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A modified series solution for structural-acoustic analysis of plates

Shi, Xianjie[1]
Institue of Systems Engineering, China Academy of Engineering Physics
No. 64 Mianshan Road, Youxian District, Mianyang, Sichuan, China

ABSTRACT

The Spectro-Geometric Method (SGM) developed recently shows great efficiency in solving solid mechanics and acoustics problems. In this paper, a modified series solution based on Spectro-Geometric Method (SGM) is presented for analysis the structural-acoustic problems consisting of a plate configuration interacting with the fluid medium. Under the current solution framework, the admissible functions for elastic plate and acoustic cavity were expressed with SGM, and all the relevant discontinuities along the boundary face were eliminated. Different boundary conditions can be simulated by setting tangential and normal restraining springs along each boundary edge. The coupling effect between the acoustic cavity and the plate with different boundary conditions can analysed with the current approach. Numerical examples are presented to demonstrate the excellent performances of the current solution framework.

Keywords: Structural-acoustic coupling, Cavity, Elastically restrained edges, Spectro-Geometric Method (SGM)
I-INCE Classification of Subject Number: 76

1. INTRODUCTION

With the increasing demands on the sound quality, such as the vehicle passenger compartments and the aircraft cabins, noise in enclosures induced by the vibration of bounding structures has becoming an important issue encountered by many engineers and acousticians. As everyone knows, the noise in most cases is caused by a coupled vibrating effect between the structure and its ambient medium. Therefore, the structural-acoustic coupling effects must be given full considerations in investing the mechanism of noise emission and vibration control.

Broadly speaking, Dowell and Voss[1] and Lyon[2] did the initial work on the investigations of plate-cavity coupled systems. Multi-modal analysis method was employed by Pretlove[3] to analyse the free vibration problem of a rectangular panel, which was backed by a closed rectangular cavity. Ref [4] reviewed the published articles on the vibration of the cavity-backed panel. A rectangular acoustic cavity closed at one end by a simply supported plate with isotropic material was investigated by Scarpa and Curti[5]. A compact matrix formulation for the steady-state analysis of structural-acoustic systems consisting of an acoustic cavity and a plate was proposed by Kim and Brennan[6]. Then, this approach was validated with experimental work.

[1] 411shixj@caep.cn

In recent years, the improved Fourier series method was utilized by Du[7] to construct the structural-acoustic model of a rectangular acoustic cavity bounded by a flexible plate with elastically restrained edges. Lau and Tang[8] established a model for a two-dimensional rectangular enclosure coupled with a flexible boundary with edges elastically restrained against translation and rotation. Then the edge restraints effect on the active sound transmission control was investigated based on this model. Chen[9] presented a general modeling method for the vibroacoustic analysis of an arbitrarily restrained rectangular plate backed by a cavity with general wall impedance. Cui[10] developed an accurate and efficient numerical implementation of acoustic-structure coupling formulations with the edge-based smoothed finite element method. Sarıgül and Karagözlü[11] presented the results of a modal structural-acoustic coupling analysis for plates with various composite parameters.

A literature review shows that as far as the study of coupled plate-cavity systems is concerned, most of the previous works are confined to simple structural boundary conditions (simply supported or clamped) and ideal cavity boundary conditions with acoustically rigid walls; moreover, the coupled system is typically modelled using rectangular plate. However, the elastically restrained annular sector plates backed by an acoustic cavity are often encountered in practical engineering applications rather than these classical boundary conditions or only rectangular plates since the support types and shapes of practical structures are always complicated. Thus, the vibro-acoustic analysis of a three-dimensional acoustic cavity bounded by an elastically restrained annular sector plate is presented by using SGM. The correctness of the established analytical model has been demonstrated by being compared with the results achieved by finite element method (FEM). On this basis, some new results and discussions are given, including the cavity depth, plate thickness, varying boundary conditions and so on, which could provide reference for future research.

