MADRID
inter.noise 2019
June 16 - 19
NOISE CONTROL FOR A BETTER ENVIRONMENT

Vibration Reduction with Additional Subsystems as absorber or bridge by Using Analytical SEA

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ABSTRACT

Noise and vibration problems have received much attention in recent years, and there is now much demand for a fundamental and non-iterative process for designing low-noise/low-vibration products, especially in the early design stage. The present authors have previously proposed a method for reducing the vibration of the main structure by adding a subsystem that acts either as a dynamic absorber of the vibration energy of the main system or a bridge to transfer vibration energy from a subsystem of the main system where vibration is harmful to one or more other subsystems where vibration can be tolerated. This paper proposes a method for locating these subsystems properly by using analytical statistical energy analysis with mathematical optimization. The feasibility of the proposed method is examined by applying it to a simplified structure comprising six plates. As an absorber, the additional subsystem stores vibration energy, thereby reducing the energy of the target subsystem. As a bridge, the additional subsystem transfers vibration energy to some other subsystem, thereby again reducing the energy of the target subsystem. Consequently, the proposed method is an effective tool for designing a low-vibration structure without changing the main structure.

Keywords: Statistical Energy Analysis, Propagation, Low-vibration
I-INCE Classification of Subject Number: 43

1. INTRODUCTION

In recent years, high-frequency noise and vibration have become more apparent because of the electrification of mechanical products, such as automobiles. This then

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necessitates a way to reduce such high-frequency noise and vibration. In conventional methods such as the finite-element method (FEM), vibration modes are analyzed and countermeasures are taken to reduce noise and vibration. However, such mode-based countermeasures affect only the frequency of interest and do not work in broadband. Worse still, the plethora of high-frequency modes makes it difficult to specify which mode is to be targeted by FEM. Consequently, the expectation is that statistical methods such as statistical energy analysis (SEA) will be used instead to assess the mean behavior of the frequency response from the perspective of vibration energy flow [1]. Also, the need to limit development costs makes it necessary to develop a non-iterative design procedure for low noise and vibration.

Against this background, we have proposed previously a two-step design process based on energy propagation analysis for problems involving broadband noise and vibration [2]. In particular, the initial design (i.e., the first design step) is intended to be a robust design that is focused on the average behavior (i.e., the baseline) of the frequency response characteristics of the target product. We use analytical SEA for this first design step because structural design lends itself to expressing SEA mathematically, especially in the early design stage [1–4]. Furthermore, because a system's vibration energy and physical parameters are directly correlated, the former can be used to obtain an outline of the structural design. However, there are few papers on using analytical SEA as an early-stage design tool and a layout design tool for low noise and vibration.

On the other hand, with an actual machine product, there are often constraints on the layout of the parts. We have therefore also developed some tools (or at least ideas) for reducing the vibration of the target system by adding subsystems to it, an example being dynamic vibration absorbers [3]. Mechanical designers would be helped if they could position such absorbers properly in the countermeasure design stage.

Given the above, the present research is aimed at the concept of additional subsystems. In particular, each additional subsystem acts as either a dynamic energy absorber (hereinafter referred to simply as an absorber) in which vibration energy is stored, thereby reducing the vibration of the main structure, or a bridge for transferring energy from a subsystem whose energy must be reduced to one whose energy can be increased without harm. A method is proposed herein for positioning the absorber or bridge correctly. The proposed method is shown to work well when designing a simplified structure comprising six plates, and the effectiveness of the method is examined through FEM calculations.

2. ROLE AND PROPER LOCATION OF ADDITIONAL SUBSYSTEM

It has been shown previously that adding a plate subsystem (the absorber) to an elongated plate (the main system) can reduce the vibration of the latter [3]. This is based on the relatively simple idea that if the input power to the target system remains constant, then increasing the number of subsystems in the target system decreases the energy of each subsystem. The absorber acts as a store for unwanted vibration energy, and if it is connected in turn to one or more subsystems of the main system for which oscillation is not a problem, then a path is formed for transmitting the vibration energy onward. In other words, the additional subsystem then acts as a bridge to transmit and ultimately dissipate energy. In the following, we show how to use analytical SEA to determine where best to connect the absorber or bridge.

