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Acoustic energy transfer characteristics of the double-plate structure coupled with a bottom acoustic cavity

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ABSTRACT

This paper is concerned with the modeling and acoustic energy transfer characteristics of the sound insulation structure composed of a double-plate structure and a bottom acoustic cavity. A general solution method is presented for predicting the acoustic characteristics of the sound insulation structure, in which the displacement of the plate and the sound pressure in the cavity are, respectively, constructed in the form of the two-dimensional and three-dimensional modified Fourier series with several auxiliary functions introduced to ensure the uniform convergence of the solution over the entire solution domain. The modal parameters of the coupled system are obtained through using the Rayleigh-Ritz procedure based on the energy expressions for the sound insulation structure. The influences of key parameters on the acoustic behaviors of the sound insulation structure are investigated, including: cavity depth and boundary conditions. The research on the acoustic energy transfer characteristics ıof the sound insulation structure provides an theoretical support for the design of architectural structures.

Keywords: Double-plate structure, Energy transfer characteristics, Modal parameters

1.INTRODUCTION

The sound insulation structures have found increasingly wide applications in modern buildings, transportation systems, vehicles, aircraft fuselage shells, and window glazing. The typical sound insulation structures are single-panel configurations [1-6]and double-plate configurations [7-9].

Dowell and Voss [1] obtain the influence of the acoustic cavity on the vibrating plate. Shahraeeni and Shakeri [2] develop a novel analytical model to address free and forced vibration of piezoelectric laminated plates coupled with rectangular acoustic cavities. Naryannan and Shanbhag[3] analyze the problem of sound transmission and structural response of a sandwich panel backed by a cavity. Panand Hansen [4] describes an experimental investigation into the active control of the noise transmission through a panel into a cavity. Kim and Brennan [5] improved control effects of the plate vibration and decreased control efforts of the actuators by using the hybrid approach. Qiu and Sha[6] used a point force actuator on the panel to actively control the noise transmission through a panel into a cavity.

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Yairi and Sakagami [7] investigated the sound radiation from a double-leaf elastic plate subjected to a point force excitation，and gained a fundamental insight into the sound radiation from an interior panel of a double-leaf structure in buildings. Chazot and Guyader [8] studied sound transmission loss of double panels with a patch-mobility method. Xin and Lu [9] investigated sound transmission loss of a rectangular double-panel partition clamp by using the method of modal function.

Compared with the sound insulation characteristics of the single-panel configurations, double-plate structures are more suitable for industrial sound insulation structures. This paper is concerned with the modeling and acoustic energy transfer characteristics of the sound insulation structure composed of a double-plate structure and a bottom acoustic cavity. The modal parameters of the coupled system are obtained using the Rayleigh-Ritz procedure based on the energy expressions for the sound insulation structure. The influences of geometric parameters and boundary conditions on the acoustic behaviors of the sound insulation structure are investigated. The research on the acoustic energy transfer characteristics of the sound insulation structure provides theoretical support for the design of architectural structures.

2. THEORETICAL FORMULATIONS

2.1 Descriptions of the model

The geometric model discussed in this paper is given in Fig. 1. The sound source in the acoustic cavity with rigid walls generates sound field, then the sound field generates a force on the lower plate through the sound pressure on the plate-cavity coupled interface. The lower plate radiates the sound field to the acoustic cavity B through the bending vibration, then the acoustic cavity B exerts a force on the upper plate through the sound pressure on the plate-cavity coupled interface. The acoustic energies are radiated into the outer space by the bending vibration of the upper plate. The boundaries of the upper and lower elastic structures are arbitrarily elastically restrained through supported springs.

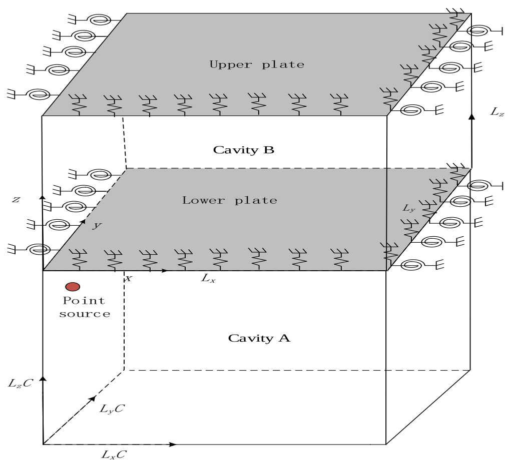


Fig.1 The double-plate structure coupled with a bottom acoustic cavity under the excitation of point sound source.

