On the reduction of sound transmission by infinite periodic plate

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ABSTRACT

Periodic structure is well known for its ability to reduce vibration and sound transmission in stop-band. As an effective tool to calculate the band structure of periodic structure as well as homogeneous structure, Wave Finite Element method(WFE) has caught much attention since it was proposed. In this paper, WFE is extended to calculate the sound transmission loss (STL) of an infinite periodic panel. Unlike homogeneous structure, modeling of the unit cell of periodic structure necessitates a sufficient number of elements inside the cell and integration of sound pressure over cell surface. In order to verify this method, the STL through an infinite homogeneous plate is calculated by analytical method and present method respectively. The results obtained by the two methods have good agreement. Then this method is applied to a periodic panel whose cell is made of three different types of materials and a significant reduction of sound transmission is found in the stop-band of the periodic panel.

Keywords: Periodic structure, reduction of sound transmission, mitigation of vibration, Wave Finite Element method
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1. INTRODUCTION

Periodic structures are shown to exhibit interesting behaviors in stop-band. All waves or a certain type of wave are attenuated in this special frequency band. Thanks to this characteristic, periodic structures can serve as vibration and sound filters. The mechanism of the former is related to attenuation of elastic wave propagation in solid and the latter involves reduction of sound wave transmission.

As to vibration mitigation, the effect of periodic structures has been thoroughly studied in the last twenty years. These studies have covered different configurations of unit cell including periodically simply supported beam [1], cellular grids [2], perforated plates and periodically rib-stiffened plate [3], sandwich panels with corrugated cores [4] and periodic composite core [5].

The sound reduction property of periodic structures has attracted much attention since the emergence of metamaterials which exhibit negative effective mass [6] and higher sound transmission loss (STL) in band gaps [7]. Thus the calculation of STL through periodic structures becomes a prerequisite for a thorough research on this topic. Analytical methods including effective mass density method and plane wave expansion method (PWE) are commonly used in the latest works of STL calculation through periodic structures. PWE has been applied to thin plates with attached resonators in [8] and [9], while effective mass density method has been used to treat the cases of thin plate with periodic piezoelectric patches [10] and plate with lateral local resonators in the presence of external mean flow [11]. A common requirement of these two methods lies in the availability of wave propagation equation of the host plate. The treatment of complex cases like sandwich or multi-domain unit cell will be a challenge for these analytical methods.

Wave finite element method has been widely used to study the wave propagation in homogeneous structure [12]. This method now extends to calculation of STL through homogeneous structure. As the unit cell is modeled with finite element method, the structure can be complex in the depth direction. The cases of thin plate (isotropic and orthotropic) and sandwich have been studied by Jean-Loup Christen [13] with lumped force and Yi Yang [14] with integrated force. Yi Yang also extends his method to multi-layered structures with fluid layers [15] and cylindrical structures [16].

Unlike homogeneous structure, an important limitation of periodic structure is that the unite cell is defined by physical problem and can not be changed to adapt to wave length. As stated by Jean-Loup Christen and Mohamed Ichchou in [13], the element size will influence the accuracy of result especially in high frequency part. If the cell size and element size are kept linking, the requirement of physical problem and accuracy can not be met at the same time. An adaption should be made to treat the case of bigger unit cell which may contain several types of materials. Consequently, we propose to mesh the unit cell with an adequate number of finite elements. The adequate number will be determined by comparison to wave length. This will lead to unit cell with internal nodes. The calculation of STL of a structure with unit cell including internal nodes has not been treated by existing work.

2. MODELING OF SOUND TRANSMISSION THROUGH AN INFINITE PERIODIC PLATE

2.1. Application of floquet-bloch theorem to unit cell with internal nodes

The unit cell of periodic structure is discretized by FEM and the governing equation is written as:

$$([\boldsymbol{K}] - \omega^2[\boldsymbol{M}])\{\boldsymbol{q}\} = \{\boldsymbol{f}\} + \{\boldsymbol{e}\} \tag{1}$$

where $\boldsymbol{K}$ and $\boldsymbol{M}$ are respectively the stiffness matrix and mass matrix; $\boldsymbol{f}$ is vector of forces between cells; $\boldsymbol{e}$ represents external force (force imposed on the plate by acoustical pressure in our case). The expression of force vector can be found in section 2.3.