2.1. Using analytical SEA to locate additional subsystem correctly

SEA is an analysis method based on balancing the vibration energy between several subsystems. In an SEA model, the system is regarded as an assembly of subsystems, and it is assumed that the energy dissipation of a subsystem is proportional to the vibration energy thereof. Furthermore, the transfer energy is taken to be proportional to the vibration energy between two subsystems. Considering the power balance leads to a set of equations, the basic SEA equation being

$$\mathbf{P}=\omega\mathbf{L}\mathbf{E} \tag{1}$$

where **P** is the external power input, **E** is the subsystem energy vector, and ω is the band center angular frequency. The matrix **L** is the matrix of loss factors and is written as

$$\mathbf{L}=\begin{bmatrix} \eta_1+\eta_{12}+\eta_{13}+\cdots & -\eta_{21} & \cdots \\ -\eta_{12} & \eta_2+\eta_{21}+\eta_{23}+\cdots & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \tag{2}$$

where η_i is the internal loss factor (ILF) of subsystem i and $\eta_{i,j}$ is the coupling loss factor (CLF) from subsystem i to subsystem j. The loss factors depend on the frequency.

In a target system comprising N subsystems, **L** is an N-dimensional matrix. If one absorber or bridge subsystem is added, then **L** becomes an N+1-dimensional matrix and the CLF is considered in the coupling subsystem (referred to as subsystem j) of the additional subsystem (referred to as subsystem a). That is, the component (a, j) of **L** is as "$-\eta_{a,j}$", the component (j, a) is "$-\eta_{j,a}$", the component (a, a) is "$\eta_a+\eta_{a,j}$ ", and "$+\eta_{j,a}$" is added to component (j, j), and the coupling subsystem is decided using the Monte Carlo method or an optimization method so that the subsystem energy to be reduced is minimized.

Herein, the analytical SEA model and the optimization process are used to determine to which subsystem the absorber or bridge should be connected. We then make the input condition the input power to the input subsystem and the objective function the subsystem energy to be reduced. The design variables are the connectivities between the absorber or bridge and the other subsystems, and the constraints the number of connections with the absorber or bridge.

2.2. Procedure for designing an absorber

An absorber subsystem reduces the vibration of the main system by absorbing and storing some of the energy of the latter. Figure 1 shows schematically the power flow in this case. The main system comprises subsystems 1–3 (enclosed by the dotted line), and we consider reducing the vibration of subsystem 1 by reducing its energy. This is done by connecting it to an absorber, whereupon energy flows from subsystem 1 to the absorber and is stored in the latter.

The energy E_a of the absorber is expressed as follows from the row of the relevant subsystem of the basic SEA equation, namely, Equation 2.

$$E_a=\frac{\eta_{1,a}E_1}{\eta_a+\eta_{a,1}} \tag{3}$$

Equation 3 shows that E_a increases when CLF ($\eta_{a,1}$) from the additional subsystem to subsystem 1 is small. Therefore, for the additional subsystem to be an absorber, the CLF ($\eta_{a,1}$) from subsystem a to subsystem 1 should be small. In the case of a plate subsystem, the CLF is expressed by Equation 4 and therefore the absorber should have a large surface area S_a and a large density ρ_a:

$$\eta_{a,i} = \frac{2L_{a,i}\tau_{a,i}}{\pi S_a}\sqrt{\frac{h_a}{\omega}}\sqrt[4]{\frac{E_a}{12\rho_a(1-\nu_a^2)}} \tag{4}$$

where S_a is the surface area, E_a is Young's modulus, ρ_a is the density, ν_a is Poisson's ratio, $L_{a,i}$ is the coupling length between the absorber and subsystem i, and $\tau_{a,i}$ is the energy transmission efficiency.