2.2 Solution procedure of double-plate structure coupled system

The displacements of two flexible panels with general elastic boundary supports could be expressed as follows:

$$
\begin{aligned}
w_i(x,y) &= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_{mn}^{p_i}\cos\lambda_{L_x m}x\cos\lambda_{L_y n}y \\
&+\sum_{m=0}^{\infty}\left[\underbrace{a_m^{p_i}\zeta_{1b}(y)+c_m^{p_i}\zeta_{3b}(y)}_{y=0}+\underbrace{b_m^{p_i}\zeta_{2b}(y)+d_m^{p_i}\zeta_{4b}(y)}_{y=L_y}\right]\cos\lambda_{L_x m}x \\
&+\sum_{n=0}^{\infty}\left[\underbrace{e_n^{p_i}\xi_{1a_1}(x)+g_n^{p_i}\zeta_{3a_1}(x)}_{x=0}+\underbrace{f_n^{p_i}\xi_{2a_1}(x)+h_n^{p_i}\xi_{4a_1}(x)}_{x=Lx}\right]\cos\lambda_{L_y n}y
\end{aligned} \tag{1}
$$

where $\lambda_{L_x m}=m\pi/L_x$ and $\lambda_{L_y n}=n\pi/L_y$.

Similarly, the sound pressure inside an acoustic cavity could be expanded as an improved Fourier series representation:

$$
\begin{aligned}
p_i(x,y,z) &= \sum_{m_x=0}^{\infty}\sum_{m_y=0}^{\infty}\sum_{m_z=0}^{\infty} A_{m_x m_y m_z}^{i}\cos\lambda_{m_x}x\cos\lambda_{m_y}y\cos\lambda_{m_z}z \\
&+\sum_{m_x=0}^{\infty}\sum_{m_y=0}^{\infty}\left[\underbrace{\zeta_{1L_z}(z)a_{m_x m_y}^{i}}_{z=0}+\underbrace{\zeta_{2L_z}(z)b_{m_x m_y}^{i}}_{z=L_z}\right]\cos\lambda_{m_x}x\cos\lambda_{m_y}y \\
&+\sum_{m_x=0}^{\infty}\sum_{m_z=0}^{\infty}\left[\underbrace{\zeta_{1L_y}(y)c_{m_x m_z}^{i}}_{y=0}+\underbrace{\zeta_{2L_y}(y)d_{m_x m_z}^{i}}_{y=L_y}\right]\cos\lambda_{m_x}x\cos\lambda_{m_z}z \\
&+\sum_{m_y=0}^{\infty}\sum_{m_z=0}^{\infty}\left[\underbrace{\zeta_{1L_x}(x)e_{m_y m_z}^{i}}_{x=0}+\underbrace{\zeta_{2L_x}(x)f_{m_y m_z}^{i}}_{x=L_x}\right]\cos\lambda_{m_y}y\cos\lambda_{m_z}z
\end{aligned} \tag{2}
$$

where $\lambda_{m_x}=m_x\pi/L_x$, $\lambda_{m_y}=m_y\pi/L_y$ and $\lambda_{m_z}=m_z\pi/L_z$. m_x, m_y and m_z are positive integers. $\zeta_{iL_x}(x)$, $\zeta_{iL_y}(y)$ and $\zeta_{iL_z}(z)$ are introduced to overcome discontinuity issues on structural–acoustic coupling surface.

The Lagrangians for two plates with general elastic boundary supports are expressed as:

$$L_{panel\ a}=U_{panel\ a}-T_{panel\ a}+W_{a_b\&p_a} \tag{3}$$

$$L_{panel\ b}=U_{panel\ b}-T_{panel\ b}-W_{a_b\&p_b}+W_{a_a\&p_b} \tag{4}$$

where $U_{panel\ a}$ and $U_{panel\ b}$ are, respectively, the potential energies of the upper and lower plates. $T_{panel\ a}$ and $T_{panel\ b}$ denote the kinetic energies of the upper and lower plates, respectively. $W_{a_a\&p_b}$, $W_{a_b\&p_a}$ and $W_{a_b\&p_b}$ are the exterior work of the lower plate done by the sound pressure inside cavity A, the exterior work of the lower plate done by the sound pressure inside cavity B, and the exterior work of cavity B done by the lower plate. The specific expression of $W_{a_j\&p_i}$ is as follows:

$$W_{a_j\&p_i}=\int_S w_i p_j \mathrm{d}S=\int_0^{Lx}\int_0^{Ly} w_i p_j \mathrm{d}x\mathrm{d}y \tag{5}$$

where p_j indicates the sound pressure inside the acoustic cavity A or B. Due to the force and reaction force, $W_{a_j\&p_i}=W_{p_i\&a_j}$.