2. THEORETICAL FORMULATIONS

2.1 Model Description

A coupled system consisting of an acoustic cavity and an annular sector plate with general boundary conditions is given in Figure 1. The geometry and dimensions are defined in an orthogonal cylindrical coordinate system (r, θ, z). A local coordinate system (s, θ, z) is also shown in the Fig. 1 which will be used in the analysis. The relationship of r and s is: s=r-a.

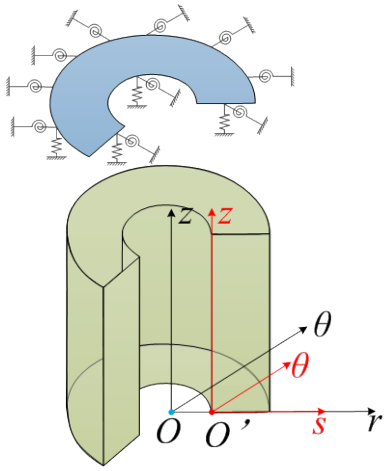


Figure 1 Coordinate system and geometry model of annular sector plate coupled with an acoustic cavity

The acoustic cavity dimensions is given as: inner radius a, outer radius b, sector angle ϕ, and height h. The elastically restrained annular sector plate with thickness h_p is bounded on the top surface (z=h) of the acoustic cavity. In addition, the difference of inner radius and outer radius is marked as R ($R\equiv b$-a). According to the artificial spring technique [12-14] and the transverse vibration model of the thin annular sector plate, two sets of elastic restraints of arbitrary stiffnesses corresponding to three translational and one rotational displacements (k and K) are generally specified on the plate's edges to simulate various boundary restraints. The stiffness distribution function for each type of elastic restraint can be specified independently and vary continuously or discontinuously along an edge. In this way, the elastic restraints not only themselves represent a class of boundary conditions of practical interest, but also can be used to unite all the classical homogeneous boundary conditions by accordingly setting the stiffness constants equal to either zero or infinity.

2.2 Series representation of field functions for the coupled system

In this investigation, a modified series expansion based on SGM is utilized to represent the admissible function of the annular sector plate and acoustic cavity. For the annular sector plate, the two-dimensional series expansion is used to indicate the plate's displacement function. Meanwhile, the three-dimensional (3-D) series expansion is utilized to indicate the acoustic pressure of the enclosure. These admissible functions can be expressed as a new form of trigonometric series as superposition of the product of cosine functions and sine functions. The introduction of sine terms can account for all the possible discontinuities on the boundary edges or the walls.

For transverse vibration problems of annular sector plate, the displacement admissible function can be expressed as[15]

$$w(s,\theta)=\sum_{m=-4}^{\infty}\sum_{n=-4}^{\infty}A_{mn}\Phi_m(s)\Phi_n(\theta) \qquad r=s+a \tag{1}$$

where A_{mn} denotes the series expansion coefficients, and

$$\begin{aligned}\Phi_m(s)&=\begin{cases}\cos\lambda_m s & m\ge 0\\ \sin\lambda_m s & m<0\end{cases} \qquad \lambda_m=m\pi/R\\ \Phi_n(\theta)&=\begin{cases}\cos\lambda_n\theta & n\ge 0\\ \sin\lambda_n\theta & n<0\end{cases} \qquad \lambda_n=n\pi/\phi\end{aligned} \tag{2}$$

Traditionally, the sound pressure can be expressed as the conventional three-dimensional Fourier series. However, the normal derivative of the pressure function will always be zero at the panel-cavity interface, failing to correctly predict the pressure and velocity distributions in the close proximity of the interface. Recently, SGM was presented to eliminate the similar discontinuity problem on arbitrary impedance wall surface for a pure acoustic cavity problem[16]. According to the Figure 1, the structural-acoustic coupling interface is located at z=L_z. Therefore, the sound pressure can be accordingly expressed as

$$p(s,\theta,z)=\sum_{m=-2}^{\infty}\sum_{n=-2}^{\infty}\sum_{l=-2}^{\infty}B_{mnl}\Phi_m(s)\Phi_n(\theta)\Phi_l(z) \qquad r=s+a \tag{3}$$

The basis function $\Phi_l(z)$ in the z-direction is also given by Equation (2) except for $\lambda_l=l\pi/h$. B_{mnl} denotes the series expansion coefficients for acoustic variables. The two sine terms introduced in each direction basis function can ensure the continuity at the walls.

