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Acoustic resonance analysis of open cavities with the boundary element method

Wan, Cheng[1]
Zheng, Chang-Jun[2]
Wang, Shuai[3]
Bi, Chuan-Xing[4]
Institute of Sound and Vibration Research, Hefei University of Technology
193 Tunxi Road, Hefei 230009, People's Republic of China

ABSTRACT

Acoustic resonances of open cavities could cause not only noise problems but also damages to structures. When using the finite element method (FEM) to do the resonance analysis of open cavities, the infinite domain outside the cavities has to be truncated, which could bring difficulties to the numerical analysis and also errors to the numerical results. In order to avoid the truncation of the infinite domain and improve the accuracy of the numerical results, this paper presents a boundary element method (BEM) for the acoustic resonance analysis of open cavities. The resulting eigenvalue problem in the BEM is a nonlinear one which is difficult to solve, and thus a contour integral method is employed in this paper to convert it into an ordinary linear one. To filter out the fictitious eigenfrequencies generated by the BEM for exterior problems, a scheme based on the combined boundary integral formulation is presented. Some numerical examples are given to verify the accuracy and validity of the present method.

Keywords: Open cavity, Eigenvalue, Boundary element method, Contour integral
I-INCE Classification of Subject Number: 70

1. INTRODUCTION

Acoustic cavity resonance could cause not only noise problems but also damages to structures, which makes it important to help control noise radiated or scattered from structures [1, 2]. Modal analysis of closed acoustic cavities can be easily implemented using the finite element method (FEM) [3]. But for open cavities, the infinite domains outside the cavities usually have to be truncated in the FEM. Koch [4] simulated the infinite domain by arranging a Perfectly Matched Layer (PML) [5], and used the FEM to do resonance analysis of open cavities. However, the PML method has some shortcomings, for example, the computational domain is enlarged considerably by the unphysical PML region, and thus requires larger solution vectors. Also, the parameters in the method affect the efficiency and accuracy of the method.

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[1]wancheng@mail.hfut.edu.cn
[2]cjzheng@hfut.edu.cn
[3]shuaiwang@hfut.edu.cn
[4]cxbi@hfut.edu.cn

In order to avoid the truncation error of the infinite domain and improve the accuracy of numerical results, this paper presents a boundary element method (BEM) for the acoustic resonance analysis of open cavities. However, since the coefficient matrices of the BEM system of equations involve the frequency implicitly, the original eigenproblem for the Helmholtz equation turns into a nonlinear eigenproblem (NEP) which is difficult to solve. In this paper, a contour integral method called the block Sakurai-Sugiura (SS) method [6] is adopted to solve the resulting NEP. In the block SS method, a NEP is converted into a generalized eigenproblem whose dimension is much smaller than the original one, and the conversion is achieved readily by solving a set of common BEM systems of equations. The method is implemented in the complex domain, consequently it can also extract the eigenvalues with imaginary parts. However, as is well-known that the BEM based on the Kirchhoff-Helmholtz boundary integral equation suffers from the fictitious eigenfrequency problem [7, 8]. In the BEM eigenvalue analysis, fictitious eigenfrequencies usually emerge along with the resonant frequencies of the problem. In order to filter out such ficitious eigenfrequencies, a scheme based on the combined boundary integral formulation which is called the Burton-Miller formulation in the BEM response analysis of exterior acoustic problems [9] is presented in this paper.

The remainder of this paper is organized as follows. In Section 2, the BEM formulations for acoustic problems are reviewed and a boundary element eigensolver based on the contour integral method is developed to do the modal analysis of acoustic open cavities. Numerical examples including a unit circle and a circle with a rectangular cavity are given in Section 3 to show the accuracy and effectiveness of the method. Section 4 concludes the paper with further discussions.