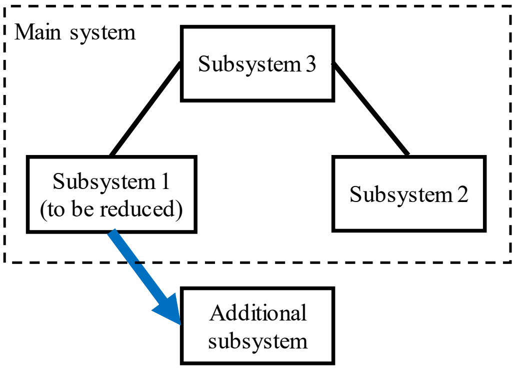


Figure 1: Power flow when additional subsystem acts as a dynamic absorber

2.3. Procedure for designing a bridge

A bridge subsystem couples subsystems in the main system and transfers energy from the subsystem whose vibration is to be reduced to one or more subsystems whose vibration is permitted. Figure 2 shows schematically the power flow in this case. Energy is transferred from subsystem 1 (whose vibration is to be reduced) to subsystem 2 via the bridge. The energy E_a of the additional subsystem in Figure 2 is therefore expressed by the following equation from the rows of the relevant subsystems of the basic SEA equation, namely, Equation 5:

$$E_a = \frac{\eta_{1,a}E_1 + \eta_{2,a}E_2}{\eta_a + \eta_{a,1} + \eta_{a,2}} \tag{5}$$

In section 2.2, the CLF ($\eta_{a,1}$) had to be smaller for the additional subsystem to act as an absorber. On the other hand, as the bridge the additional subsystem is paid attention to CLF ($\eta_{a,2}$) because the main energy path is from the upper location of subsystem 1 (subsystem 3) and then CLF ($\eta_{a,1}$) affects small. Therefore, the CLF ($\eta_{a,2}$) should be

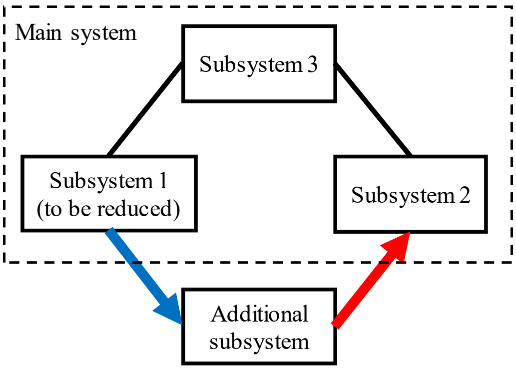


Figure 2: Power flow when additional subsystem acts as a bridge

made larger by setting a smaller surface area S_a and a smaller density ρ_a for the additional subsystem in Equation 4, which is contrary to the case of the absorber.

Note that the transfer subsystem rather than the storage system should be located downstream of the energy transfer because energy is transferred from the higher-energy subsystem to the lower-energy subsystem via the bridging subsystem. Thus, if the energy level of the subsystem downstream of the energy flow is not lower than that of the subsystem to be reduced, the bridging subsystem cannot be used.

2.4. Determining location of additional subsystem

The optimization using analytical SEA determines to which subsystem in the main system the absorber or bridge should be connected. The procedure for doing so is as follows.

1) Construct the analytical SEA model of the target system.
2) Determine the role of the additional subsystem (i.e., absorber or bridge).
3a) For an absorber, decrease the CLF from the absorber to the connected subsystem. This is done in practice by making either the surface area or the density of the absorber large.
3b) For a bridge, increase the CLF from the bridge to the connected subsystem. This is done in practice by making either the surface area or the density of the bridge small.
4) Use the analytical SEA model with optimization to determine the location of the absorber or bridge.

3. COMPARING ANALYTICAL SEA AND FEM

In this section, analytical SEA model of simple plate structure system is constructed and kinetic energy up to 10000 Hz is estimated. The result is compared with the kinetic energy calculated by FEM, so to validate the first step design.

3.1. Simple structure and analytical SEA modeling

Consider a simple structure (outer size: 0.4 m × 0.3 m × 0.2 m) comprising six plates as shown in Figure 3. This structure is modeled by six plate SEA subsystems. The numbers in the figure are the subsystem numbers; subsystems 1 and 2 are made of 20-mm-thick aluminum plate (density: 2680 kg/m^3; Young's modulus: 70 GPa; Poisson's ratio: 0.34), and subsystems 3–6 are made of 10-mm-thick steel plate (density: 7900 kg/m^3; Young's modulus: 210 GPa; Poisson's ratio: 0.3).

