periodic excitations for its stability and fitness. Besides, the adaptive algorithms with secondary path identification are especially advantangeous to achieve vibration attenuation under periodic and white-noise excitations [1, 2]. Owing to the time-varying behavior of structure, the secondary path is usually pre-identified before or during active vibration process. Secondary path identification is added in FxLMS [3] to filter the reference signal of LMS [4]. However, as it is difficult to guarentee the accuracy in modelling the secondary path, many improved strategies about secondary path identification are proposed to coordinate the imbalance between convergence speed and steady state during the active control process. To enhance the identification accuracy, Ericsson [5] proposed a strategy, that is, introducing additional white noise for online secondary path identification. This strategy was adopted by Akhtar [6], who proposed a variable step size in secondary path identification process, so as to decrease the influence between identification process and control process. And Davari [7] improved this strategy by describing a variable power method of additional noise to reduce the influence. Meanwhile, more strategies are proposed for the steady state, which contribute to obtaining better convergence performance. Richard [8] proposed FxLMP (filtered-x mean p-power algorithm) to obtain better vibration attenuation effect under impulsive excitation compared with FxLMS, by ensuring the minimizing stable error. Lopes [9] proposed a new bandwidth limited modified FxLMS and found that it was less sensitive to secondary path modelling errors than the FxLMS. Li [10] proposed a time-domain FxNewton (filtered-x Newton narrowband algorithm) with inverse secondary-path filtering the extracted sinusoidal reference signal to deal with more complex excitations,

such as swept and multiple high-level harmonics in broadband vibration. Although many efforts have been paid to deal with the influence of secondary path, FxLMS strategies are not perfect yet. Parra [11] found that the strategy making use of additional white noise is not as effective as expected when the difference between identified FIR (finite impulse response) filter and the actual secondary path is unavoidable.

Nowadays many researches have been focused on improving the secondary path identification strategy. Based on these researchments, secondary path identification of FxLMS can be improved to enhance the accuracy in concerned frequency band and obtain excellent active vibration attenuation for most excitations. In this paper, An improved adaptive algorithm is proposed so as to enhance active vibration attenuation effect under narrow-band Gaussian disturbance covering nature frequencies of structure.

## 2. EXISING ALGORITHM WITH SECONDARY PATH IDENTIFICATION

In a common active vibration control system, sensor and actuator are used to pick up the residual vibration signal  $e(n)$  and receive the control signal  $y(n)$  respectively. Based on the least mean square (LMS) algorithm, the control signal  $y(n)$  is obtained and transmitted by secondary path  $S(z)$ , which is shown in Figure 1(a). The uncontrolled vibrational signal  $d(n)$  is generated by  $x_1(n)$  through primary path  $P(z)$  and expected to be offset by  $y_s(n)$ , which passes through  $S(z)$ . Here the secondary path  $S(z)$  is unknown and has an effect on the convergence direction of adaptive algorithm. In order to pre-estimate the influence of  $S(z)$ , the identified secondary path  $\hat{S}(z)$  of FxLMS is obtained, as shown in Figure 1(b). In the active vibration control process,  $\hat{S}(z)$  is copied to filter coefficient signal  $x(n)$ . Thus the decreasing process of  $e(n)$  can be obtained by making use of LMS and the pre-estimating secondary path  $\hat{S}(z)$ .



(a) adaptive control process (b) Secondary path identification process  
Figure 1: Flow diagram about FxLMS

Based on LMS algorithm, the control filter  $\mathbf{W}_n$  of FxLMS is derived as

$$\mathbf{W}_{n+1} = \mathbf{W}_n - \mu e(n) \hat{\mathbf{X}}_n \quad (1)$$

$$\hat{\mathbf{S}}_{n+1} = \hat{\mathbf{S}}_n - \mu e(n) \mathbf{X}_{sn} \quad (2)$$