2. THEORY

2.1 BEM formulations

Time-harmonic exterior acoustic problems are considered in this paper. Given a harmonic time dependence $e^{-\mathrm{i}\omega t}$, the differential equation governing the steady-state linear acoustics is the Helmholtz equation

$$\nabla^2 p(\mathbf{x}) + k^2 p(\mathbf{x}) = 0\,, \tag{1}$$

where $p(\mathbf{x})$ is the complex sound pressure at point $\mathbf{x}$ inside the exterior domain S, ∇^2 is Laplace's operator, $k = \omega / c$ is the wave number, ω is the circular frequency and c is the speed of sound in the undisturbed medium.

Eq. (1) can be reformulated into a Kirchhoff-Helmholtz boundary integral equation defined on the structural boundary Γ as follows:

$$c(\mathbf{x})p(\mathbf{x}) + \int_\Gamma q^*(\mathbf{x},\mathbf{y})p(\mathbf{y})\mathrm{d}\Gamma(\mathbf{y}) - \int_\Gamma p^*(\mathbf{x},\mathbf{y})q(\mathbf{y})\mathrm{d}\Gamma(\mathbf{y}) = 0\,, \tag{2}$$

where the coefficient $c(\mathbf{x})$ depends on the geometry of boundary Γ at point $\mathbf{x}$. It is 1/2 when $\mathbf{x}$ is located on a smooth part of the boundary. $p^*(\mathbf{x},\mathbf{y})$ the fundamental solution of the Helmholtz equation, $q(\mathbf{y})$ and $q^*(\mathbf{x},\mathbf{y})$ the normal derivatives of $p(\mathbf{y})$ and $p^*(\mathbf{x},\mathbf{y})$, and $\mathbf{y}$ the field point. For 2D acoustic problems, we have

$$p^*(\mathbf{x},\mathbf{y}) = \frac{\mathrm{i}}{4} H_0^{(1)}(kr)\,, \tag{3}$$

$$q^*(\mathbf{x},\mathbf{y}) = \frac{\partial p^*(\mathbf{x},\mathbf{y})}{\partial n(\mathbf{y})} = -\frac{k\mathrm{i}}{4} H_1^{(1)}(kr)\frac{\partial r}{\partial n(\mathbf{y})}\,, \tag{4}$$

where $r = |\mathbf{x}-\mathbf{y}|$, $n(\mathbf{y})$ is the normal direction on the boundary at the point $\mathbf{y}$, $H_0^{(1)}$ and $H_1^{(1)}$ are first kind Hankel functions of the zeroth and first order, respectively.

The boundary conditions (BC) on Γ can be classified into the Dirichlet BC when the sound pressure is known, the Neumann BC when the normal particle velocity is known and the impedance BC when the acoustic impedance is given on the boundary.

In this paper, Eq. (2) is referred to as the conventional boundary integral equation (CBIE) and it can be utilized to calculate the unknown boundary values. However, the BEM based on it suffers from the fictitious eigenfrequency problem when solving exterior acoustic problems. In the BEM eigenvalue analysis, such fictitious eigenfrequencies can also be obtained. In order to filter them out, a scheme based on the combined integral formulation which is called the Burton-Miller formulation in the BEM response analysis of exterior acoustic problems [9] is given in this paper.

The normal derivative boundary integral equation (NDBIE) is obtained by taking the normal derivative of Eq. (2) at the point $\mathbf{x}$ as

$$c(\mathbf{x})q(\mathbf{x})+\int_{\Gamma}\tilde{q}^{*}(\mathbf{x},\mathbf{y})p(\mathbf{y})\mathrm{d}\Gamma(\mathbf{y})-\int_{\Gamma}\tilde{p}^{*}(\mathbf{x},\mathbf{y})q(\mathbf{y})\mathrm{d}\Gamma(\mathbf{y})=0\,, \tag{5}$$

where $\tilde{p}^{*}(\mathbf{x},\mathbf{y})$ and $\tilde{q}^{*}(\mathbf{x},\mathbf{y})$ are the normal derivatives of $p^{*}(\mathbf{x},\mathbf{y})$ and $q^{*}(\mathbf{x},\mathbf{y})$ at point $\mathbf{x}$, respectively.