It is noteworthy that with all unknown series expansion coefficients being solved, the displacement and the sound pressure functions are capable of representing the exact solution to the problem.

2.3 Solution for the coupling system

Due to the sufficiently smooth characteristic of the displacement and acoustic pressure functions of the coupled system, the weak solution based on energy principle is also equavilent to a solution determined from directly solving the governing equations and the accompanying boundary and coupling conditions. Therefore, the energy formulation description will be given to derive the governing equation for the coupled systems.

Firstly, the Lagrangian function of the annular sector plate can be described as

$$L_{\mathrm{p}} = V_{\mathrm{p}} + V_{\mathrm{s}} - T_{\mathrm{p}} + W_{\mathrm{a\&p}} \tag{4}$$

where V_{p} denotes the total potential energy associated with the bending deformation of the plate; V_{s} represents the potential energy stored in the boundary restraining springs; T_{p} is the total kinetic energy of the plate; and $W_{\mathrm{a\&p}}$ represents the work done by the sound pressure acting on the structural-acoustic interface which can be determined from

$$W_{\mathrm{a\&p}} = \int_S wpdS_1 = \int_0^R \int_0^\phi wp(s+a)dsd\theta \tag{5}$$

For small amplitude vibration, the elastic strain energy of the annular sector plate can be written as

$$\begin{aligned} V_{\mathrm{p}} = \frac{D}{2}\int_0^\phi \int_0^R \Bigg[& \left(\frac{\partial^2 w}{\partial s^2} + \frac{1}{s+a}\frac{\partial w}{\partial s} + \frac{1}{(s+a)^2}\frac{\partial^2 w}{\partial \theta^2} \right)^2 \\ & -2(1-\mu)\frac{\partial^2 w}{\partial s^2}\left(\frac{1}{s+a}\frac{\partial w}{\partial r} + \frac{1}{(s+a)^2}\frac{\partial^2 w}{\partial \theta^2} \right) \\ & +2(1-\mu)\left\{ \frac{\partial}{\partial s}\left(\frac{1}{s+a}\frac{\partial w}{\partial \theta} \right)^2 \right\}^2 \Bigg](s+a)dsd\theta \end{aligned} \tag{6}$$

where $D=Eh_{\mathrm{p}}^3/(12(1-\mu^2))$is the flexural rigidity of the plate; μ is the Poisson's ratio; E denotes the Young's modulus.

The potential energy stored in the elastic boundary restraints is

$$\begin{aligned} V_{\mathrm{s}} = & \frac{1}{2}\int_0^\phi \left\{ a\left[k_{s0}w^2 + K_{s0}(\frac{\partial w}{\partial s})^2 \right]_{s=0} + b\left[k_{s1}w^2 + K_{s1}(\frac{\partial w}{\partial s})^2 \right]_{s=R} \right\} d\theta \\ & + \frac{1}{2}\int_0^R \left\{ \left[k_{\theta 0}w^2 + K_{\theta 0}(\frac{\partial w}{(s+a)\partial \theta})^2 \right]_{\theta=0} + \left[k_{\theta 1}w^2 + K_{\theta 1}(\frac{\partial w}{(s+a)\partial \theta})^2 \right]_{\theta=\phi} \right\} ds \end{aligned} \tag{7}$$

where k_{s0} and k_{s1} ($k_{\theta 0}$ and $k_{\theta 1}$) are linear spring constants, and K_{s0} and K_{s1} ($K_{\theta 0}$ and $K_{\theta 1}$) are the rotational spring constants at s=0 and R (θ=0 and ϕ), respectively.

