



The use of the simplex approach for analysis of semi-infinite and finite sound barriers

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ABSTRACT

Sound barriers design applies the physics of sound diffraction radiated from a source towards a chosen control point, around barrier edges over a semi-infinite domain. Maekawa backed by experiments Kirchhoff's diffraction solution and added modifications. The result involved only geometry and Fresnel number. Solving insertion loss (IL) caused by an infinite barrier necessitates location of a source point, control point and a point at the top of the barrier and a vertical plane that includes the sight line between the source and the control point. The three points constitute vertices of a triangle with dependent lengths and angles. Three of them given and three calculated. If the three sides of the triangle are measured, one can find its angles by calculation (in any case - three unknowns out of six variables). Different angles appear in solving IL for a finite sound barrier of vertical sides. Given the triangle sides lengths, the author calculated in a previous paper the angles from the control point to the two ends of the finite barrier by introducing Heron's formula. The same can be done with the shadow zone angle. We show here that this approach is useful in other environmental acoustics problems, where sound barriers are involved. We have used here the trigonometric properties of the triangle and the n-simplex which is 2-simplex for a triangle. Such shapes can apply vector analysis to define angles and locations in IL calculations. Solved examples are added.

Keywords: Sound Barriers, propagation, transmission and scattering of sound, outdoor noise control, analytical methods.

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1. INTRODUCTION

Much of the theoretical research was performed on sound barriers that protect residential areas near highways, railway lines, airports, industrial premises, recreation areas and other noisy zones from excessive noise levels, bringing them down to acceptable levels. The variety of solutions generally include external long (infinite) or finite sound barrier walls along noisy roads, or acoustic isolation at the receiver.

The use of a finite sound barrier is a possible solution especially along such intercity roads and railways, where most of the highway length passes near rural areas. While buildings height in the vicinity does not exceed two floors, it is necessary to protect the residents near local segments of the lanes. Thus, planning finite sound barriers (sometimes combined with specific topographic conditions) can be useful locally even when covering a relatively small horizontal view angle α , yielding the necessary noise reduction and acoustic protection, while the costly "infinite" barrier that covers 180 degrees is not needed along most of its length.

It is possible to find the effect of the barrier's view angle α on the amount of noise reduction caused by using the geometrical data of the problem – See figures 1 and 2. However, since the barrier's location, dimensions and distances to the receiver and sources are given as linear dimensions and not directly by the angle α , we introduced Heron's formula in the calculations of the barrier insertion loss (IL) in a previous publication – see Rosenhouse¹.

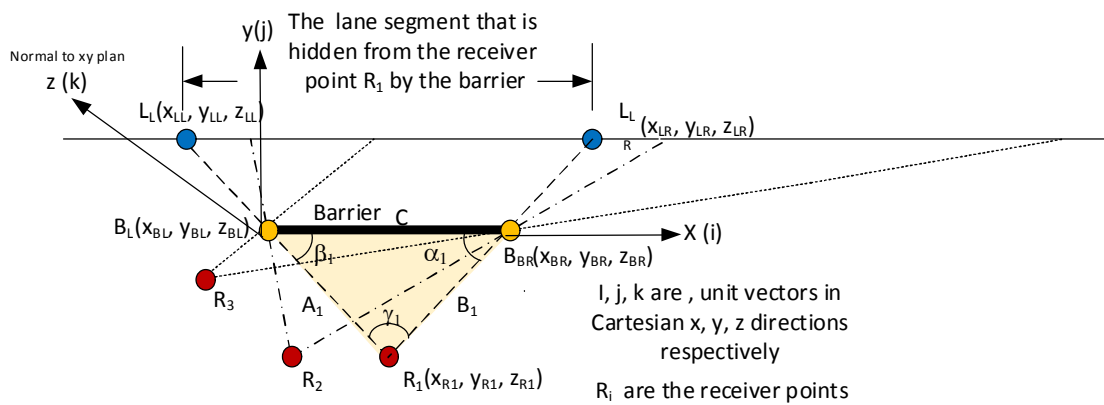


Figure 1. Location of a finite sound barrier and control points by a road.