$$\hat{x}(n) = \hat{\mathbf{S}}_n \tilde{\mathbf{X}}_n^T \quad (3)$$

In which,  $\mathbf{X}_{sn} = \begin{bmatrix} x_s(n) & \tilde{\mathbf{X}}_{n-1}(1:\hat{N}_s-1) \end{bmatrix}$ ,  $\tilde{\mathbf{X}}_n = \begin{bmatrix} x(n) & \tilde{\mathbf{X}}_{n-1}(1:\hat{N}_s-1) \end{bmatrix}$ ,  $\hat{\mathbf{X}}_n = \begin{bmatrix} \hat{x}(n) & \hat{\mathbf{X}}_{n-1}(1:N_w-1) \end{bmatrix}$ ,  $\hat{N}_s$  and  $N_w$  are the length of identified filter and controlling filter, respectively.  $x(n)$  is obtained by the reference sensor. The step size coefficient  $\mu$  [12], which is related with the convergence direction of active control process, is determined as

$$\mu \propto \frac{1}{P_x \hat{S}S} \quad (4)$$

In which  $P_x$  is the power of reference  $x(n)$ . It can be found that the accuracy of identified secondary path  $\hat{S}(z)$  has a negative impact on the convergence direction of controlling filter  $\mathbf{W}$ . And the poor accuracy of  $\hat{S}(z)$  can even lead to the failure of active vibration control. Based above, existing adaptive algorithms with secondary path identification process are described as follows.

As shown in Figure 1(b), the pre-estimating secondary path is traditionally identified by adding additional noise  $x_s(n)$ . Ericsson [5] proposed that an on-line secondary path identification algorithm can make contributions to an accurate  $\hat{S}(z)$ . In the on-line adaptive algorithm, the additional random noise  $v(n)$  is applied to the structure along with control signal  $y(n)$  during the active control process. Here the filter weight of FxLMS is rewritten as

$$\mathbf{W}_{n+1} = \mathbf{W}_n - \mu_w f(n) \hat{\mathbf{X}}_n \quad (5)$$

$$\hat{\mathbf{S}}_{n+1} = \hat{\mathbf{S}}_n - \mu_s f(n) \mathbf{V}_{sn} \quad (6)$$

In which,  $\mathbf{V}_{sn} = [v(n) \quad \mathbf{V}_{s(n-1)}(1: \hat{N}_s - 1)]$ ,  $\mu_w$  and  $\mu_s$  are related to the power of  $\hat{x}(n)$  and  $v(n)$ , respectively.  $f(n)$  is the summation of identification error and control error, which is calculated as

$$f(n) = e_v + e(n) \quad (7)$$

It can be found that  $f(n)$  is related with the identification error  $e_v$  and control error  $e(n)$ . When  $e_v$  is close to zero,  $f(n)$  will be greatly affected by  $e(n)$ . Namely, the identification result can be modified by the active control effect. Thus the control effect can be decreased by the additional random noise, which means that the vibration attenuation effect is associated with the power of additional noise  $v(n)$ .

In order to weaken the influence of additional random noise and reduce the power of additional noise, Akhtar [6] proposed a variable step size (VSS) FxLMS, as shown in Figure 2. Compared with Ericsson's algorithm, the variable step size during the secondary path identification and the variable power of additional noise are added.

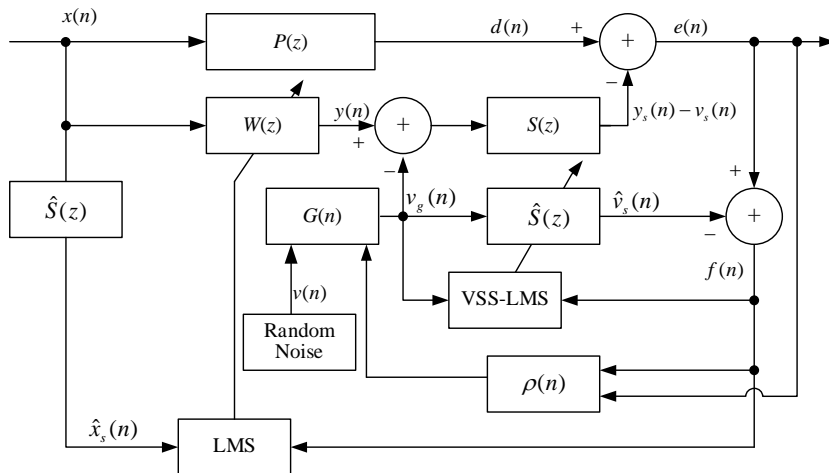


Figure 2: FxLMS with on-line secondary path identification by Akhtar[6]