The total potential energy and total kinetic energy of the upper and lower plates are expressed as follows:

$$\begin{aligned}U_{panel\ i}=&\frac{D_i}{2}\int_0^{L_x}\int_0^{L_y}\left\{\left(\frac{\partial^2 w_i}{\partial x^2}\right)^2+\left(\frac{\partial^2 w_i}{\partial y^2}\right)^2+2\mu i\frac{\partial^2 w_i}{\partial x^2}\frac{\partial^2 w_i}{\partial y^2}+2\left(1-\mu_i\right)\left(\frac{\partial^2 w_i}{\partial x\partial y}\right)^2\right\}\mathrm{d}x\mathrm{d}y\\&+\frac{1}{2}\int_0^{L_y}\left[k_{x0}w_i^2+K_{x0}\left(\frac{\partial w_i}{\partial x}\right)^2\right]_{x=0}\mathrm{d}y+\frac{1}{2}\int_0^{L_y}\left[k_{xL_x}w_i^2+K_{xL_x}\left(\frac{\partial w_i}{\partial x}\right)^2\right]_{x=L_x}\mathrm{d}y\\&+\frac{1}{2}\int_0^{L_x}\left[k_{y0}w_i^2+K_{y0}\left(\frac{\partial w_i}{\partial y}\right)^2\right]_{y=0}\mathrm{d}x+\frac{1}{2}\int_0^{L_x}\left[k_{yL_y}w_i^2+K_{yL_y}\left(\frac{\partial w_i}{\partial y}\right)^2\right]_{y=Ly}\mathrm{d}x\end{aligned} \tag{6}$$

$$T_{panel\ i}=\frac{1}{2}\int_0^{L_x}\int_0^{L_y}\rho_i h_i\left(\frac{\partial w_i}{\partial t}\right)^2\mathrm{d}x\mathrm{d}y=\frac{1}{2}\rho_i h_i\omega^2\int_0^{L_x}\int_0^{L_y}w_i^2\mathrm{d}x\mathrm{d}y \tag{7}$$

where the subscript *i* is 1 or 2. ρ_i and h_i are, respectively, the density and thickness of the plate.

The Lagrangian functions of the acoustic cavities A and B are expressed as:

$$L_{cavity\ a}=U_{cavity\ a}-T_{cavity\ a}-W_{ext\ a} \tag{8}$$

$$L_{cavity\ b}=U_{cavity\ b}-T_{cavity\ b}-W_{ext\ b} \tag{9}$$

where $U_{cavity\ a}$ and $U_{cavity\ b}$ are, respectively, the total potential energies of the acoustic cavities A and B. $T_{cavity\ a}$ and $T_{cavity\ b}$ are, respectively, the total kinetic energies of the acoustic cavities A and B. $W_{ext\ a}$ mainly contains $W_{p_b\&a_a}$ and W_S. $W_{ext\ b}$ mainly contains $W_{p_a\&a_b}$ and $W_{p_b\&a_b}$.

The total acoustic potential energy in the acoustic cavity is

$$V_j=\frac{1}{2\rho_{0j}c_{0j}^2}\int_V p_j^2\mathrm{d}V_j=\frac{1}{2\rho_{0j}c_{0j}^2}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}p_j^2\left(x,y,z\right)\mathrm{d}x\mathrm{d}y\mathrm{d}z \tag{10}$$

The total kinetic energy in the acoustic cavity is

$$T_j = \frac{1}{2\rho_{0j}\omega^2}\int_{V_j}\left[\left(\frac{\partial p_j}{\partial x}\right)^2+\left(\frac{\partial p_j}{\partial y}\right)^2+\left(\frac{\partial p_j}{\partial z}\right)^2\right]\mathrm{d}V_j$$
$$= \frac{1}{2\rho_{0j}\omega^2}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\left[\left(\frac{\partial p_j}{\partial x}\right)^2+\left(\frac{\partial p_j}{\partial y}\right)^2+\left(\frac{\partial p_j}{\partial z}\right)^2\right]\mathrm{d}x\mathrm{d}y\mathrm{d}z \tag{11}$$

where V_j is the potential energy of cavity A or B; T_j represents the kinetic energy of the acoustic cavity A or B; ρ_{0j} denotes the density of the medium in the acoustic cavity A or B; c_{0j} is the sound velocity of the medium inside cavity A or B.