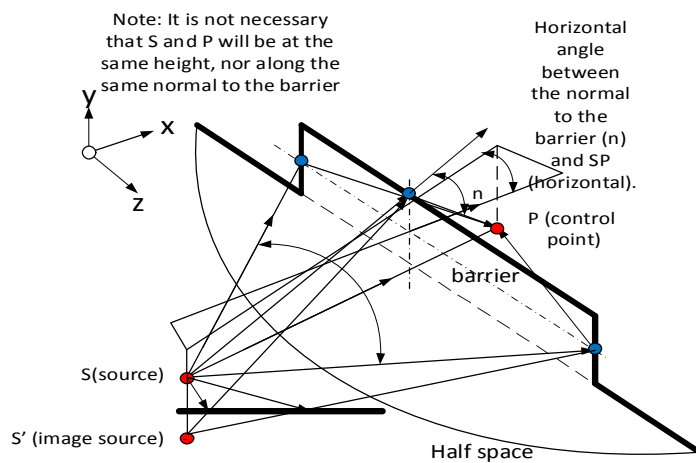


Figure 2. Finite sound barrier over a semi-infinite plane, a point source, image source and a point observation point.

Heron's formula entirely eliminates the direct use of angles. As a result, only data of linear dimensions are needed as input data for the calculations.

We expanded here the solution method using the trigonometric properties of the triangle and the n-simplex which is 2-simplex for a triangle (following Maekawa's model^{2, 3}) and irregular convex quadrilateral forms for calculation of wide barriers (following Pierce's model⁴), as well as definition of shadow zones and other problems. Such shapes can apply vector analysis to define the desired unknowns from the given data.

2. SOUND BARRIERS ANALYSIS – PHYSICS CONTRIBUTION AND THE SIMPLIFIED SOLUTIONS THAT FOLLOW

The fundamental model of the environmental sound barrier is the rigid, thin half plane (see Fig. 2). It separates the noise source S and the receiver P by interception of the sight line SP, and thus reduces the free field noise level at P. Due to diffraction at the edges of the screen it cannot entirely avoid the penetration of noise into the "shadow zone", even when taking into account an approximation of infinite transmission loss through the wall.

The key to sound waves diffraction is the discovery of wave characterization of sound in water and air by the stoics⁵. (Marcus Vitruvius Pollio, c. 80-70 BC to c. 15 BC)⁶. Vitruvius said in his book, in ch. III, section 6, pp. 138-139, that " ...sound moves in an endless number of circular rounds like the innumerably increasing circular waves which appeared when a stone is thrown into water Unless interrupted by narrow limits or by some obstruction, which prevents such waves from reaching their end due to formation The first waves flowing back, break up the formation of those which follow."

Due to his investigation of waves in optics, vision and acoustics, Leonardo da Vinci (1452-1519)^{7,8,9} was the first to report about light and diffraction, and it was accurately described by Grimaldi (1618-1663)¹⁰. However, the first explanation was given by Fresnel^{11,12} in 1818, being based on Huygens' construction^{11,12,13,14} and Young's principle of interference¹². Then the era of mathematical solutions in the insertion loss analysis of sound barriers began.

Indeed, in the 18th century Fresnel^{11,12} had a rigorous mathematical solution for a 2-D problem. In 1882 Kirchhoff put Fresnel's analysis on a sound mathematical base introducing his integral theorem - Born and Wolf.¹¹

The first exact solution for the diffraction of a plane wave by a semi-infinite screen was shown by Sommerfeld in 1896¹⁵. Sommerfeld's exact solution can analyze also the acoustic effect of a wedge shaped screens considering the angle of the wedge. Macdonald¹⁶ gave in 1915 the first exact solution for the diffraction of a spherical wave by a semi-infinite screen, which Kirchhoff's solution cannot do. Fresnel and Kirchhoff also solved the problem of fully absorbent screens - Born and Wolf¹¹. Macdonald's solution was used by Jonasson¹⁷ to solve the diffracted sound pressure from the edge of a plane, as being a best approximation.