$\boldsymbol{q}$ is a vector of DOFs of the unit cell and is divided into 9 parts according to the following order:

$$\{\boldsymbol{q}\} = \left\langle \; \boldsymbol{q}_{lb} \quad \boldsymbol{q}_{rb} \quad \boldsymbol{q}_{rt} \quad \boldsymbol{q}_{lt} \quad \boldsymbol{q}_{b} \quad \boldsymbol{q}_{r} \quad \boldsymbol{q}_{t} \quad \boldsymbol{q}_{l} \quad \boldsymbol{q}_{i} \; \right\rangle^T \tag{2}$$

The division map is shown in figure 1.

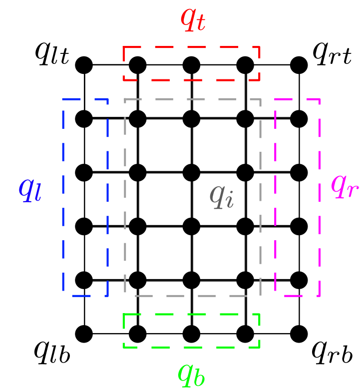


Figure 1: The unit cell and distribution of DOFs

The whole DOFs of the unit cell can be reduced to the base $\{\widetilde{\boldsymbol{q}}\}$ by applying floquet-bloch theorem and continuity of displacements [17]:

$$\{\boldsymbol{q}\} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}e^{-ik_x\triangle_x} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}e^{-ik_x\triangle_x - ik_y\triangle_y} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}e^{-ik_y\triangle_y} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{I}e^{-ik_x\triangle_x} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}e^{-ik_y\triangle_y} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \{\widetilde{\boldsymbol{q}}\} = [N(k_x, k_y)]\{\widetilde{\boldsymbol{q}}\} \tag{3}$$

where

$$\{\widetilde{\boldsymbol{q}}\} = \left\langle \; \boldsymbol{q}_{l} \quad \boldsymbol{q}_{b} \quad \boldsymbol{q}_{lb} \quad \boldsymbol{q}_{i} \; \right\rangle^T \tag{4}$$

k_x and k_y are wave number along x and y direction, $\triangle_x$ and $\triangle_y$ are length and width of the unit cell.

Multiply Equation (1) by the conjugate transpose of matrix N, we can obtain:

$$\begin{aligned}&[\boldsymbol{N}^H(k_x,k_y)]([\boldsymbol{K}]-\omega^2[\boldsymbol{M}])[\boldsymbol{N}(k_x,k_y)]\{\widetilde{\boldsymbol{q}}\}\\ =&[\boldsymbol{N}^H(k_x,k_y)]\{\boldsymbol{f}\}+[\boldsymbol{N}^H(k_x,k_y)]\{\boldsymbol{e}\}\\ =&[\boldsymbol{N}^H(k_x,k_y)]\{\boldsymbol{e}\}\end{aligned} \tag{5}$$

where $[\boldsymbol{N}^H(k_x,k_y)]\{\boldsymbol{f}\}$ vanishes by applying again floquet-bloch theorem and equilibrium of forces between cells.

As sound pressure is also harmonic in space and continuous between cells, the treatment of $\{\boldsymbol{e}\}$ can be similar to displacement—i.e., $\{\boldsymbol{e}\}=[\boldsymbol{N}(k_x,k_y)]\{\widetilde{\boldsymbol{e}}\}$. As a result, $[\boldsymbol{N}^H(k_x,k_y)]\{\boldsymbol{e}\}=[\boldsymbol{N}^H(k_x,k_y)][\boldsymbol{N}(k_x,k_y)]\{\widetilde{\boldsymbol{e}}\}=\langle 2\boldsymbol{e}_l\ 2\boldsymbol{e}_b\ 4\boldsymbol{e}_{lb}\ \boldsymbol{e}_i\rangle^t$. Since we will integrate sound pressure over cell surface and introduce directly distributed force, this manipulation is currently not useful in our case but may interest other readers.

2.2. Incident plane sound wave, reflection and transmission

This subsection will be dedicated to sound transmission theory and deduction of external force $\{\boldsymbol{e}\}$.