$$\tilde{p}^{*}(\mathbf{x},\mathbf{y})=\frac{\partial p^{*}(\mathbf{x},\mathbf{y})}{\partial n(\mathbf{x})}=-\frac{\mathrm{i}k}{4}H_1^{(1)}(kr)\frac{\partial r}{\partial n(\mathbf{x})}\,, \tag{6}$$

$$\tilde{q}^{*}(\mathbf{x},\mathbf{y})=\frac{\partial q^{*}(\mathbf{x},\mathbf{y})}{\partial n(\mathbf{x})}=\frac{\mathrm{i}k}{4r}H_1^{(1)}(kr)n_i(\mathbf{x})n_i(\mathbf{y})+\frac{\mathrm{i}k^2}{4}H_2^{(1)}(kr)\frac{\partial^2 r}{\partial n(\mathbf{x})\partial n(\mathbf{y})}\,, \tag{7}$$

where $H_2^{(1)}$ is first kind Hankel functions of the second order, n_i is the Cartesian component of the vector $n(\mathbf{x})$ or $n(\mathbf{y})$, and the Einstein summation convention is used here, so repeated indices imply summation over their range.

Combining the CBIE and NDBIE, the integral equation for the Burton-Miller's method can be written as follows,

$$\begin{aligned}&c(\mathbf{x})p(\mathbf{x})+\int_{\Gamma}q^{*}(\mathbf{x},\mathbf{y})p(\mathbf{y})\mathrm{d}\Gamma(\mathbf{y})-\int_{\Gamma}p^{*}(\mathbf{x},\mathbf{y})q(\mathbf{y})\mathrm{d}\Gamma(\mathbf{y})\\&+\alpha c(\mathbf{x})q(\mathbf{x})+\alpha\int_{\Gamma}\tilde{q}^{*}(\mathbf{x},\mathbf{y})p(\mathbf{y})\mathrm{d}\Gamma(\mathbf{y})-\alpha\int_{\Gamma}\tilde{p}^{*}(\mathbf{x},\mathbf{y})q(\mathbf{y})\mathrm{d}\Gamma(\mathbf{y})=0\end{aligned}\,, \tag{8}$$

where α is the coupling coefficient of the Burton-Miller's method. Discretizing Eq. (8) by collocation method allows us to formulate the system matrices $\mathbf{H}$ and $\mathbf{G}$ as

$$H^{ij}=c(\mathbf{x}_i)\delta_{ij}+\int_{\Gamma}\left[q^{*}(\mathbf{x}_i,\mathbf{y})+\alpha\tilde{q}^{*}(\mathbf{x}_i,\mathbf{y})\right]\mathrm{d}\Gamma(\mathbf{y})\,, \tag{9}$$

$$G^{ij}=-c(\mathbf{x}_i)\delta_{ij}+\int_{\Gamma}\left[p^{*}(\mathbf{x}_i,\mathbf{y})+\alpha\tilde{p}^{*}(\mathbf{x}_i,\mathbf{y})\right]\mathrm{d}\Gamma(\mathbf{y})\,, \tag{10}$$

where δ_{ij} is the Dirac delta function.

Then, the system of equations can be obtained as

$$\mathbf{Hp}-\mathbf{Gq}=0\,. \tag{11}$$

By applying the BC and rearranging the unknowns to the left hand side, the final BEM system of equations can be obtained. Because the coefficient matrices of the BEM system of equations involves the wave number implicitly, we obtain a NEP in the BEM eigenvalue analysis to find the eigenpairs to satisfy

$$\mathbf{A}(k_j)\mathbf{x}_j=0\,, \tag{12}$$

where $\mathbf{A}$ is the coefficient matrix, $(k_j,\mathbf{x}_j)$ are the eigenpairs which satisfy Eq.(12). In general, it is not an easy task to solve such a NEP, so that different methods have been developed in the past decades. Next, a recently developed contour integral approach proposed in [6] is introduced to form a boundary element eigensolver for the resonance analysis of open acoustic cavities.