Over the years, the theory has been further developed and generalized and has obtained integral forms, for example by applying the Wiener-Hopf method. Tolstoy¹⁸ obtained an exact explicit solution for the sound diffracted by a wedge, represented by a sum of infinite series.

Redfearn¹⁹ has shown in 1940 an approximate solution for sound wave diffraction in case of a source at a prescribed distance from a semi-infinite screen. While Sommerfeld's exact solution depends on 5 independent parameters, Redfearn's solution depends on two parameters which are h/λ and the diffraction angle φ - see figure 3. λ is the wave length (m). Thus, it has errors that can be estimated by Keller's asymptotic solution. Indeed, Keller²⁰ GTD (geometrical theory of diffraction) simplified the diffraction formulae very much, by combining Kirchhoff's approximation with rigorous Sommerfeld's-type solutions. The analytical approximation of the curves in figure 3 is:

$$\Delta L_p = 20 \lg_{10} \left\{ 2 \frac{\pi \left[(2h/\lambda) \tan \frac{\varphi}{2} \right]^{0.5}}{\tanh \pi \left[(2h/\lambda) \tan \frac{\varphi}{2} \right]^{0.5}} \right\}; dB \quad (1)$$

This approximation matches also the exact solution by Pierce⁴, within 1 dB error.

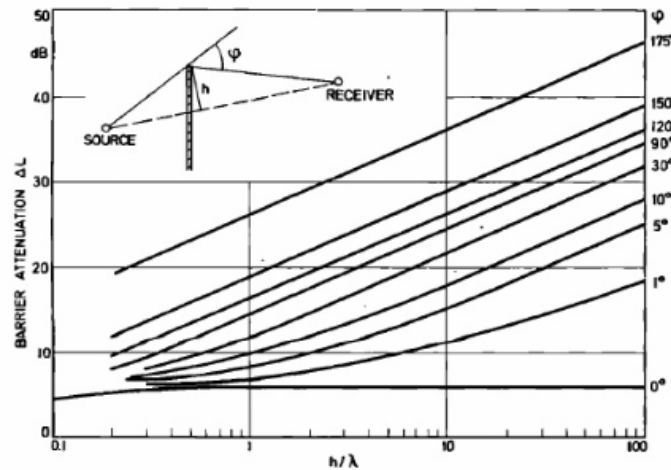


Figure 3. Redfearn's IL (Insertion Loss) chart of a thin rigid screen.

For reviewing generally other solutions of this topic, readers are referred to works by authors such as Baker and Copson,¹³ Rossing²¹, Skudrzyk²² and Attenborough, Li and Horoshenkov²³. Several available theories find the noise reduction achieved by a barrier, by using the concept "optical diffraction" and superposition of waves scattered at the edge of the barrier and the part of the incident waves which is not blocked by it. Many papers were published about the subject; see for example the review by Li and Wong,²⁴ among many others.

The simplest solution for noise reduction by a thin half plane that fits the Fresnel-Kirchhoff diffraction approximation model was presented by Maekawa^{2,3} and Pierce⁴ although this approach was already known (see Redfearn¹⁹, Rettinger²⁵⁻²⁷), and its fundamental physical model was borrowed from optics (Born and Wolf¹). Yet, some publications still use the classical solutions – e.g. Rosenhouse²⁸ and there are programs for mapping sound fields by exact formulation and numerical techniques. The latter are less attractive and the popular solutions tend to use only a small number of geometrical parameters, which lead to the use of simplex mathematics and Heron's formulae. See figures 1 and 2.

Maekawa's Formula reads:

$$\Delta L_p = \begin{cases} 10 \lg_{10}(3+20N) & \text{for } N > -0.05 \\ 0 & \text{for } N < -0.05 \end{cases}; \quad N = \frac{2\Delta x}{\lambda}; \quad (2)$$

$$\Delta x = r_s + r_r - R.$$

N – Fresnel number, P – control point. R – the length of the sight line, SP, m.

r_s – the distance between the source and the top of the barrier, 2-D analysis, m.

r_r – the distance between the control point and the top of the barrier, 2-D analysis, m.