Substituting Eqs. (1) and (2) into the Lagrangians, Eqs. (3) and (4), and applying the Rayleigh–Ritz procedure against each of the unknown Fourier series coefficients, the matrix forms of structural systems may be described as

$$\left[K_{p_a}-\omega^2 M_{p_a}\right]W_a + C_{a_b\&p_a}P_b = 0 \tag{12}$$

$$\left[K_{p_b}-\omega^2 M_{p_b}\right]W_b - C_{a_b\&p_b}P_b + C_{a_a\&p_b}P_a = 0 \tag{13}$$

where K_{p_a} and K_{p_b} are, respectively, the stiffness matrices of the upper and lower plates. M_{p_a} and M_{p_b} denote the mass matrices of the upper and lower plates, respectively. $C_{a_b\&p_a}$, $C_{a_b\&p_b}$ and $C_{a_a\&p_b}$ represent the coupling matrix.

Substituting Eqs. (1) and (2) into the Lagrangians, Eqs. (8) and (9), and applying the Rayleigh–Ritz procedure against each of the unknown Fourier series coefficients, the matrix forms of acoustical cavity systems may be described as

$$K_a P_a - \omega^2 M_a P_a + \omega^2 C_{p_b\&a_a} W_b = Q \tag{14}$$

$$K_b P_b - \omega^2 M_b P_b + \omega^2 C_{p_a\&a_b} W_a - \omega^2 C_{p_b\&a_b} W_b = 0 \tag{15}$$

where K_a and K_b are the stiffness matrices of the acoustic cavity systems, respectively. M_a and M_b are, respectively, the mass matrices of the acoustic cavity systems. $C_{p_b\&a_b}$, $C_{p_a\&a_b}$ and $C_{p_b\&a_a}$ represent the coupling matrices.

Combining the above matrices, the matrix equation of the coupled system composed of the double-plate structure and a bottom acoustic cavity is

$$\left\{\begin{bmatrix} K_{p_a} & 0 & 0 & C_{a_b\&p_a} \\ 0 & K_{p_b} & C_{a_a\&p_b} & -C_{a_b\&p_b} \\ 0 & 0 & K_a & 0 \\ 0 & 0 & 0 & K_b \end{bmatrix} - \omega^2 \begin{bmatrix} M_{p_a} & 0 & 0 & 0 \\ 0 & M_{p_b} & 0 & 0 \\ 0 & -C^T_{a_a\&p_b} & M_a & 0 \\ -C^T_{a_b\&p_a} & C^T_{a_b\&p_b} & 0 & M_b \end{bmatrix}\right\} \begin{bmatrix} W_a \\ W_b \\ P_a \\ P_b \end{bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ Q \\ 0 \end{Bmatrix} \tag{16}$$

Substituting the obtained vectors W_a, W_b, P_a and P_b into the upper plate, the lower plate and the acoustic cavities A and B, the expressions for the sound field and the vibration displacements can be obtained. If the right term of the equation (16) is set to zero, the modal characteristics of the cavity-plate coupled system can be obtained according to the matrix eigenvalue problem.