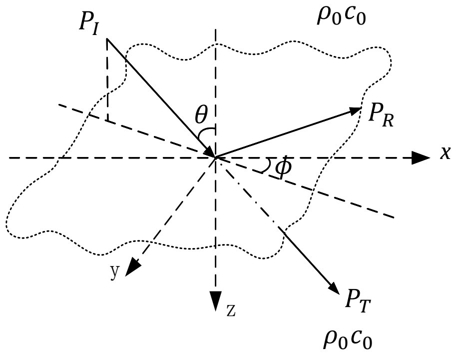


Figure 2: Incident plane wave, transmission and reflection

We consider a simple situation in which a plane wave is incident from air onto an infinite periodic plate with elevation angle θ and azimuth angle ϕ. This oblique incident plane wave interacts with the plate and creates reflected wave in this side and transmitted wave in the other air half-space. As the fluid on the two sides is assumed identical, the z-component of the wave vector will be same for both incident wave and transmitted wave. Thus the incident, reflected and transmitted wave can be written as

$$\begin{aligned}P_I&=p_I e^{(i\omega t-ik_x x-ik_y y-ik_z z)}\\ P_R&=p_R e^{(i\omega t-ik_x x-ik_y y+ik_z z)}\\ P_T&=p_T e^{(i\omega t-ik_x x-ik_y y-ik_z z)}\end{aligned} \tag{6}$$

For the sake of simplicity, the term $e^{i\omega t}$ will be omitted in the following deduction.

According to the continuity of velocity on the interface of fluid and structure—i.e., the upper and lower face of the infinite plate, we have:

$$\begin{aligned}-\frac{1}{\rho_0}\frac{\partial P_1}{\partial z}\bigg|_{z=0}&=i\omega v\bigg|_{z=0}=-\omega^2 W\\ -\frac{1}{\rho_0}\frac{\partial P_2}{\partial z}\bigg|_{z=0}&=i\omega v\bigg|_{z=0}=-\omega^2 W\end{aligned} \tag{7}$$

where P_1 represents the pressure field of the upper half-space and equals the sum of P_I and P_R. While P_2 represents the pressure field of the lower side and equals P_T.

Substituting Equation (6) for P_1 and P_2, we can establish a relation between the amplitudes of sound pressure and amplitude of transverse displacement of the plate:

$$\begin{aligned} \frac{1}{\rho_0}(-ik_z p_I + ik_z p_R)e^{(-ik_x x - ik_y y)} = \omega^2 w_0 e^{(-ik_x x - ik_y y)} \Rightarrow p_I - p_R = \frac{i\rho_0\omega^2 w_0}{k_z} \\ \frac{1}{\rho_0}(-ik_z p_T)e^{(-ik_x x - ik_y y)} = \omega^2 w_0 e^{(-ik_x x - ik_y y)} \Rightarrow p_T = \frac{i\rho_0\omega^2 w_0}{k_z} \end{aligned} \tag{8}$$

where w_0 is the amplitude of the plate displacement.

The force imposed on the plate can be written as:

$$F = P_1\Big|_{z=0} - P_2\Big|_{z=0} = (P_I + P_R)\Big|_{z=0} - P_T\Big|_{z=0} = (2p_I - \frac{2i\rho_0\omega^2 w_0}{k_z})e^{(-ik_x x - ik_y y)} = 2p_I e^{(-ik_x x - ik_y y)} - \frac{2i\rho_0 c_0 \omega W}{cos\theta} \tag{9}$$

2.3. Finite Element discretization of the unit cell

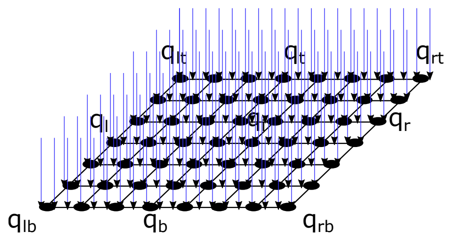


Figure 3: Schematic diagram of discretization of unit cell

The stiffness matrix, mass matrix and external force vector of the unite cell will be obtained by finite element discretization. A schematic diagram of the discretization is shown in figure 3.