Making use of the power ratio  $\rho(n)$  of  $f(n)$  and  $e(n)$ , the variable power of additional noise is calculated by

$$v_g(n) = \sqrt{(1 - \rho(n))\sigma_{v_{\min}}^2 + \rho(n)\sigma_{v_{\max}}^2} \cdot v(n) \quad (8)$$

In which,  $\rho(n) = P_f(n)/P_e(n)$ , and the variable step size can be determined by

$$\mu_S(n) = \rho(n)\mu_{S_{\min}} + (1 - \rho(n))\mu_{S_{\max}} \quad (9)$$

Where  $\mu_{S_{\max}}$  and  $\mu_{S_{\min}}$  are pre-estimated by related simulations, as described in reference [6].  $\sigma_{v_{\min}}$  and  $\sigma_{v_{\max}}$  are the variance of additional random noise. According to Equation 7 and Equation 8, when the active control process is stable, the power of additional noise and  $\rho(n)$  are tend to  $\sigma_{v_{\max}}$  and 1, respectively. Obviously, the minimum additional noise  $\sigma_{v_{\min}}$  is hardly to access.

Algorithms mentioned above were designed to obtain the accurate secondary path and pursue the excellent active vibration attenuation. These improvements about FxLMS are technically achieved by means of reducing the mutual influence between secondary path identification and the active control effect. Nevertheless, the frequency response property of FIR cannot yet perfectly describe the complex secondary path over the whole frequency range. So it is difficult to ensure identification accuracy and control effectiveness. In addition, the frequency response characteristics of the structure are largely varied at different natural frequencies. It is difficult to obtain an accurate identified secondary path within the whole frequency band. Therefore some improvements about secondary path identification are required to ensure the vibration attenuation effect for different controlled modes.

### 3. IMPROVED ALGORITHM

A block diagram of adaptive control system employing the improved algorithm is shown in Figure 3. As shown below, the improved algorithm is implemented under the framework of FxLMS. The controlling filter  $\mathbf{W}_n$  and the identified filter  $\hat{\mathbf{S}}_n$  are obtained by LMS algorithm. And the controlling filter  $\mathbf{W}_n$  is updated by the identified secondary path  $\hat{\mathbf{S}}_n$  and error signal  $f(n)$ , as described in Equation 5. The identified filter  $\hat{\mathbf{S}}_n$  is calculated according to Equation 6.

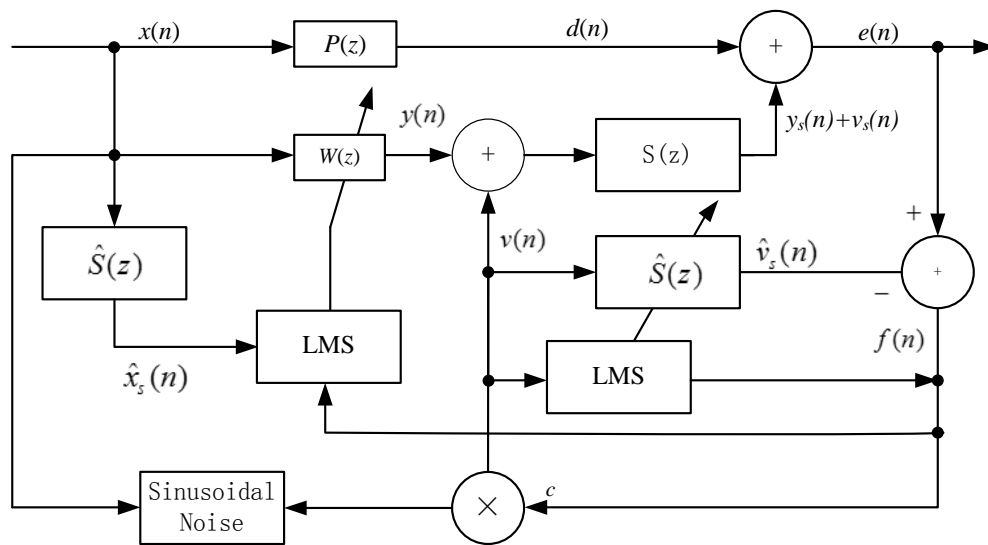


Figure 3: Improved algorithm scheme

As shown in Figure 3,  $v(n)$  is the additional noise. A sinusoidal signal is adopted here, as shown below.