In this analytical SEA model, the CLFs are calculated from Equation 4 and the ILF is fixed at 0.05 for all subsystems. The energy transmission efficiency τ_{ij} is given by

$$\tau_{ij} = \left[\frac{\Delta^{-5/4} + \Delta^{-3/4} + \Delta^{3/4} + \Delta^{5/4}}{\Delta^{-2}/2 + \Delta^{-1/2} + 1 + \Delta^{1/2} + \Delta^{2}/2} \right]^2 \tag{6}$$

where $\Delta = h_j/h_i$ is the plate thickness ratio and h_i is the plate thickness of subsystem i.

We also built an FEM model of the structure, comprising 16,800 elements and 17,446 nodes. We performed frequency response analysis to calculate the subsystem energy under a point excitation. The number of excitations is one and the number of responses for evaluating the subsystem energy is four for one subsystem.

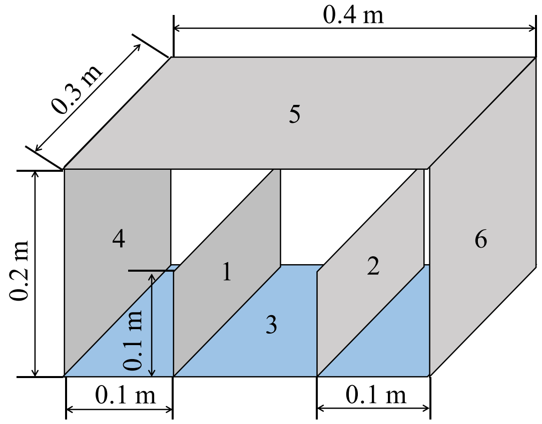


Figure 3: Test shell structure with six subsystems

3.2. Comparison of responses

To verify the constructed SEA model, we compare the SEA predictions and the results of FEM calculations. For the SEA, the power input into subsystem 1 was set to 1 W and the energy of subsystem 2 was calculated. For the FEM calculations, the material damping was set to 0.03 and a force of 1 N was applied at the center of subsystem 1. The responses at several points on subsystem 2 were calculated to evaluate the spatially averaged kinetic energy normalized by the input power.

Figure 4 compares the subsystem energy (kinetic energy) calculated by the analytical SEA with that calculated by FEM. The results of the analytical SEA clearly agree well with the frequency average of the FEM results.

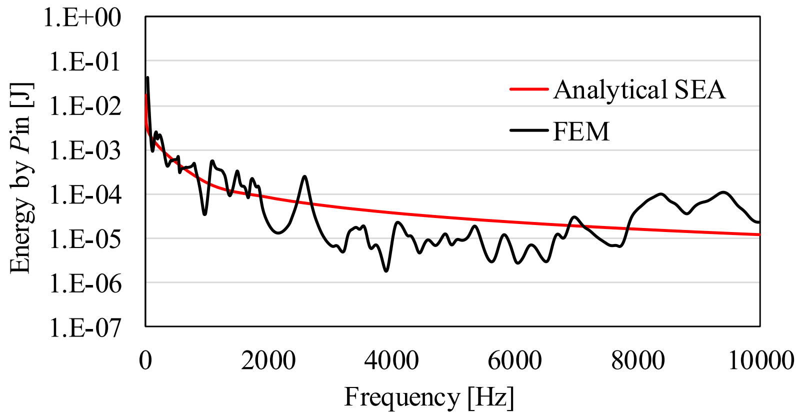


Figure 4: Comparison of subsystem 2 energies by analytical SEA and FEM

4. APPLYING PROPOSED METHOD TO A SIMPLE STRUCTURE

In this section, we present the results of applying the method proposed in Section 2 to the simple structure shown in Figure 3, first in the case of an absorber and then in the case of a bridge. The structures designed using the proposed method are then examined by FEM calculations.

4.1. Absorber

4.1.1. Proper connection

The external input power is applied to subsystem 1 of the target system in Figure 3, and the objective is to minimize the energy of subsystem 2 by somehow connecting an absorber. To simplify the discussion, the absorber is connected to only one subsystem. The design parameter is then the connectivity between the absorber and one of subsystems 1–6.