S – sound source point, λ - source sound wave length, m.

ΔL_p – reduction of sound level by the screen, at the control point, dBA. See figure 4.

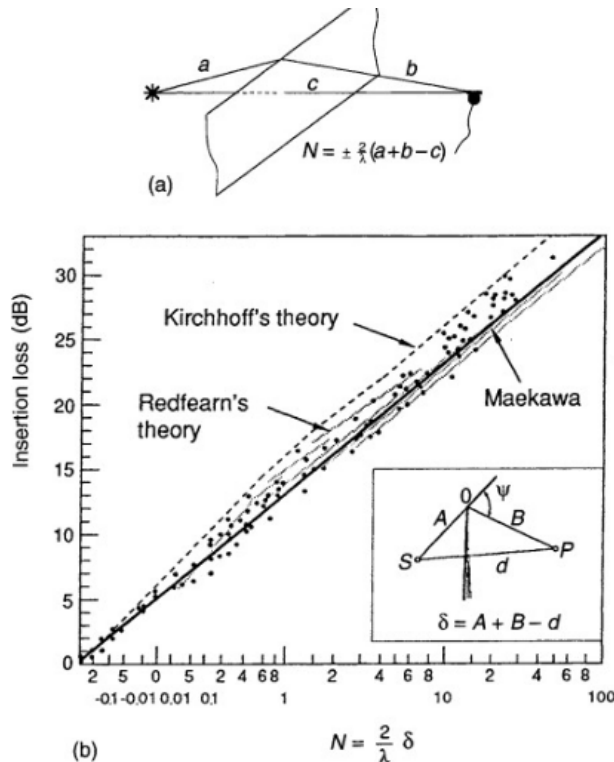


Figure 4. Insertion loss by a thin semi-infinite sound barrier using Kirchhoff's theory, Redfearn's theory, and Maekawa's formula and experiments.

Menounou²⁹ has shown corrections for Maekawa's formulae, involving various kinds of sources (spherical, line, transportation).

Another well-known formula is that by Kurze and Anderson³⁰, which deviates by about ∓ 1.5 dBA from Maekawa's curve for $N < 1$:

$$\Delta L_p = 5 + 20 \lg_{10} \left[\frac{\sqrt{2\pi N}}{\tanh(\sqrt{2\pi N})} \right] \quad \text{for } N > 0. \quad (3)$$

Yamamoto and Takagi³¹ formulated four types of more accurate solutions none of which deviates from Maekawa's formula by more than 0.5 dB. The four solution types are respectively:

$$\Delta L_p = \begin{cases} 10 \lg_{10} [(1 + G(N))(N + 0.3)] & \text{for } N > -0.3; \\ 0 & \text{for } N < -0.3 \end{cases}; \quad N = \frac{2\Delta x}{\lambda};$$

$$\Delta x = r_s + r_r - R. \quad (4a)$$

$$G(N) = 3.621 \left\{ \tan \left(\frac{N - 0.005}{0.0145} \right) + \frac{\pi}{2} \right\} + 6.165 \{1 - \exp[-0.205(N + 0.3)]\} + 2.354.$$

$$\Delta L_p = \begin{cases} 10 \lg_{10} N + 13 & \text{for } N > 1 \\ 5 + 8 \frac{N}{|N|^{0.55+0.143|N|}} & \text{for } -0.3 < N < 1; \\ 0 & \text{for } N < 0.3 \end{cases}; \quad N = \frac{2\Delta x}{\lambda}; \quad (4b)$$

$$\Delta x = r_s + r_r - R.$$

$$\Delta L_p = \begin{cases} 10 \lg_{10} N + 13 & \text{for } N > 1 \\ 5 \pm 8|N|^{0.438} & \text{for } -0.3 < N < 1; \\ 0 & \text{for } N < 0.3 \end{cases}; \quad N = \frac{2\Delta x}{\lambda}; \quad (4c)$$