3. NUMERICAL CALCULATION AND ANALYSIS

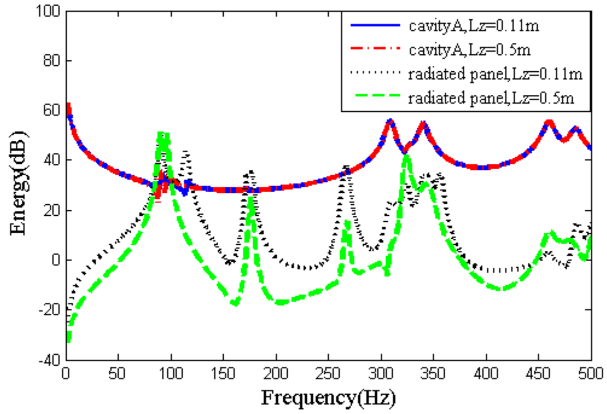
Firstly, the method is validated by the results [10] of double-plate structure coupled with a bottom cavity. Lx=0.5m，Ly=0.35m，LzC=0.55m，the thickness of the upper plate h_a and the lower plate are, respectively, 0.002 m and 0.003 m. The depth of the acoustic cavity B is 0.33 m. The density ρ_{air} is 1.21 kg/m^3, and sound velocity c_0 is 340 m/s. The density of the upper and lower plates is ρ =2720 kg/m^3, the Poisson's ratio is $\nu = 0.33$, the Young's modulus is E=7.1*10^{10} N/m^2, and the boundaries of the double plates are simply supported. In this model, the upper plate is the radiated panel. The truncated series M of the acoustic cavity (Mx=My=Mz=M) and the truncated series N of the plates (Mp=Np=N) are 4 and 12, respectively. Table 1 shows the natural frequencies of the first 500 Hz for double-plate structure coupled system. It can be seen from Table 1 that the natural frequencies from current method agrees well with the results obtained by the method of literature [10], and the maximum difference does not exceed 0.13%. It can be determined that the truncated series for the acoustic cavity and the plate can satisfy higher calculation accuracy.

Table 1 Natural frequencies of double-plate structure coupled with a bottom cavity (Hz)

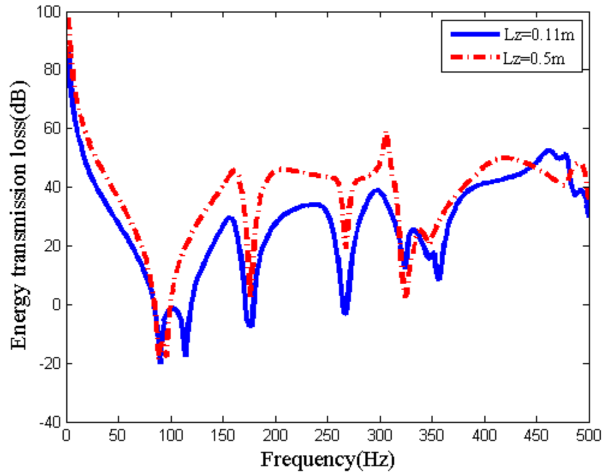
Modal order	Literature [10] (Hz)	Current method (Hz)	deviation (%)
1	67.3	67.3	0
2	96.7	96.7	0
3	117.2	117.3	0.09
4	175	174.9	0.06
5	178.5	178.3	0.11
6	216.1	215.9	0.09
16	438.3	437.9	0.09
17	459.9	459.9	0
18	473.1	473.7	0.13

3.1 Effect of cavity depth on the coupled system

To reveal the effect of the depth of the acoustic cavity B on the energy transfer loss of the system, Figure 2 gives the energies of the plate-cavity coupled system with different acoustic cavity depths. The depths of the selected acoustic cavity B are 0.11 m and 0.5 m, respectively. The energy transmission loss is defined as the difference between the sound potential energy of cavity A and the sound power of the radiated plate, where the volume velocity and position of the point source in the cavity A are $Q_0 = 2\times10^{-5} m^3/s$ and $\left(Lx/10, Ly/10, 9LzC/10\right)$, respectively. Figure 2(a) shows, under the two kinds of cavity depths, the acoustic potential energy of cavity A and radiated sound power of the radiated plate. It can be seen from Fig. 2(a) that the potential energy of the acoustic cavity A is unevenly distributed within 0-500 Hz, and the peaks of potential energy mainly occur at acoustic modes. The cavity depths have less influence on the acoustic potential energy. For the change of the coupled modes, the radiated sound power of radiated plate decreased with the increase of depth of cavity B. Fig. 2(b) shows the energy transmission loss. The energy transmission loss increase with the increase of the depth of the cavity B by the influence of the cavity depth on the acoustic potential energy of cavity A and radiated acoustic power.



(a) Acoustic potential energy inside cavity A and radiated sound power of radiated panel



(b) Energy transmission loss

Fig. 2 Energy of the double-plate structure coupled with a bottom acoustic cavity having different acoustic cavity depths

3.2 Influence of boundary conditions on the energy of coupled system

In order to observe the influence of the boundary conditions on the coupled system energy, Figures 3 and 4 show the energy transmission loss under different boundary conditions. The geometrical and physical parameters of the coupled system are the same as above. The depths of the acoustic cavities A and B are, respectively, 0.55 m and 0.33 m. The boundaries of the upper and lower plates are uniformly changed.