Applying the proposed method shows that the absorber should be connected to subsystem 2, which in this case is the target subsystem. Figure 5 shows the energy of subsystem 2 both without and with the absorber, confirming the vibration reduction with the absorber. Figure 6 shows the subsystem connection diagram with the absorber (a). This result is typical because subsystem 2 is the most downstream position and then its energy should be transferred to the other subsystem (the absorber).

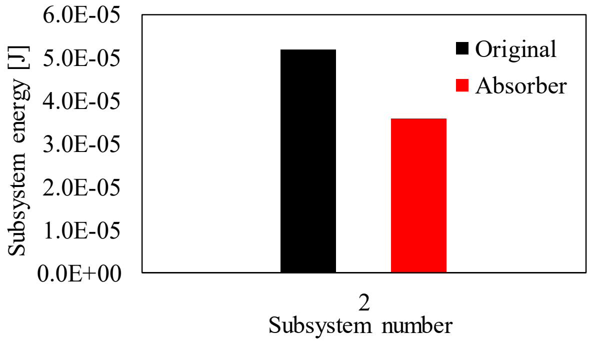


Figure 5: Comparison of subsystem energies without and with the absorber

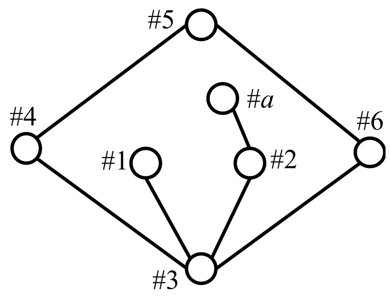
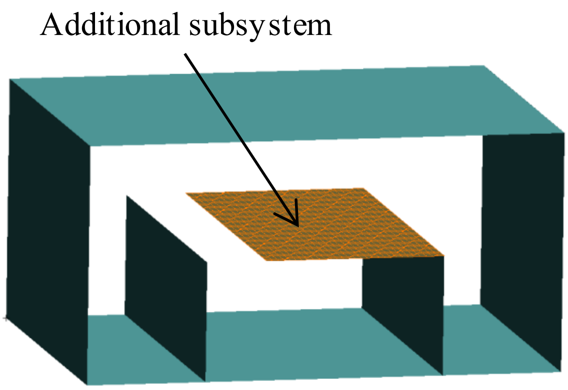


Figure 6: Diagram of subsystem connection after adding the absorber

4.1.2. Validation with FEM

Here, the vibration reduction by the absorber is verified by FEM calculations before and after adding the absorber. In SEA, the shape of the absorber are not determined, but the surface area and coupling length to the connected subsystem are. We consider the plate system shown in Figure 7(a), in which the additional plate subsystem is connected

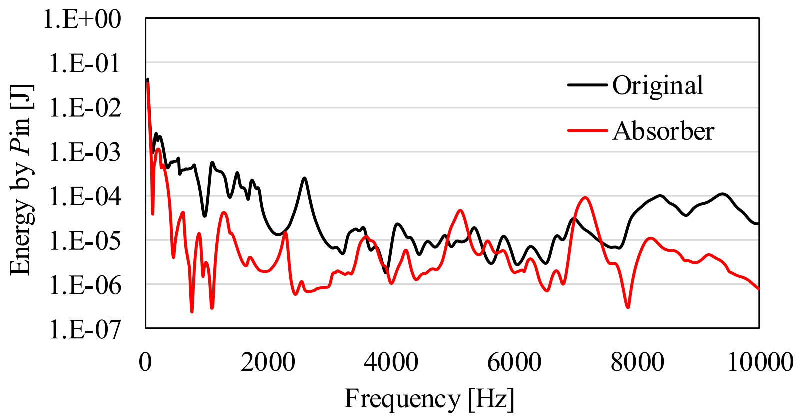


(a) Adding subsystem as absorber

Figure 7: Verification results for simple shell structure by FEM

to subsystem 2.

Figure 7(b) compares the energy of subsystem 2 both without and with the absorber plate. The vibration is reduced in almost all of the frequency range except at a few frequencies associated with the local natural modes of the absorber plate. It is difficult to avoid such peaks.