$$\Delta x = r_s + r_r - R.$$

$$\Delta L_p = \begin{cases} 10 \lg_{10} N + 13 & \text{for } N > 1 \\ 5 + 9.07674 \times \sinh^{-1}(|N|^{0.485}) & \text{for } -0.3 < N < 1; \\ 0 & \text{for } N < 0.3 \end{cases}; \quad N = \frac{2\Delta x}{\lambda}; \quad (4d)$$

$$\Delta x = r_s + r_r - R.$$

The TL calculated by equations 2, 3, 4 depend only on two independent geometrical parameters: N and Δx .

3. THE USE OF SIMPLEX THEORY IN CALCULATING INSERTION LOSS

Triangles can be used frequently to analyze the insertion loss of straight finite and semi-infinite noise barriers in 2-D and 3-D problems. See figures 1-3, but regularly only 3 of the 6 data that define the triangle are applied – there are three sides and three angles in each triangle and any three of them can define the others as well. See figures 3 and 4. The use of any of the sides and angles of the triangle can be useful in finding the effectivity of the sound barriers – for example, the shadow zone angle. Triangles and tetrahedrons are forms of simplices, defined as a finite collection of affinely independent vertices. The n simplex is a convex hull of a set of n+1 such points in an Euclidean space of dimension n or more. Examples of simplices are shown in figure 5.

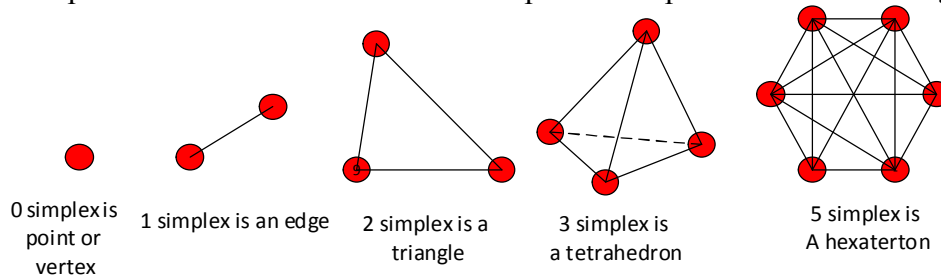


Figure 5. examples of simplices

The simplex method solves problems in linear programming. It is highly mathematical and historically it was developed by Clifford (1845-1879)³², **describing the triangle and the tetrahedron as simplest forms of confine of area and volume.** It is one of the most influential algorithms. The mathematician Pieter Hendrik Schoute (1846-1923) coined the word simplex in 1902 (written by him in Latin as "simplest"). Dantzig (1914-2005)³³ created the simplex algorithm for linear programming that solves problems for many applications in science, engineering and the arts.

4. PIERCE INSERTION LOSS ANALYSIS FOR WIDE BARRIERS BY IRREGULAR CONVEX QUADRILATERAL FORMULATION AND MAEKAWA'S APPROACH

Two different methods for estimating IL were published in the literature: that of Maekawa^{2,3} and that of Pierce⁴, with difference in Δx – see equation 2 and figure 6:

$$\Delta x(\text{Maekawa}) = \overline{S_1 B'_1} + \overline{B'_1 P} - \overline{S_1 P} \quad (4)$$

$$\Delta x(\text{Pierce}) = \overline{S_1 B_2} + \overline{B_1 B_2} + \overline{B_2 P} - \overline{S_1 P} \quad (5)$$

Now, for Maekawa triangle:

$$\overline{S_1 B'_1} = 5.0 \text{ m}; \overline{B'_1 P} = 5.4 \text{ m}; \overline{S_1 P} = 9.00 \text{ m}; \Delta x(\text{Maekawa}) = 5 + 5.4 - 9 = 1.4 \text{ m.}$$