The kinetic energy of the annular sector plates, by neglecting rotary inertia, is given by

$$T_{\mathrm{p}} = \frac{1}{2}\iint_A \rho_{\mathrm{p}} h_{\mathrm{p}} (\partial w/\partial t)^2 dA = \frac{1}{2}\rho_{\mathrm{p}} h_{\mathrm{p}} \omega^2 \int_0^R \int_0^\phi w^2 (s+a)\,\mathrm{d}s\,\mathrm{d}\theta \tag{8}$$

where ρ_{p} is the mass density for the plate; ω denotes the circular frequency.

Then, the Lagrangian function of the acoustic cavity can be defined as

$$L_{\mathrm{a}} = V_{\mathrm{a}} - T_{\mathrm{a}} - W_{\mathrm{wall}} - W_{\mathrm{p\&a}} \tag{9}$$

in which the term V_{a} denotes the acoustic potential energy of the enclosure; T_{a} is the kinetic energy of the cavity; W_{wall} is the dissipated acoustic energy caused by the impedance walls; $W_{\mathrm{p\&a}}$ represents the work associated with the vibration of the annular sector plate.

The acoustic potential energy of the enclosure can be calculated from

$$V_{\mathrm{a}} = \frac{1}{2\rho_0 c_0^2}\int_V p^2(s,\theta,z)dV = \frac{1}{2\rho_0 c_0^2}\int_0^R\int_0^\phi\int_0^h p^2(s+a)dsd\theta dz \tag{10}$$

in which ρ_0 and c_0 are the mass density and the sound speed of the acoustic medium in the cavity.

The kinetic energy can be explicitly expressed as

$$T_{\mathrm{a}} = \frac{1}{2\rho_0\omega^2}\int_0^R\int_0^\phi\int_0^h \left[\left(\frac{\partial p}{\partial s}\right)^2 + \left(\frac{\partial p}{(s+a)\partial\theta}\right)^2 + \left(\frac{\partial p}{\partial z}\right)^2\right](s+a)dzd\theta ds \tag{11}$$

where ω denotes the circular frequency.

According to the coupled system model, the dissipated acoustic energy can be determined with

$$W_{\mathrm{wall}} = \sum_{i=1}^{5}\int_{s_i}\tilde{W}_a ds_i = -\frac{1}{2}\sum_{i=1}^{5}\int_{s_i}\frac{p^2}{j\omega Z_i}ds_i \tag{12}$$

where Z_i and S_{i} denote the complex acoustic impedance of the i'th wall's surface and the i'th impedance-wall surface. The surface (z=h) is the coupled surface. Therefore, the number of impedance-wall surface is five.

The work associated with plate vibration is equal to the work associated with the sound pressure of the acoustic cavity, that is

$$W_{\mathrm{a\&p}} = W_{\mathrm{p\&a}} \tag{13}$$

The Rayleigh-Ritz technique is utilized to determine the solution. In this case, minimizing the Lagrangian functions against the unknown series expansion coefficients, one can be able to obtain the finial system in matrix form as

$$\left[\mathbf{K}_{\mathrm{p}} - \omega^2\mathbf{M}_{\mathrm{p}}\right]\mathbf{E} + \mathbf{C}_{\mathrm{a\&p}}\mathbf{F} = \mathbf{0} \tag{14}$$

$$\left[\mathbf{K}_{\mathrm{a}} + \omega\mathbf{Z}_{\mathrm{s}} + \omega^2\mathbf{M}_{\mathrm{a}}\right]\mathbf{F} + \omega^2\mathbf{C}_{\mathrm{p\&a}}\mathbf{E} = \mathbf{0} \tag{15}$$