$$v(n) = c \sin(2\pi f_0 t) \quad (10)$$

In which,  $c = \begin{cases} 0, P_f(n) < \varepsilon \\ 1, P_f(n) > \varepsilon \end{cases}$ ,  $f_0$  is the controlled natural frequency of structure. Assume

the power of  $e(n)$  is  $P_{1e}$  under the primary excitation  $x(n)$ . When the controlling filter  $\mathbf{W}_n$  is ideal, the power of  $e(n)$  is  $P_{2e}$  under the primary excitation  $x(n)$  and the additional noise  $v(n)$ . Let  $\varepsilon$  be a relative value, which is between  $P_{1e}$  and  $P_{2e}$ . Here  $f_0$  can be obtained based on the characteristics of the reference signal  $x(n)$ . During the actual application, the structural resonance will be excited by  $x(n)$ . According to Equation 10, the additional sinusoidal signal can be shutdown when the identification process is effective. It is found that the control effect has few influence on the additional noise if the active control system with improved algorithm is tend to stable. And the secondary path identification can be artificially directed at controlled natural frequencies.

As shown in Figure 3, the detailed steps of the proposed improved algorithm are described below.

a) Excite the structure by the primary actuator with the narrowband Gaussian noise  $x(n)$ . And denote the uncontrolled vibration response of structure at the position of error sensor as  $d(n)$ .

b) Generate the additional sinusoidal noise according to the spectral peak of vibration response. And collect the reference signal associated with  $x(n)$  by the reference sensor.

c) Start active control process and secondary path identification process with proper step size  $\mu_w$  and  $\mu_s$ .

d) Calculate the controlling signal  $y(n)$ , which is shown as below.

$$y(n) = \mathbf{W}_n \mathbf{X}_n^T \quad (11)$$

Here  $\mathbf{W}_n$  is obtained by Equation 5, and  $\mathbf{X}_n = [x(n) \quad \mathbf{X}_n(1:N_w-1)]$ .

e) Apply the sum of the additional noise  $v(n)$  and controlling signal  $y(n)$  to the secondary actuator. Further, calculate  $f(n)$  as Equation 8 according to the error signal  $e(n)$  to verify the identification result. And then update the identified secondary path  $\hat{\mathbf{S}}(n)$  according to Equation 7.

f) Change the additional noise  $v(n)$  according to the coefficient  $c$ . When  $c$  is equal to 0, cut off the additional noise as Equation 10 to avoid weakening the control effect. Then the identified secondary path  $\hat{\mathbf{S}}(n)$  is almost unchanged.

When the steps mentioned above are accomplished,  $f(n)$  and  $e(n)$  can be gradually decreased to a stable value which is close to zero. Finally active vibration control is achieved.

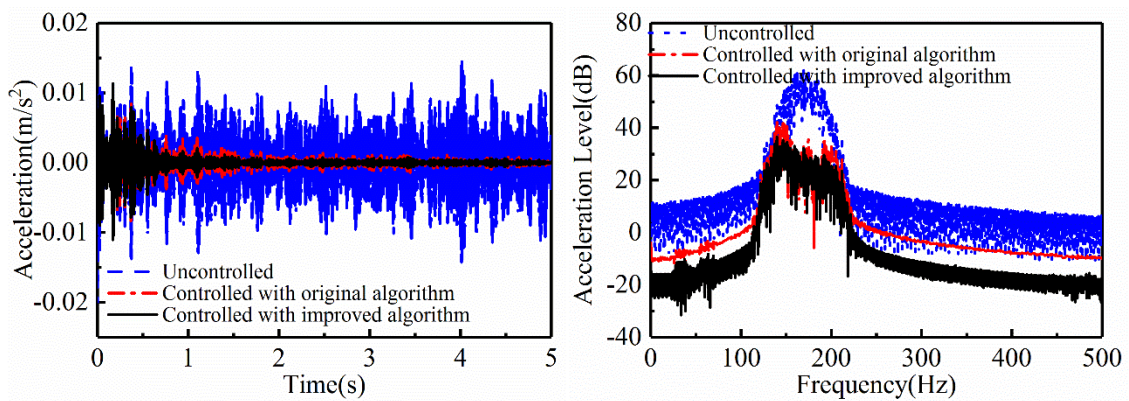
#### 4. SIMULATION RESULTS

As mentioned above, the improved algorithm shown in Figure 3 is proposed based on the original algorithm in Figure 2. Both original algorithm and the improved algorithm are carried out to deal with the resonance vibration generated by complex excitations. To verify the superiority of the improved algorithm, some simulations are carried out in this section. In the simulations, the performance of the improved algorithm is compared with that of the original algorithm under the same conditions. A steel rectangular plate is chosen as the controlled target. The plate with dimensions of 500 mm×600 mm×1mm under free boundary condition can be excited by a primary actuator and a secondary actuator to perform primary excitation and secondary excitation, respectively. The