For Pierce quadrilateral:

$$\overline{S_1 B_2} = 4.4 \text{ m}; \overline{B_1 B_2} = 1.4 \text{ m}; \overline{B_2 P} = 4.6 \text{ m}; \overline{S_1 P} = 9.00 \text{ m.}$$

$$\Delta x(\text{Pierce}) = \overline{S_1 B_2} + \overline{B_1 B_2} + \overline{B_2 P} - \overline{S_1 P} = 4.4 + 1.4 + 4.6 - 9 = 1.1 \text{ m.}$$

Now, for calculation of the insertion loss (IL) of the barrier of "infinite length" equation 2 can be used, which for $\lambda=1$ m becomes $IL = -10 \lg_{10}(3 + 40 \Delta x)$. Thus:

$$IL(\text{Maekawa}) = -17.7 \text{ dB}; \quad IL(\text{Pierce}) = -16.3 \text{ dB} \text{ and the}$$

difference between the two becomes -1.4 dB. This difference might be much higher if each of the source S and the control P will be closer to the wall. All the difference, $\Delta(\Delta x) = \overline{B_1 B} + \overline{B_2 B} - \overline{B_2 B_1}$, between Maekawa and Pierce models is in the triangle, $B_1 B_2 B$. If the distance between S and the barrier and P and the barrier $\gg \lambda$, the difference is small and so the difference in IL, and if those distances tend to zero the difference tends to infinity. In the last case, Pierce's formulation is the more logical one.

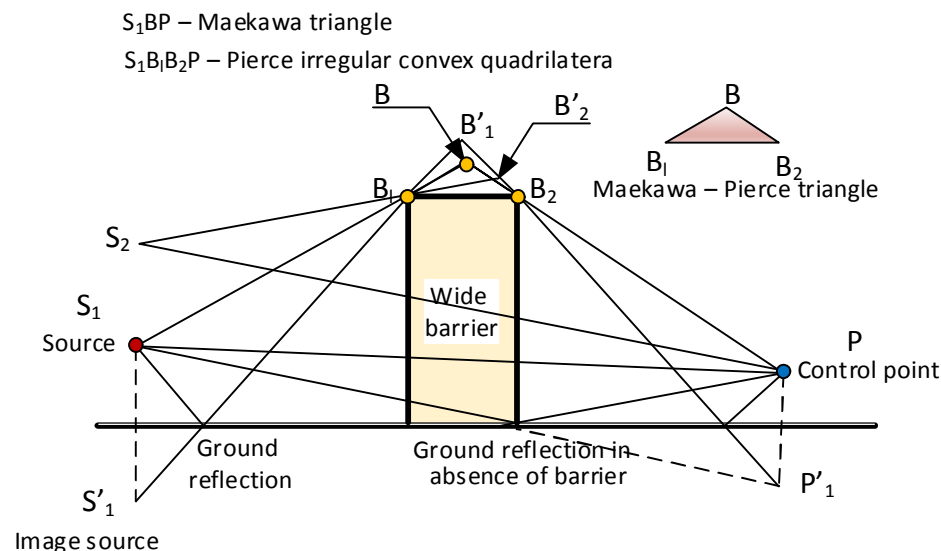
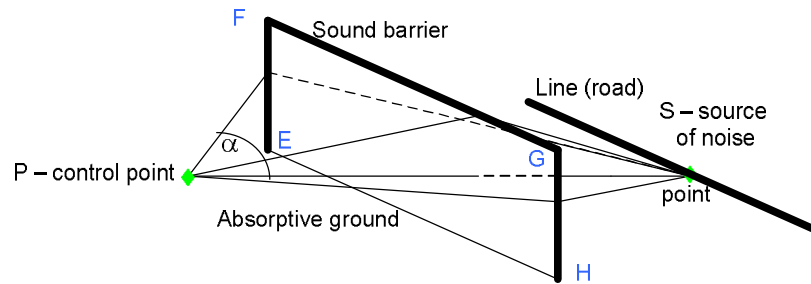