where $\mathbf{K}_{\mathrm{p}}$ and $\mathbf{M}_{\mathrm{p}}$ are, respectively, stiffness and mass matrices of the annular sector plate; $\mathbf{K}_{\mathrm{a}}$ and $\mathbf{M}_{\mathrm{a}}$ denote the stiffness and mass matrix of the acoustic cavity, respectively; $\mathbf{C}_{\mathrm{a\&p}}$ is the coupling matrix between the acoustic cavity and panel structure, $\mathbf{C}_{\mathrm{p\&a}} = \mathbf{C}_{\mathrm{a\&p}}^{\mathrm{T}}$; **E** and **F** represent the unknown series expansion coefficients for plate and acoustic cavity, respectively.

It is not difficult to find that there is a nonlinear term for the circular frequency, which make it difficult to solve directly. Therefore, it needs further transformations and the circular frequency can be transfered into a linear matrix equation which can be solved directly [17].

$$\left(\mathbf{R} - \omega\mathbf{S}\right)\mathbf{Y} = \mathbf{0} \tag{16}$$

where

$$\mathbf{R}=\begin{bmatrix} 0 & \mathbf{K}_{\mathrm{p}} & 0 & 0 \\ \mathbf{K}_{\mathrm{p}} & 0 & \mathbf{C}_{\mathrm{a\&p}} & 0 \\ 0 & 0 & 0 & -\mathbf{K}_{\mathrm{a}} \\ 0 & 0 & \mathbf{K}_{\mathrm{a}} & \mathbf{Z}_{\mathrm{s}} \end{bmatrix} \tag{17}$$

$$\mathbf{S}=\begin{bmatrix} \mathbf{K}_{\mathrm{p}} & 0 & 0 & 0 \\ 0 & \mathbf{M}_{\mathrm{p}} & 0 & 0 \\ 0 & 0 & -\mathbf{K}_{\mathrm{a}} & 0 \\ 0 & -\mathbf{C}_{\mathrm{p\&a}} & 0 & -\mathbf{M}_{\mathrm{a}} \end{bmatrix} \tag{18}$$

$$\mathbf{Y}=\begin{bmatrix}\mathbf{E} & \omega\mathbf{E} & \mathbf{F} & \omega\mathbf{F}\end{bmatrix}^{\mathrm{T}} \tag{19}$$

It is easy to find that the circular frequency is generalized eigenvalue of Equation (16). Moreover, the generalized eigenvectors is **Y**, which can be introduced into the displacements and the acoustic pressure, and the mode shape can be obtained immediately. The presented model can be directly extended to forced vibration analysis of coupled system and an acoustic cavity bounded by multiple panel structures.

3. NUMERICAL RESULTS AND DISCUSSIONS

In this section, numerous numerical examples will be presented to validate the accuracy and effectiveness of the current approach in predicting the structural-acoustic coupling behaviour of the coupled system. On this basis, some new results and discussions are given which includes the cavity depth and various boundary conditions. Since little result can be found in the open literature for the annular sector plate-acoustic cavity model, the results calculated with FEM is employed for comparisons.

As the first problem, for testing the validity of the current solutions, coupled system with geometry dimensions (a=0.4m, b=1m， $\phi=\pi/3$, h=0.5m, h_{p}=5mm). The boundary conditions for the annular sector plate is simply supported (denoted with SSSS). The material parameters of the plate structure are given as: mass density ρ_{p}=7800kg/m^3, Young's modulus E=200GPa, and Poisson's ratio v =0.3. The density of and sound speed in the cavity are ρ_0=1.21kg/m^3 and c_0=340m/s, respectively. In the present solution framework, the simply supported boundary condition can be readily realized by setting the stiffness of the rotation and translational springs to zero and infinity (represented by 10^{10} in the numerical calculation), respectively. The five walls (except for z=h surface) are acoustically rigid. The first 10 natural frequencies of the coupled system are listed in Table 1. The difference listed in this table is defined as $|f_{\mathrm{SGM}}-f_{\mathrm{FEM}}|/f_{\mathrm{FEM}}\times 100$.