As the kinetic energy and strain energy can be expressed in a common way, we focus on the virtual work of the force imposed by sound pressure. According to Equation (9), the discretizaiton of the terms $p_I e^{(-ik_x x - ik_y y)}$ should be discussed firstly. This term can either by integrated analytically or numerically. As this term vary the same way along x and y with W, its distribution can be approximated by the shape function of W.

$$p_I e^{(-ik_x x - ik_y y)} = p_I \langle \boldsymbol{N} \rangle \{\boldsymbol{p}^e\} \tag{10}$$

where $\{\boldsymbol{p}^e\}$ is a vector of nodal value of $e^{(-ik_x x - ik_y y)}$ which can be calculated directly before integration.

The virtual work of the sound pressure can be written as:

$$\begin{aligned} \delta W_s &= \int_A \delta W F dA \\ &= \int_A \delta W (2p_I e^{(-ik_x x - ik_y y)} - \frac{2i\rho_0 c_0 \omega W}{cos\theta}) dA \\ &= \langle \boldsymbol{q}^e \rangle 2p_I \int_A \{\boldsymbol{N}\}\langle \boldsymbol{N} \rangle dA \{\boldsymbol{p}^e\} - \langle \boldsymbol{q}^e \rangle \frac{2i\rho_0 c_0 \omega}{cos\theta} \int_A \{\boldsymbol{N}\}\langle \boldsymbol{N} \rangle dA \{\boldsymbol{q}^e\} \\ &= \langle \boldsymbol{q}^e \rangle \{\boldsymbol{e}^e\} - \langle \boldsymbol{q}^e \rangle \frac{2i\rho_0 c_0 \omega}{cos\theta} [\boldsymbol{M}_p^e] \{\boldsymbol{q}^e\} \end{aligned} \tag{11}$$

Equation (11) indicates that the sound pressure in fact not only contributes to the vector of force but also contribute to mass matrix. Besides the normal mass matrix derived from kinetic energy, a supplementary part M_p^e derived from virtual work of sound pressure should not be ignored. As to e^e, it is the force vector of an element. The force vector $\{e\}$ in section 2.1 can be easily derived by assemble process.

3. CASE OF AN HOMOGENEOUS PLATE-VALIDATION

Although the original goal of this method is to solve the problem of periodic structure, the case of homogeneous plate can be used to verify this method.

We consider an infinite plate made of glass whose properties are $E = 0.69\times10^{11} Pa, h = 0.005m, \nu = 0.29, \rho = 2500kg/m^3, \eta = 0.002$. The fluids on the two sides are identical and both air with density $\rho_0 = 1.29kg/m^3$ and wave speed $c_0 = 340m/s$. A plane wave is incident upon the plate with $\theta = 45°$ and $\phi = 0°$. The plate is discretized by Kirchhoff plate element with 3 DOFs $(w, w_{,x}, w_{,y})$ on each node. The size of the unit cell is 10mm*10mm.

The unit cell is meshed by 100*100 finite elements. The link between cell size and element size has been broken, thus cell size can satisfy physical limitation and element size can meet accuracy requirement. In figure 4, we can see that present method (marked by □) agrees well with the analytical solution.

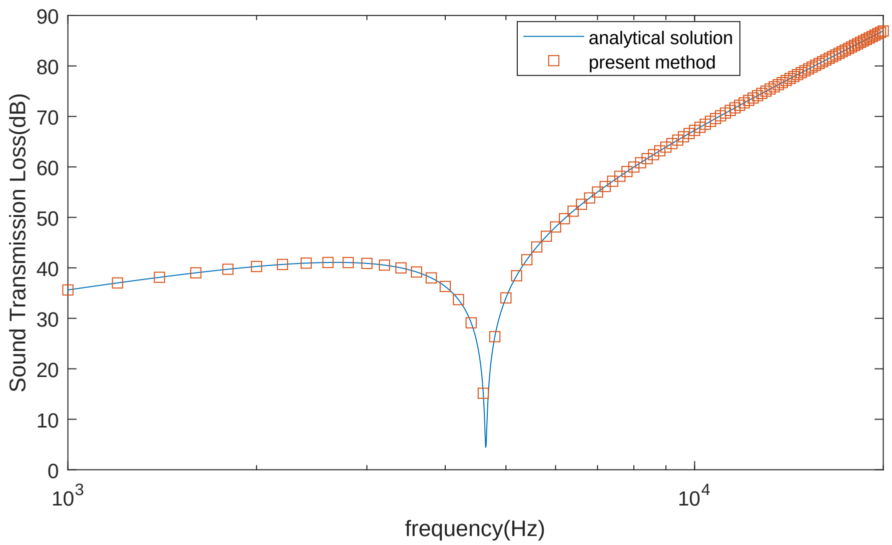


*Figure 4: STL calculated by different methods:—— Analytical solution;□ Cell contains 100*100 elements, cell size 10mm*10mm (present method)*