2.2 Boundary element eigensolver based on the contour integral method

In this section, a contour integral method called the block SS method [6] is introduced to solve the resulting NEP of the BEM. The method is a projection method which can extract eigenvalues inside a region bounded by a closed Jordan curve. In this method, we first form the following matrix:

$$\mathbf{M}_r = \frac{1}{2\pi \mathrm{i}} \int_C z^r \mathbf{V}^H \mathbf{A}(z)^{-1} \mathbf{V} \mathrm{d}z \,, \tag{13}$$

where C is a closed Jordan curve in the complex plane, $r = 0,1, \cdots, 2K-1$, K are positive integers, $(.)^H$ denotes the conjugate transpose, $\mathbf{V} \in \mathbb{C}^{n\times L}$ is an arbitrary nonzero matrix, $\mathbf{A}(z) \in \mathbb{C}^{n\times n}$ is the coefficient matrix of Eq. (12), n is the number of degrees of freedom of the problem and L should be superior to the maximum algebraic multiplicity of the eigenvalues lying inside C. In addition, $KL \geq m$ and m is the number of eigenvalues lying inside C.

Then two Hankel matrices $\mathbf{H}_1$ and $\mathbf{H}_2$ can be formed by using the matrices $\mathbf{M}_r$

$$\mathbf{H}_1 = \left[\mathbf{M}_{j+r-2} \right]_{j,r=1}^{K}, \tag{14}$$

$$\mathbf{H}_2 = \left[\mathbf{M}_{j+r-1} \right]_{j,r=1}^{K}. \tag{15}$$

By calculating the eigenvalues of the matrix pencil $\mathbf{H}_2 - k\mathbf{H}_1$, the eigenvalues k_j and the eigenvector $\mathbf{w}_j$ located in the closed curve C can be obtained. After obtaining the eigenvalue of the matrix pencil, the eigenvectors for the original NEP can be calculated by

$$\mathbf{x}_j = \mathbf{S}\mathbf{w}_j, \; j = 1, 2, \cdots, m, \tag{16}$$

where $\mathbf{S} = \left[\mathbf{S}_0, \mathbf{S}_1, \cdots, \mathbf{S}_{K-1} \right]$, and

$$\mathbf{S}_r = \frac{1}{2\pi \mathrm{i}} \int_C z^r \mathbf{A}(z)^{-1} \mathbf{V} \mathrm{d}z, \;\; r = 1, 2, \cdots, K-1. \tag{17}$$

In the numerical eigenvalue analysis, the contour integrals in Eqs. (13) and (17) can be calculated numerically by using the N-point trapezoidal rule. If we use a circular integral path $z^C = \gamma + \rho e^{i\theta} \left(0 \leq \theta < 2\pi \right)$, the matrices can be calculated numerically as

$$\hat{\mathbf{M}}_r = \frac{1}{N} \sum_{j=0}^{N-1} \rho \left(\frac{z_j - \gamma}{\rho} \right)^{r+1} \mathbf{V}^H \mathbf{A}(z)^{-1} \mathbf{V} \,, \tag{18}$$

$$\hat{\mathbf{S}}_r = \frac{1}{N} \sum_{j=0}^{N-1} \rho \left(\frac{z_j - \gamma}{\rho} \right)^{r+1} \mathbf{A}(z)^{-1} \mathbf{V} \,. \tag{19}$$

where $z_j^C = \gamma + \rho e^{\mathrm{i}\theta_j}$ and $\theta_j = \left(2\pi / N \right)\left(j + 1/2 \right)$.