primary actuator and secondary actuator are attached in different position on the plate as the location of primary excitation is always unknown in practical application. A narrowband Gaussian disturbance is applied to the primary actuator, to generate the primary vibration response  $d(n)$ . After that, some typical natural frequencies of plate are obtained firstly and selected as the central frequency of narrowband Gaussian disturbance. The reference sensor and error sensor paired with secondary actuator is used to obtain  $x(n)$  and  $e(n)$ , as shown in Figure 2 and Figure 3. The algorithms mentioned above are carried out to obtain the controlling force. It is expected that the vibration response of the plate can be decreased and close to zero under the superposition of the primary excitation  $x(n)$  and the proper secondary excitation  $y(n)$ .

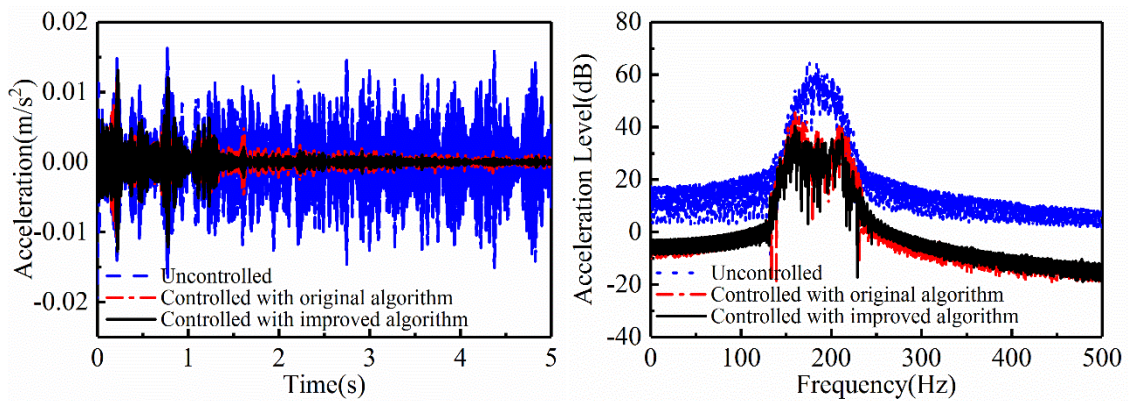
In subsequent simulations, some representative natural frequencies of the plate will be chosen as central frequency of narrowband Gaussian disturbance, such as 169Hz, 185Hz, or 300Hz, to verify the performance of the improved algorithm. And the bandwidth of Gaussian disturbance is set to be 12.5Hz, 25Hz and 25Hz, respectively. FIR filters are used to describe both the identified filter and controlling filter of algorithms respectively. In order to obtain the optimal active control effect of these algorithms, the optimum identified step size and controlling step size [13] are adopted according to the power of the reference signal and the additional signal, respectively.

The primary excitations are the narrowband Gaussian disturbances which central frequencies are 169Hz, 185Hz, and 300Hz respectively. The comparison results of different algorithms are shown in Figure 4-6.