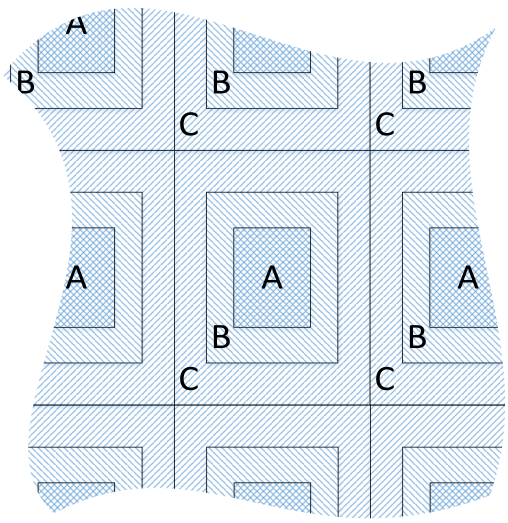
4. APPLICATION TO AN INFINITE PERIODIC PANEL

This method is applied to the case of an infinite periodic plate whose unit cell is made of three types of materials (figure 5a). The properties and geometry of the different domains are listed in table 1. The elevation angle θ is $-54.7°$ and azimuth angle ϕ is 45°.

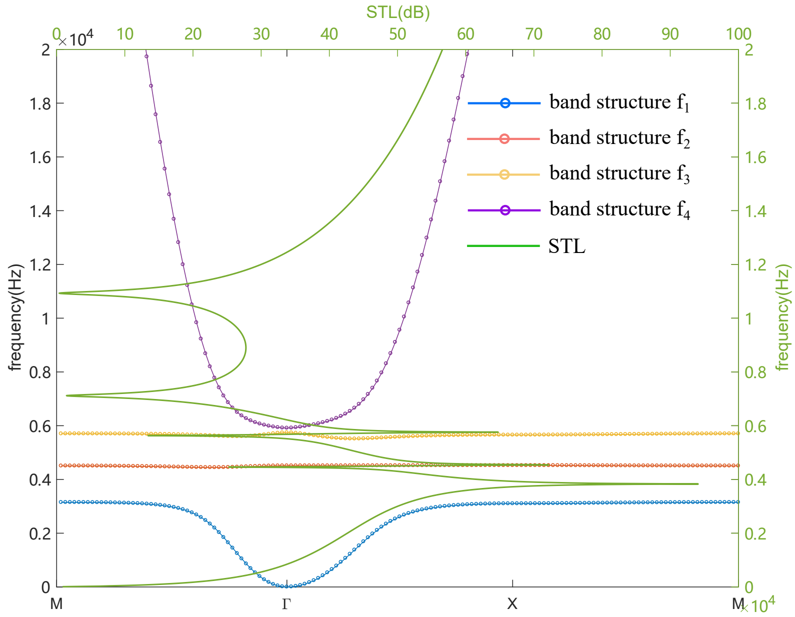
	length	width	thickness	material	E	ν	ρ
	mm	mm	mm	-	Mpa	-	kg/m^3
domain A	4	8	5	steel	200000	0.2	7800
domain B	6	10	5	rubber	0.12	0.47	1300
domain C	10	14	5	epoxy	4350	0.3679	1180

Table 1: Mechanical and geometric properties

The STL through this periodic plate is calculated with the present method and the result is shown in figure 5b. In order to figure out the the relation between band gap and pics of STL, the band structure is also calculated and shown in the same figure. We can see that in the range of audible frequencies there are three band gaps. In each of band gap, the STL raises steeply.