Table 1 – The first 10 modal frequencies for the coupled plate-cavity system

Mode order	FEM, Hz	SGM, Hz	Difference, %
1	57.33	57.40	0.12
2	120.15	120.21	0.05
3	160.26	160.12	0.09
4	216.38	216.30	0.04
5	222.57	221.97	0.27
6	238.08	238.14	0.03
7	292.03	291.90	0.04
8	329.13	327.76	0.41
9	339.09	338.68	0.12
10	341.10	341.03	0.02

A nice agreement can be observed between these two sets of results with the largest difference being less than 0.5%. In the present calculation, the trigonometric series expansion is truncated to M_p=N_p=12 for the displacement expression of the panel, and to M=N=L=3 for the acoustic pressure expression.

Table 2 gives the first seven natural frequencies of the annular sector plate-cavity coupled system with various sector angles and cavity depths. The geometry dimensions and materials for the coupled system are kept the same with the previous example. The results obtained with SGM agree well with the FEM results. Table 2 shows the first seven narutal frequencies of the coupling system varying with various sector angles and cavity depths. It can be observed that the first order natural frequency decrease with the increase of the sector angle ϕ.

Table 2 – The first 7 modal frequencies for the coupled plate-cavity system with various sector angles and cavity depths

h	ϕ	Method	Mode number						
			1	2	3	4	5	6	7
0.8	$\pi/3$	SGM	56.94	120.24	160.12	212.06	217.08	222.44	238.13
		FEM	56.89	120.33	160.44	212.05	217.78	222.38	238.64
	π	SGM	37.56	43.58	56.32	74.47	81.19	94.96	120.37
		FEM	37.58	43.61	56.31	73.07	80.08	95.01	120.53
	$3\pi/2$	SGM	36.25	37.13	43.96	50.82	54.87	61.49	73.49
		FEM	36.24	37.60	43.92	50.86	54.97	61.48	73.52
	2π	SGM	34.40	35.77	39.58	41.66	44.99	50.20	56.36
		FEM	34.00	35.72	39.57	41.82	44.88	49.44	56.36
1.6	$\pi/3$	SGM	56.39	106.69	120.23	160.13	212.24	216.68	222.36
		FEM	56.34	106.71	120.33	160.44	212.23	217.42	222.30
	π	SGM	36.93	43.73	56.22	73.75	79.66	94.45	106.81
		FEM	36.97	43.70	56.24	73.22	79.60	94.94	106.74
	$3\pi/2$	SGM	35.62	37.94	43.87	51.09	54.14	61.46	73.46
		FEM	35.64	37.95	43.86	51.08	54.14	61.46	73.47
	2π	SGM	34.92	35.13	39.51	41.19	44.27	49.43	56.30
		FEM	34.94	35.15	39.52	41.20	44.28	49.44	56.31

It is further noticed that the boundary conditions of the panel have influence on the mode characteristics of the coupled system. Therefore, the effect of boundary restraints of the annular sector plate on the modal characteristics of coupled system will be analyzed. As mentioned before, it is convienient to obtain all kind os boundary conditions for the elastic plate through modifying the stiffness of translational and rotational springs along each panel edge. Different boundary conditions of the plate are considered in Table 3. The geometry dimensions for the coupled system is defined as: a=0.5m, b=1m， $\phi=\pi/6$, h=0.6m, h_p=3mm. The material property for plate and acoustic medium is kept the same with the previous example.