Thus, after obtaining the shifted and scaled eigenvalues λ_j, the original eigenvalues k_j can be recovered by

$$k_j = \gamma + \rho \lambda_j \,. \tag{20}$$

In order to calculate the eigenvalue k_j and the eigenvector $\mathbf{w}_j$ for the matrix pencil $\mathbf{H}_2 - k\mathbf{H}_1$, we utilize the singular value decomposition (SVD) on the $\mathbf{H}_1$ to obtain

$$\mathbf{H}_1 = \mathbf{U}\boldsymbol{\Sigma}\mathbf{W}^H \,, \tag{21}$$

where $\mathbf{U}, \mathbf{W} \in \mathbb{C}^{KL\times KL}$ are unitary matrices, $\boldsymbol{\Sigma} = \mathrm{diag}\left(\sigma_1, \sigma_2, \cdots, \sigma_{KL} \right)$ and $\sigma_1, \sigma_2, \cdots, \sigma_{KL}$ are nonnegative real numbers in descending order. Let δ be a positive threshold value and omit the small singular values $\sigma_{m+1} < \delta \bullet \sigma_1$.

The original NEP is now converted into an ordinary linear eigenproblem for finding the eigenpairs $(\lambda_j, \mathbf{y}_j)$ of the matrix $\mathbf{H}_m = \mathbf{H}_3\left(1:m, 1:m \right)$, where $\mathbf{H}_3 = \mathbf{U}^H \mathbf{H}_2 \mathbf{W} \boldsymbol{\Sigma}^{-1}$.

After obtaining the eigenpairs $(\lambda_j, \mathbf{y}_j)$ of $\mathbf{H}_3$, the eigenvalues k_j can be recovered by Eq. (20). The eigenvectors $\mathbf{x}_j$ for the original NEP can be calculated by

$$\mathbf{x}_j = \mathbf{S}\mathbf{W}_m\mathbf{\Sigma}_m^{-1}\mathbf{y}_j,\ j = 1, 2, \cdots, m. \tag{22}$$

Furthermore, we have to calculate $\mathbf{A}^{-1}\mathbf{V}$ for a set of complex wave numbers along the integration path C in Eqs. (18) and (19). Instead of evaluating $\mathbf{A}^{-1}$, the following system of equations can be solved as:

$$\mathbf{AX} = \mathbf{V}, \tag{23}$$

where $\mathbf{X}, \mathbf{V} \in \mathbb{C}^{n\times L}$. The solution of Eq. (23) is still very expensive in the conventional BEM and can be accelerated by using the techniques like the H-matrix method [10] to improve the computational efficiency.

3. NUMERICAL EXAMPLES

Numerical examples are presented in this section to demonstrate the accuracy and efficiency of the proposed method for calculating acoustic resonance frequencies of open cavities. The medium of the acoustic field is air with density of ρ_m = 1.20 kg/m^3 and the sound speed of c = 340.0 m/s. All integrals in the BEM are evaluated numerically through the 10-point Gauss-Legendre quadrature rule.

3.1 A unit circle example

A unit circle with the origin of coordinates at its center is taken as the first example in this section. From the general solution of the problem, the analytical eigenvalues are determined by the roots of $H_n'(k) = 0$ for the Neumann BC [1]. The multiplicity of the eigenvalues is 2. In the numerical eigenvalue analysis, the circumference is divided into 360 constant elements. The contour integral path C is defined as a circle with γ= (5.1, 0), ρ= 3.0 and the parameters in the block SS method are set as $N = 512$, $K = 5$, $L = 15$ and $\delta = 10^{-10}$.