(b) Energy of subsystem 2 to be reduced

Figure 7: Verification results for simple shell structure by FEM

4.2. Bridge

4.2.1. Proper connection

Next, we consider adding a bridge. To simplify the discussion, the bridge is connected to only two subsystems to transfer vibration energy from one to the other. The optimization algorithm is used to determine the location of the bridge that minimizes the energy of subsystem 2. The result is that the bridge should be connected between subsystems 2 and 5, the former being the target subsystem and the latter being a subsystem whose vibration is not a concern. Figure 8 shows the subsystem energies both without and with the bridge, confirming that adding the bridge decreased the energy of subsystem 2 and increased the energy of subsystem 5, both as intended. However, the reduction is smaller than that in the case of the absorber shown in Figure 5. It seems that the energy propagates through subsystems 5, 6, 3, and 2 as if

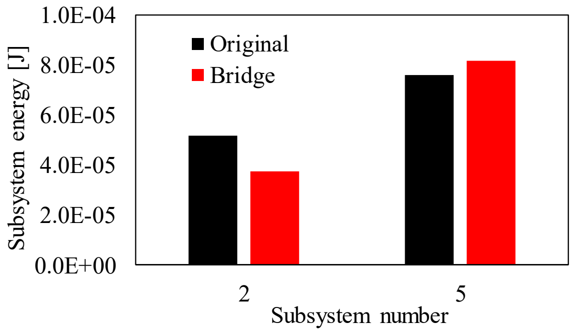


Figure 8: Comparison of subsystem energies without and with the bridge

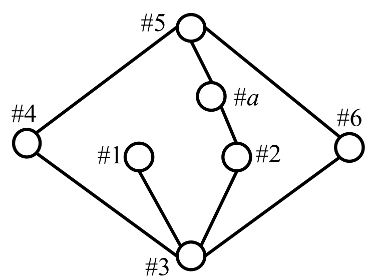
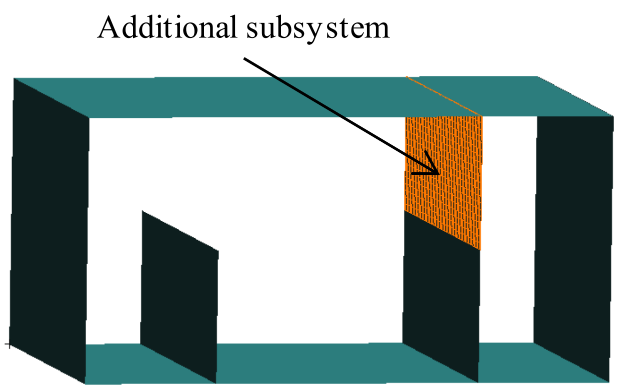


Figure 9: Diagram of subsystem connection after adding a bridge

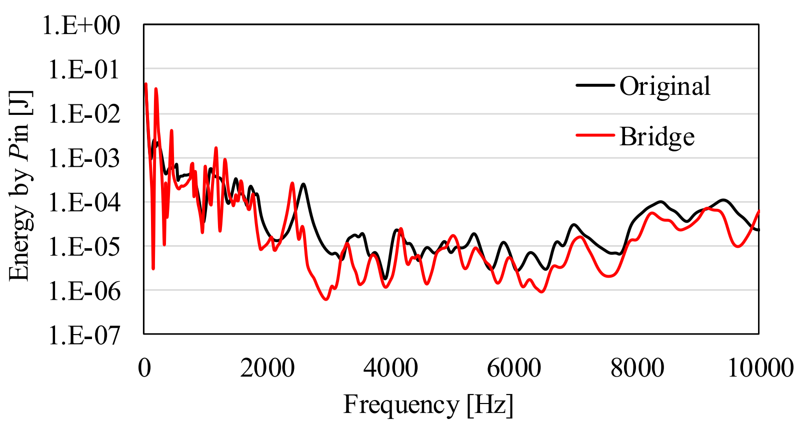
another energy path has been added. Figure 9 shows the subsystem connection diagram after adding the absorber. This indicates that adding the bridge made a path for the vibration energy and promoted energy propagation from the target subsystem (to reduce its vibration) to another downstream subsystem (a subsystem far from the target subsystem). In addition, subsystem 5 was identified because it sits between the input subsystem 1 and the target subsystem 2.