Table 3 – The first 10 modal frequencies for the coupled plate-cavity system with elastic restraint

Mode order	$k=10^{10}$N/m K=0Nm/rad		$k=10^{10}$N/m $K=10^2$Nm/rad		$k=10^{10}$N/m $K=10^6$Nm/rad		$k=10^{10}$N/m $K=10^{10}$Nm/rad	
	SGM	FEM	SGM	FEM	SGM	FEM	SGM	FEM
1	76.43	76.35	77.11	77.03	139.81	139.86	140.46	140.50
2	166.71	166.62	167.34	167.26	248.37	248.50	249.45	249.58
3	203.07	203.31	203.78	204.03	283.80	283.77	283.80	283.77
4	283.58	283.55	283.58	283.56	303.18	303.68	304.60	305.12
5	312.36	312.88	312.97	313.49	346.22	346.17	346.26	346.17
6	320.44	320.56	321.14	321.26	404.03	403.90	404.05	403.90
7	346.17	346.12	346.17	346.12	415.46	416.50	417.17	418.22
8	394.55	396.20	395.26	396.92	439.68	440.05	441.72	442.10
9	403.98	403.84	403.98	403.84	447.40	447.21	447.37	447.21
10	447.38	447.20	447.38	447.21	493.52	493.22	493.57	493.23

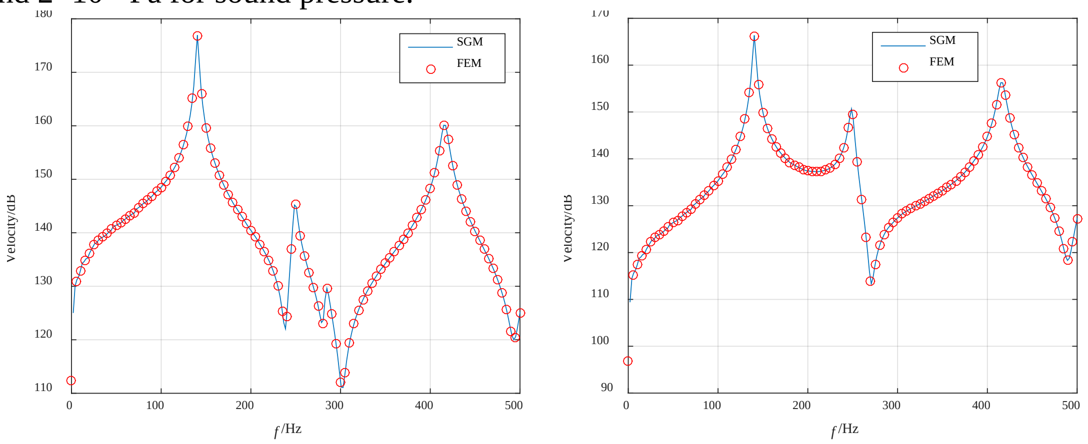
It can be seen from Table 3 that the SGM results agree well with FEM results for coupled system with general elastic condition. It can also be observed that the stiffness of the rotational springs have influence to the natural frequencies of the coupled system than.

Besides air, water is another common medium in life. Table 4 lists the first ten natural frequencies of the annular sector plate-cavity coupled system filled with water. The geometry dimensions and the material property of the annular sector plate is kept the same with the previous example. The mass density and the speed of sound propagation in the water isρ_{air}=1000kg/m^3 and c_{air}=1500m/s. The results calculated by FEM are also given in Table 4 as a reference. The maximum error of the two methods in natural frequency study is 0.95%. This shows that although the proposed method is slightly different from the FEM in natural frequency calculation, it shows good consistency in revealing the strong coupling between the plate and water medium.

Table 4 – The first 10 modal frequencies for the coupled plate-cavity system filled with water