The CBIE and the combined boundary integral formulation are used and the coupling parameter in the combined boundary integral formulation is set as $\alpha = \mathrm{i}/k$ [7]. Relative errors $\varepsilon_{\mathrm{Re}}$ and $\varepsilon_{\mathrm{Im}}$ are defined as

$$\varepsilon_{\mathrm{Re}} = \left|[\mathrm{Re}(k_i^n) - \mathrm{Re}(k_i^a)] / \mathrm{Re}(k_i^a)\right|, \tag{24}$$

$$\varepsilon_{\mathrm{Im}} = \left|[\mathrm{Im}(k_i^n) - \mathrm{Im}(k_i^a)] / \mathrm{Im}(k_i^a)\right|, \tag{25}$$

where k_i^a and k_i^{n} denote the analytical solution and the numerical result, respectively.

The numerical results for the unit circle are given in Table 1, where the symbol ∗ in top right of a number denotes the fictitious eigenvalue. It is noted that the eigenvalues of CBIE in Table 1 are divided into two categories, and one of them contains the eigen-values with large negative imaginary parts, which are the actual eigenvalues of the problem. When considering the acoustic resonance problems in the external field, the energy is transmitted to infinite, which leads to the acoustic radiation damping and represents by the negative imaginary parts of the complex eigenvalues [11]. In addition to the actual eigenvalues, another group of eigenvalues which have very small imaginary parts are the fictitious eigenvalues introduced by using the BEM based on the CBIE.

In Table 1, ‘---’ indicates that the data do not exist since the analytical fictitious eigenvalues have no imaginary parts and the error cannot be calculated. It can be observed from Table 1 that the numerical results agree very well with the analytical eigenvalues, and the multiplicity of the eigenvalues can also be obtained. It is also observed that the fictitious eigenvalues have been removed from the real axis to the complex plane with positive imaginary parts. Since the actual eigenvalues of interest are the complex number with negative imaginary parts, it makes us very easy to filter out the fictitious eigenvalues after using the the combined boundary integral formulation.

Table 1. Numerical eigenvalues for the unit circle.

i	CBIE			BM		
	k_i^{n}	$\varepsilon_{\mathrm{Re}}$	$\varepsilon_{\mathrm{Im}}$	k_i^{n}	$\varepsilon_{\mathrm{Re}}$	$\varepsilon_{\mathrm{Im}}$
1	2.37391-0.96755i	2.5E-05	1.6E-05	2.37392-0.96755i	2.4E-05	1.6E-05
2	2.37391-0.96755i	2.5E-05	1.6E-05	2.37392-0.96755i	2.4E-05	1.6E-05
3	$2.40491+1.1\times10^{-7}$i*	3.6E-05	---	2.98059+1.27956i*	2.4E-01	---
4	3.32218-1.07277i	3.1E-05	2.2E-05	3.32218-1.07276i	2.9E-05	3.5E-05
5	3.32218-1.07277i	3.1E-05	2.2E-05	3.32218-1.07276i	2.9E-05	3.5E-05
6	$3.83185-6.1\times10^{-6}$i*	3.7E-05	---	4.35902+1.49106i*	1.4E-01	---
7	$3.83185-6.1\times10^{-6}$i*	3.7E-05	---	4.35902+1.49106i*	1.4E-01	---
8	4.27703-1.16123i	3.5E-05	1.7E-05	4.27703-1.16119i	3.2E-05	5.0E-05
9	4.27703-1.16123i	3.5E-05	1.7E-05	4.27703-1.16119i	3.2E-05	5.0E-05
10	$5.13581-2.6\times10^{-5}$i*	3.8E-05	---	5.47955+1.57806i*	6.7E-02	---
11	$5.13581-2.6\times10^{-5}$i*	3.8E-05	---	5.47955+1.57806i*	6.7E-02	---
12	5.23683-1.23831i	4.1E-05	6.1E-06	5.23681-1.23824i	3.7E-05	6.4E-05
13	5.23683-1.23831i	4.1E-05	6.1E-06	5.23681-1.23824i	3.7E-05	6.4E-05
14	$5.52028+5.8\times10^{-7}$i*	3.8E-05	---	6.17563+1.61832i*	1.2E-01	---
15	6.20045-1.30708i	4.7E-05	1.4E-05	6.20043-1.30696i	4.3E-05	7.6E-05
16	6.20045-1.30708i	4.7E-05	1.4E-05	6.20043-1.30696i	4.3E-05	7.6E-05
17	$6.38039-6.2\times10^{-5}$i*	3.7E-05	---	6.57462+1.53575i*	3.0E-02	---
18	$6.38039-6.2\times10^{-5}$i*	3.7E-05	---	6.57462+1.53575i*	3.0E-02	---
19	$7.01585-5.4\times10^{-6}$i*	3.8E-05	---	7.63120+1.73656i*	8.8E-02	---
20	$7.01585-5.4\times10^{-6}$i*	3.8E-05	---	7.63120+1.73656i*	8.8E-02	---
21	7.16712-1.36946i	5.5E-05	3.6E-05	7.16709-1.36928i	5.0E-05	9.2E-05
22	7.16712-1.36946i	5.5E-05	3.6E-05	7.16709-1.36928i	5.0E-05	9.2E-05
23	$7.58861-1.2\times10^{-4}$i*	3.7E-05	---	7.70618+1.46837i*	1.6E-02	---
24	$7.58861-1.2\times10^{-4}$i*	3.7E-05	---	7.70618+1.46837i*	1.6E-02	---