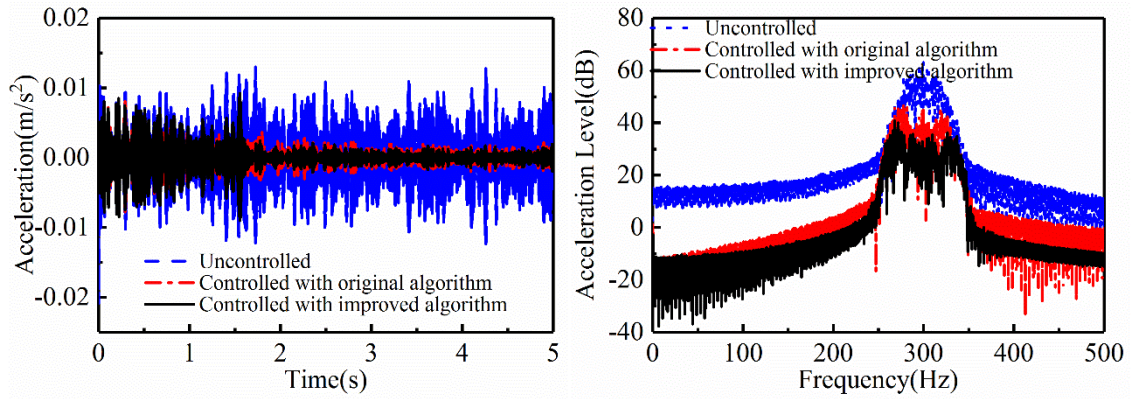
(a) Comparison in time domain (b) Comparison in frequency domain

Figure 4: performance under narrow band excitation  
(central frequency: 169 Hz, bandwidth: 12.5Hz)



(a) Comparison in time domain (b) Comparison in frequency domain

Figure 5: performance under narrow band excitation  
(central frequency :185 Hz , bandwidth: 25Hz)



(a) Comparison in time domain (b) Comparison in frequency domain  
 Figure 6: performance under narrow band excitation  
 (central frequency: 300 Hz, bandwidth: 25Hz)

As shown in Figure 4(a)-6(a), the amplitude of acceleration decreases gradually with the increasing of time. It can be seen that the convergence speed of the improved algorithm is faster than that of the original algorithm. When the control performance in time domain is stable, the control effect of different algorithms in frequency domain can be obtained by the frequency spectrum analysis, as shown in Figure 4(b)-6(b). It is found that the active control effect of the improved algorithm is better than that of the original algorithm. The acceleration level in frequency domain is reduced about 20dB. Considering that the simulation conditions of active control process are performed under ideal conditions, experiments of active vibration control would be designed to further verify the performance of the improved algorithm.

## 5. EXPERIMENTAL RESULTS

In order to keep consistent with simulations described above, the experimental system is built up, as shown in Figure 7. The primary excitation is generated by the multi-channel analyzer system controlled by PC. Then the excitation signal is transmitted to primary actuator through the power amplifier. The vibration response of the plate is measured by an error accelerometer and a reference accelerometer. Meanwhile, the measurement results of accelerometers are transmitted to development board, so as to generate the control signal based on the control algorithm. Next, measurements of accelerometers are transmitted to PC through multi-channel analyzer system, so as to monitor the vibration attenuation results.

The natural frequencies of the plate are measured before the active vibration control, some of which are 160 Hz, 191 Hz and 307 Hz, not exactly the same as the calculated frequencies due to the minor difference of thickness. The produced Gaussian disturbance signals, whose bandwidth is consistent with simulations at the same mode, are regarded as the primary excitations in the experiment. Controlled signals are calculated through the original algorithm or the improved algorithm. As shown in Figure 8-10, the vibration attenuations are achieved through the interaction of primary excitation and secondary excitation on the plate.

As shown in Fig 8-10, the active vibration control is achieved, and the acceleration levels are effectively reduced. Furthermore, it should be noted that compared with original algorithm, a better vibration attenuation is achieved by the improved algorithm. The characteristic rules in the experiments are consistent with that in the simulations. According to Fig 8-10, the vibration attenuation effect is considerable at different



frequency range. However the vibration attenuation of the experiments is less effective than that of simulations, due to the limit of real-time performance of experiment instruments.