(a) Periodic structure whose unit cell is made of three different types of materials



(b) Band structure and STL

Figure 5: Periodic plate, band structure and STL calculated by present method

5. CONCLUSIONS

A wave finite element method to calculate the sound transmission loss of infinite periodic structure is presented in this paper. The adaption to bigger and multi-domains unit cell which is common case for periodic structure, is realized by meshing the unit cell with a sufficient number of finite elements and integration of sound pressure over the whole unit cell surface. This method consists in extending the WFEM to the case of unit cell with internal nodes. The accuracy of this method is demonstrated by comparison with analytical solution of a homogeneous plate. In the application to an infinite periodic panel, steep raises of STL has been found in each band gap, which agrees well with the conclusion of other works.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

[1] Parikshit Sonekar, Mira Mitra, By Parikshit, and Sonekar And. A wavelet-based model of one-dimensional periodic structure for wave-propagation analysis. *Proc. R. Soc. A Proc. R. Soc. A*, 466:263–281, 10 2009.

[2] Massimo Ruzzene, Fabrizio Scarpa, and Francesco Soranna. Wave beaming effects in two-dimensional cellular structures. *Smart Materials and Structures*, http://dx.doi.org/10.1088/0964-1726/12/3/307:363, 06 2003.

[3] Changwei Zhou. *Wave and modal approach for multi-scale analysis of periodic structures*. PhD thesis, Ecole Centrale de Lyon, 2014.

[4] Zhi-Jing Wu, Feng-Ming Li, and Yi-Ze Wang. Vibration band gap behaviors of sandwich panels with corrugated cores. *Computers & Structures*, 129:30–39, 2013.

[5] X.-F Liu, Yan-Feng Wang, Yue-Sheng Wang, and Ch Zhang. Wave propagation in a sandwich plate with a periodic composite core. *Journal of Sandwich Structures and Materials*, 16:319–338, 04 2014.

[6] Mourad Oudich, B Djafari-Rouhani, Y Pennec, Badreddine Assouar, and Bernard Bonello. Negative effective mass density of acoustic metamaterial plate decorated with low frequency resonant pillars. *Journal of Applied Physics*, 116:184504, 11 2014.

[7] Zhengyou Liu, Xixiang Zhang, Yiwei Mao, Y. Y. Zhu, Zhiyu Yang, C. T. Chan, and Ping Sheng. Locally resonant sonic materials. *Science*, 289(5485):1734–1736, 2000.

[8] Yong Xiao, Jihong Wen, and Xisen Wen. Sound transmission loss of metamaterial-based thin plates with multiple subwavelength arrays of attached resonators. *Journal of Sound and Vibration*, 331:5408–5423, 12 2012.

[9] Badreddine Assouar, Mourad Oudich, and Xiaoming Zhou. Acoustic metamaterials for sound mitigation. *Comptes Rendus Physique*, 02 2016.

[10] Hao Zhang, Jihong Wen, Yong Xiao, Gang Wang, and Xisen Wen. Sound transmission loss of metamaterial thin plates with periodic subwavelength arrays of shunted piezoelectric patches. *Journal of Sound and Vibration*, 343, 02 2015.

[11] Ting Wang, Meiping Sheng, and Qinghua Qin. Sound transmission loss through metamaterial plate with lateral local resonators in the presence of external mean flow. *The Journal of the Acoustical Society of America*, 141:1161–1169, 02 2017.

[12] Brian Mace and Elisabetta Manconi. Modelling wave propagation in two-dimensional structures using finite element analysis. *Journal of Sound and Vibration*, 318:884–902, 12 2008.

[13] Jean-Loup Christen, Mohamed Ichchou, Abdelmalek Zine, and Bernard Troclet. Wave finite element formulation of the acoustic transmission through complex infinite plates. *Acta Acustica united with Acustica*, 102:984–991, 11 2016.

[14] Yi Yang, Brian R. Mace, and Michael J. Kingan. Prediction of sound transmission through, and radiation from, panels using a wave and finite element method. *The Journal of the Acoustical Society of America*, 141(4):2452–2460, 2017.

[15] Yi Yang, Brian Mace, and Michael Kingan. Wave and finite element method for predicting sound transmission through finite multi-layered structures with fluid layers. *Computers & Structures*, 204, 04 2018.

[16] Michael Kingan, Yi Yang, and Brian Mace. Application of the wave and finite element method to calculate sound transmission through cylindrical structures. *Journal of Physics: Conference Series*, 744:012240, 09 2016.

[17] RS Langley. A note on the force boundary conditions for two-dimensional periodic structures with corner freedoms. *Journal of Sound and Vibration*, 167(2):377–381, 1993.