3.2 A circular model with a rectangular cavity

A circular model with a rectangular cavity as shown in Figure 1 is considered in this subsection. The Neumann BC is prescribed on the boundary of the model. The size parameters of the model are set as R=1m, l/R= 1/3, and l/d = 2. In the numerical eigenvalue analysis, the boundary of the model is divided into 333 constant elements. The circular contour path with γ= (12, 0) and ρ= 2.0 is employed and the parameters in the block SS method are set as $N=512$, $K=5$, $L=15$ and $\delta=10^{-10}$.

Numerical eigenvalues are given in Table 2. Since the combined boundary integral formulation is used here, the fictitious eigenvalues indicated by stars have positive imaginary parts, which makes it easy to filter them out from the numerical results. By comparing with the eigenvalues of the unit circle, we can easily find the resonance frequencies of the cavity. Figure 2(a) shows the mode shape of k_2^n corresponding to the mode number (1, 0) [4]. The mode shapes corresponding to the mode numbers (3, 0) and (4, 0) obtained by using other contour paths are also depicted in Figure 2(b) and 2(c). The frequencies with respect to them are 29.08-0.25i and 38.34-0.22i, respectively. The resonance frequencies and the corresponding mode shapes of the open cavity agree well with the FEM results reported in [3, 4]. Therefore, it is shown that the present boundary element eigensolver is valid in the numerical resonance analysis of acoustic open cavities.

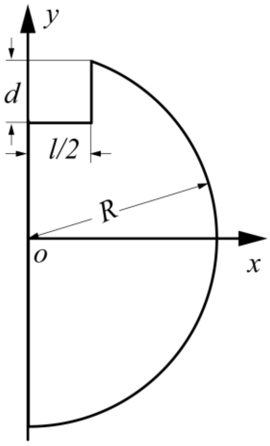

Figure 1. Circular model with a rectangular cavity

Table 2. Numerical eigenvalues for the circular model with a rectangular cavity

i	k_i^n	i	k_i^n
1	11.01489-1.66373i	10	12.27486+1.84795i*
2	11.23632-0.69483i	11	12.37476+1.38832i*
3	11.24166+1.43629i*	12	12.39945-1.78559i
4	11.29337+1.3603i*	13	12.43130+1.37002i*
5	11.33078-1.55869i	14	12.45943+1.88850i*
6	11.33753+1.66025i*	15	12.60031+1.58679i*
7	11.51159+1.69666i*	16	12.82975+1.70661i*
8	12.01693-1.71069i	17	13.01597-1.75295i
9	12.17069+1.93313i*		