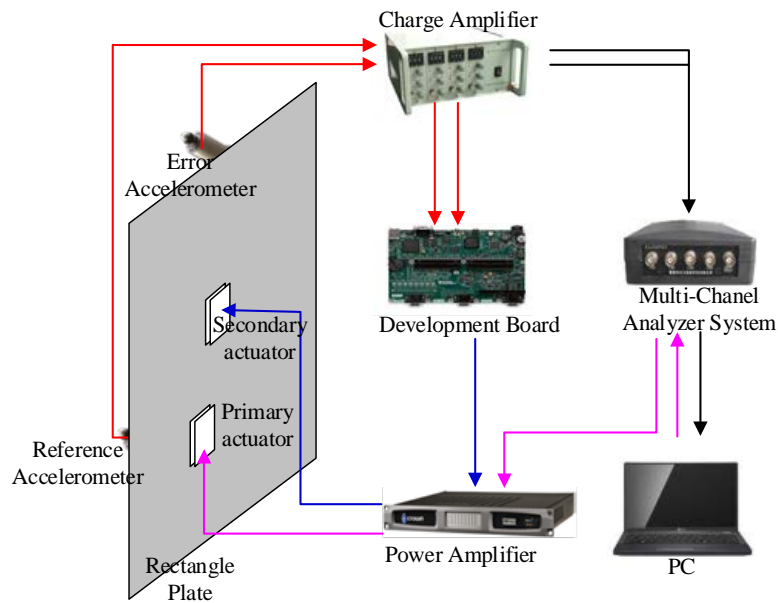


Figure 7: Schematic diagram of experimental system

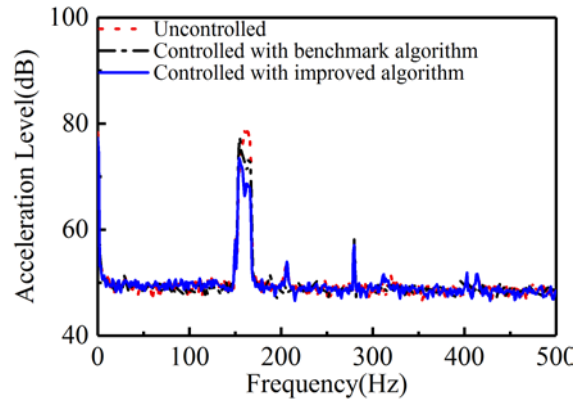


Figure 8: performance under narrow band excitation (central frequency: 160 Hz, bandwidth: 12.5 Hz)

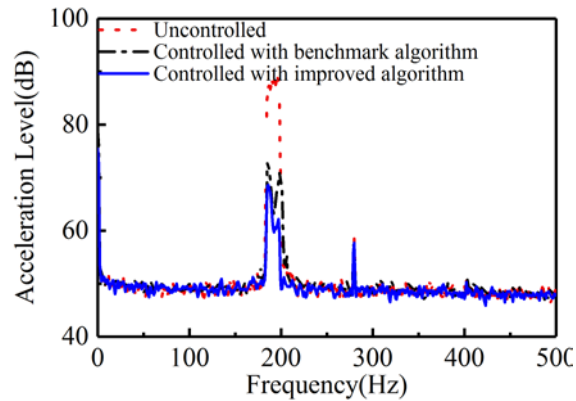


Figure 9: performance under narrowband excitation (central frequency: 191 Hz, bandwidth: 25 Hz)

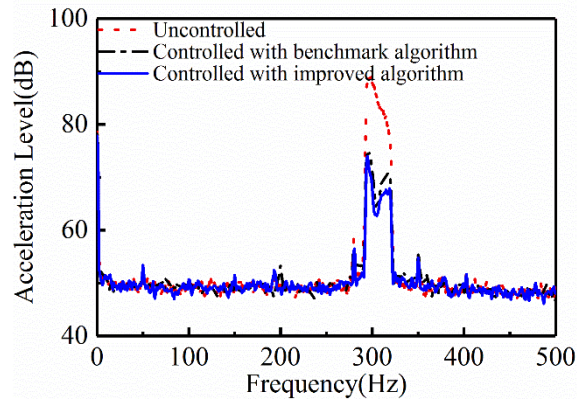


Figure 10: performance under narrow band excitation  
(central frequency: 307 Hz, bandwidth: 25Hz)

## 6. CONCLUSIONS

Since the influence of vibration attenuation caused by FIR filters is not completely eliminated by the original algorithm, to enhance the vibration attenuation and improve identification accuracy, an improved adaptive algorithm making use of additional sinusoidal signal is proposed in this paper. The secondary path identification of the improved algorithm is artificially directed at the controlled natural frequency. The vibration attenuations of original algorithm and the improved algorithm are compared by simulations and experiments under narrowband Gaussian disturbance. Both simulation and experimental results show that the vibration attenuation effect of the improved algorithm at controlled frequency range is better than that of the original algorithm. Moreover, the improved algorithm is effective under narrowband Gaussian disturbance with different central frequencies. Meanwhile, faster convergence speed is displayed for the improved algorithm, which further proves a better performance of the improved algorithm.

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