Figure 3 shows the effect of the translational restraining stiffness *k* on the energy transmission loss. When the translational restraining stiffness *k* is changed from zero to 1e5, the energy transfer loss within considered frequency range substantially remains unchanged. When the translational restraining stiffness *k* is changed from 1e5 to 1e8, the fundamental frequencies of the coupled system shift and the energy transmission loss increase with the increase of the translational restraining stiffness *k*. When the translational restraining stiffness *k* is changed from 1e8 to 1e15, the energy transfer loss within considered frequency range substantially remains unchanged. The effect of the rotational restraining stiffness K on the energy transmission loss is shown in Figure 4. When the rotational restraining stiffness K is changed from 1e3 to 1e6, the fundamental frequencies of the coupled system shift and the energy transmission loss increase with the increase of the rotational restraining stiffness K. Besides, the rotational restraining stiffness K has less influence on the energy transmission loss within considered frequency range.

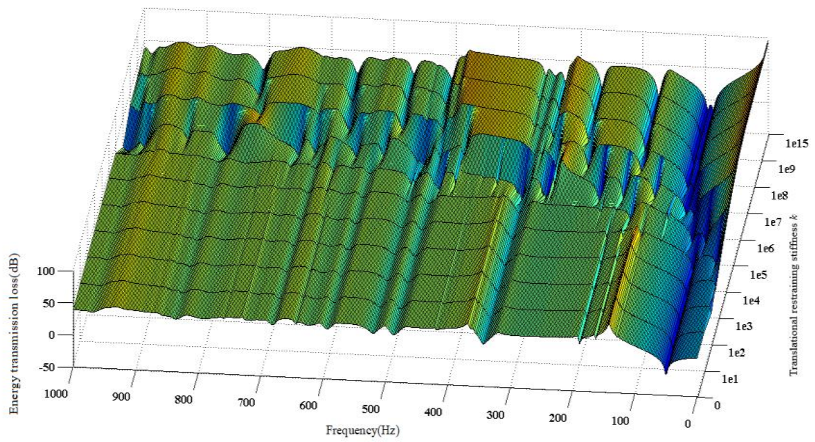


Fig. 3 Effect of translational restraining stiffness on the energy transmission loss of the coupled system

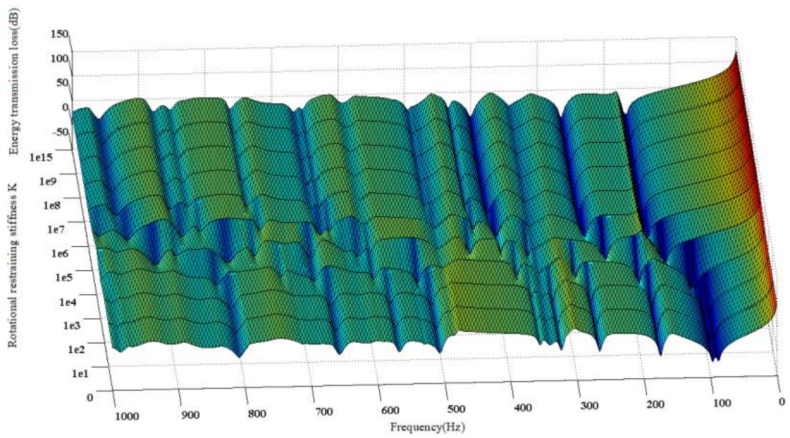


Fig. 4 Effect of rotational restraining stiffness on the energy transmission loss of the coupled system

4. CONCLUSIONS

An analytical model has been developed for investigating the double-plate structure and a bottom acoustic cavity. The displacement of the plate and the sound pressure in the cavity are, respectively, described as the two-dimensional and three-dimensional modified Fourier series with several auxiliary functions. The Rayleigh-Ritz procedure against based on the energy expressions for the sound insulation structure is utilized to obtain the modal parameters of the coupled system. The energy transmission loss of the structure increase with the increase of the depth of the cavity between double plates. The effects of the elastically restraining stiffness on the energy transmission loss are mainly focused within certain stiffness range.

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