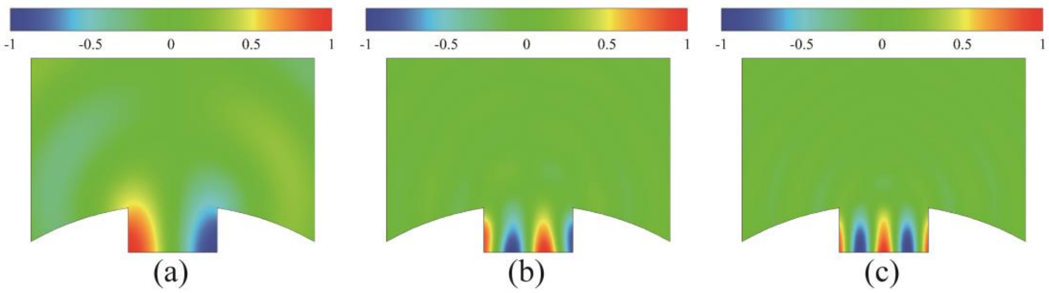

Figure 2. The mode shapes of the open cavity. (a) The eigenvalue $k_1 = 11.23\text{-}0.69i$. (b) The eigenvalue $k_2 = 29.08\text{-}0.25i$. (c) The eigenvalue $k_3 = 38.34\text{-}0.22i$.

4. CONCLUSIONS

This paper presents a numerical method for the acoustic resonance analysis of open cavities based on the BEM. The method avoids the truncation of the infinite domain and improves the accuracy of the numerical solutions. The resulting eigenvalue problem in the BEM is a NEP, which is solved by using the contour integral method called the block SS method. The combined boundary integral formulation is employed to impose positive imaginary parts on the fictitious eigenvalues, which make it very easy to filter out the fictitious eigenvalues from the actural resonance frequencies of exterior acoustic problems. Numerical examples of a unit circle and a circular model with a rectangular cavity are used to show the accuracy and validity of the propose method. Further studies are to be carried out for more complicated models.

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6. REFERENCES

1. S. Hein, T. Hohage, W. Koch and J. Schöberl, *"Acoustic Resonances in a High Lift Configuration",* J. Fluid Mech, (2007)
2. S. Ortiz, C. L. Plenier and P. Cobo, *"Efficient modeling and experimental validation of acoustic resonances in three-dimensional rectangular open cavities",* Appl. Acoust. (2013)
3. L.M. González, P. Cobo, V. Theofilis and E. Valero, *"Acoustic resonances in 2D open cavities",* Acta Acust. Acust. (2013)
4. W. Koch, *"Acoustic resonances in rectangular open cavities",* AIAA J. (2005)
5. J. Berenger, "*A perfectly matched layer for the absorption of electromagnetic waves*", J.Comp. Phys. (1994)
6. J. Asakura, T. Sakurai, H. Tadano, T. Ikegami and K. Kimura, *"A numerical method for nonlinear eigenvalue problems using contour integrals"*, JSIAM Lett. (2009)
7. C. J. Zheng, H. B. Chen, H. F. Gao and L. Du, *"Is the Burton–Miller formulation really free of fictitious eigenfrequencies?",* Eng. Anal. Bound. Elem. (2015)
8. S. Marburg, *"Boundary element method for time-harmonic acoustic problems",* In: M. Kaltenbacher (Ed), Computational Acoustics, Springer, (2018)
9. A. J. Burton and G. F. Miller, *"The application of integral equation methods to the numerical solution of some exterior boundary-value problems",* Proc. R. Soc. Lond. (1971)
10. M. Bebendorf, *"Hierarchical Matrices",* Lect. Notes Comput. Sci. Eng. (2008)
11. C. E. Wallace, *"The acoustic radiation damping of the modes of a rectangular panel"* J. Acoust. Soc. Am., (1987)