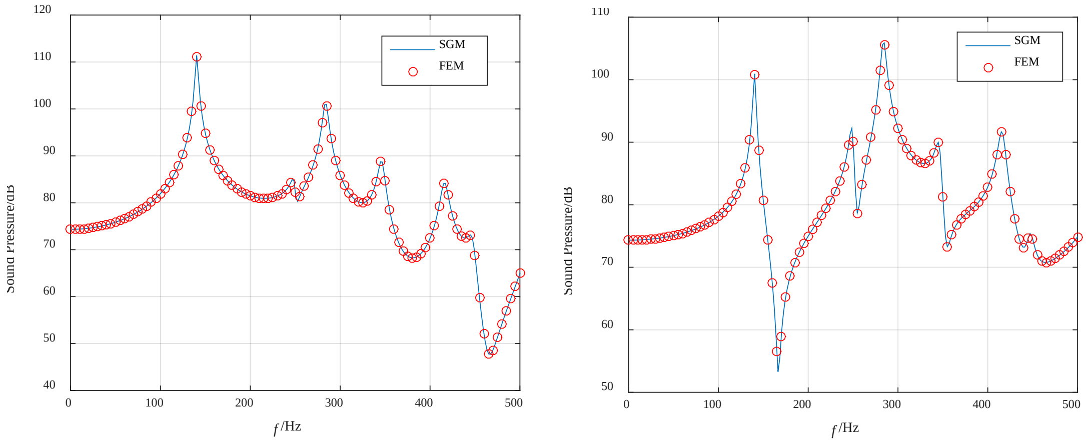
Mode order	FEM, Hz	SGM, Hz	Difference, %
1	111.47	112.54	0.95
2	143.75	143.34	0.29
3	204.19	205.72	0.75
4	237.29	236.87	0.18
5	282.62	281.57	0.37
6	352.27	352.61	0.10
7	371.92	370.76	0.31
8	427.23	427.86	0.15
9	488.80	487.14	0.34
10	514.73	515.56	0.16

Finally, the forced response analysis of the annular sector plate-cavity coupled system is investigated based on the modal analysis of the coupled system. The geometry dimensions of the coupled system is kept with the same configuarations of the example for Table 4. The acoustic medium is set as air. The point force is located at (0.25, 15°) whose amplitude is F=1 N. In order to mitigate the effect of resonance to a certain extent, the values of the sound speed is defined as $c_1=c_0(1+j\xi)$, which ξ is the damping ratio, ξ=0.01. Figure 2 shows the velocity responses at two separate locations (0.25, 15°) and (0.4, 22°). Plotted in Figure 3 are the sound pressure inside the cavity at (0.15, 10°, 0.2) and (0.4, 20°, 0.5). The reference values in the dB scales are 10^{-9} m/s for velocity and 2×10^{-5} Pa for sound pressure.



(a) (0.25, 15°) (b) (0.4, 22°)

Figure 2 Velocity response of the plate



(a) (0.15, 10°, 0.2)   (b) (0.4, 20°, 0.5)

Figure 2 Sound pressure response of the cavity

It is seen that both the resonant peaks and magnitude of the calculated vibration and acoustic responses match well with the FEM results. So the analytical model constructed in this artile can be used to predict the forced response of the annular sector plate-acoustic cavity coupled system.

4. CONCLUSIONS

In this investigation, the structural-acoustic coupling analysis of a coupled structural-acoustic system consisting of a three-dimensional cavity bounded by an annular sector plate with general restrained edges is performed with SGM. Under the current solution framework, the transverse vibration displacement of the annular sector plate and the sound pressure in the cavity is invariantly expressed as a new form of trigonometric series expansions. The introduction of the sine terms not only eliminates the potential discontinuities at the coupling surface or elastically restrained edges but also accelerate the convergence of the series expansions. Rayleigh-Ritz procedure is utilized to derive the motion equation of the coupled system. The correctness of the established analytical model has been demonstrated by comparing the current results with FEM results.

The current model can be effectively utilized to predict the modal behaviour and the vibro-acoustic response of an annular sector plate-cavity system. Unlike the conventional Fourier series or modal expansions, all the classical homogeneous boundary conditions for the bounding plate can be treated in a unified manner simply by setting the stiffness for each elastic restraint to either zero or infinity. In addition, some new results and discussions are given, which could be the benchmark for future research. Although the vibro-acoustic system in this study includes only one elastic plate, the proposed method can be readily extended to those involving annular sector plates and shell.

5. ACKNOWLEDGEMENTS

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