4.2.2. Validation with FEM

As in section 4.1.2, we built an FEM model with the bridge. Using the proposed method, we can identify the location of the bridge, whereupon we model the bridge by the plate shown in Figure 10(a). This bridging plate is located between subsystems 2 and 5 in the FEM model. Figure 10(b) and (c) are similar to the results by SEA. The energy of subsystem 2 is reduced in a wide frequency range, while that of subsystem 5 is slightly increased. However, the vibration reduction of subsystem 2 is smaller than the case of the absorber in the previous section as in the SEA results.

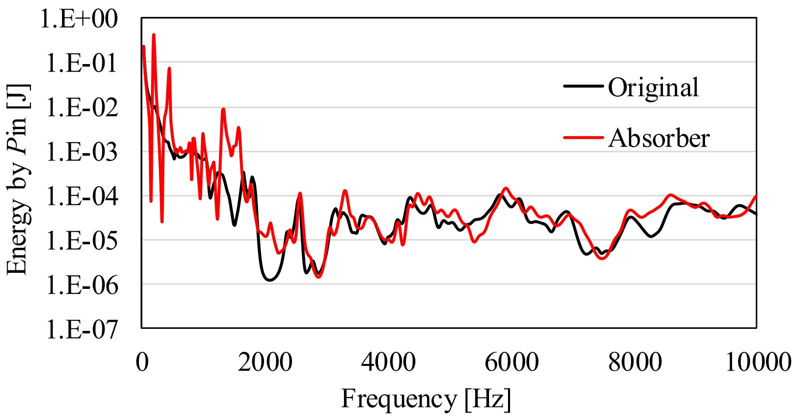


(a) Adding subsystem as bridge



(b) Energy of subsystem 2 to be reduced

Figure 10: Verification results for simple shell structure by FEM



(c) Energy of subsystem 5 to be enhanced

Figure 10: Verification results for simple shell structure by FEM

5. CONCLUSIONS

This paper has proposed a method for using analytical SEA to design an additional subsystem to reduce the vibration of an existing main structure. The research findings are summarized below.

1) The vibration energy of the target subsystem in the main structure can be reduced by adding another subsystem that acts as either (i) an absorber of the vibration energy of the target or (ii) a bridge to transfer vibration energy from the target to another subsystem.
2) Analytical SEA was used to demonstrate the proposed method. The specifications of the absorber and the bridge were determined, and an optimization algorithm was used to determine their proper locations.
3) Having applied the proposed method to a simple structure comprising six shell subsystems, numerical analysis verified that the absorber and bridge could reduce the vibration of the target subsystem.

6. REFERENCES

[1] T. Miyama, A. Miyazaki, H. Nakamura and T. Yamazaki, *"Low vibration structure design based on promotion and interruption of vibration energy flow"*, Proceedings of Dynamics and Design Conference, Transactions of the Society of Automotive Engineers of Japan, 20175061, pp.336-341, 2017-5. (in Japanese)

[2] K. Murata, T. Miyama, T. Murayama, H. Nakamura, T. Nakamitu, N. Kawabata and T. Yamazaki, *"Experimental Verification of Two-Step Design for Reduction of Noise and Vibration"*, Proceedings of the Society of Automotive Engineers of Japan, 20176077, pp.431-436, 2017-10. (in Japanese)

[3] K. Murata, T. Miyama, Y. Tanimoto, T. Nakamitu, N. Kawabata and T. Yamazaki, *"Vibration Reduction by Adding Vibration Absorbing Sub-Structure"*, Proceedings of the Society of Automotive Engineers of Japan, 20185149, 2018-5. (in Japanese)

[4] T. Murayama, J. Henmi, H. Nakamura and T. Yamazaki, *"Fundamental Study on Structure Layout Method by Using Analytical SEA"*, Proceedings of Dynamics and Design Conference 2018, 2018-8. (in Japanese)