Figure 6. Maekawa – Pierce triangle

5. Finite Sound Barriers

While the horizontal view angle of the infinite barrier is $\alpha=180^\circ$ (fully blocked horizontal view), the finite barrier is characterized by a smaller angle α , which deteriorates its noise reduction ability due to side leaks (See figure 2.). There are tables and graphs that show the correction. Some publications show curves in 2-D that enable one to find the noise reduction by the finite noise screen as a function of the horizontal angle α of the observer that the barrier length occupies. At first, this correction was done for point sources, but later the approach was extended for use in line sources (transportation applications). Pioneers in doing that are Scholes and Sargent³⁴ (See also

Scholes, Salvidge and Sargent³⁵). The results enable easy interpolation for practical cases. It is also possible to build a table that applies the percentage of area occupied by openings and slits in the wall as a parameter for estimating noise reduction.



$$\Delta L_{p,tot} = 10 \times \lg_{10} \left\{ \sum_{i=1}^3 \left[10^{-0.1 \times \Delta L_{p,i}} \right] \right\}; \text{ dB}$$

Figure 7. Finite sound barrier, a point source and an observation point.

However, in cases of point sources, it is more accurate to use the formula in figure 7 and here comes the use of the simplex method. This approach is explained by the example in figure 8 and the following equation:

$$L_b - L_f = IL = +10 \text{ Lg}_{10} \sum_{i=1}^{i=3} \left(\frac{1}{3 + 20N_i} \right) \text{ dBA}$$

$$L_b - L_f = IL = 10 \text{ Lg}_{10} (D_i) \text{ dBA} \quad (6)$$

In figure 8, the barrier AB separates between the source S₁ and the two control points P₁ and P₂. The Insertion Loss (IL) of free field caused by S at the control points can be calculated by equation (6) that represents the IL of the top of the barriers and the two vertical sides of the barrier. All three depend on Fresnel number, and consequently on Δx, so that the main task in the calculations is to find all Δx values of the problem, which involves finding sides of triangles and quadrilaterals – see figures 1-4, 6-8. In figure 8 there are three relevant triangles: S₁AP₂, S₁BP₁, S₁AP₁, S₁BP₂.

In each of the two first triangles, the sides are known, and in the two last triangles the length of the sides $\overline{AP_1}$ and $\overline{BP_2}$ are missing, which yields respectively:

$$\Delta x_1 = 476.3 + 24.61 - 500 = 0.91 \text{ m}, \quad \Delta x_2 = 689.17 + 41.59 - 726.78 = 3.98 \text{ m}$$

$$\Delta x_3 = AP_1 + 24.61 - 726.28 = \blacksquare \text{ m}, \quad \Delta x_4 = 500 + 41.59 - \overline{BP_2} = \blacksquare \text{ m}$$

The following calculation solves $\overline{AP_1}$: $\angle P_1S_1A = 130^\circ - 24^\circ 56' = 105^\circ 04'$. Thus 3 data for the triangle – two sides and an angle are available: a=24.61 m, b= 726.78 and $\gamma = 104.07^\circ$ – see figure 9 notation. This allows for calculating the other three unknowns, using the general law of cosines: $b^2 = a^2 + c^2 - 2ac \cos \beta$. Thus $c^2 + 12.465c - 527603.516 = 0$. The result is $AP_1 = 720.16 \text{ y m}$, out of the two solutions.

The following calculation solves $\overline{BP_2}$: First we use the triangle AS₁B, with the known length of the sides: $\overline{AS_1} = 24.61 \text{ m}$, $\overline{S_1B} = 41.59 \text{ m}$, $\overline{AB} = 60.425 \text{ m}$ and $\angle AS_1B = 130^\circ$ to find $\angle BAS_1$, using the law of cosines: $\beta = \arccos \left(\frac{b^2 - a^2 - c^2}{2ac} \right)$ and the result is: $\angle BA+S_1 = 31.82^\circ$. Thus: $\angle P_1AS_1 = 164 - 31.82 - 0.78 = 121.4^\circ$. This information Supplies sufficient data to solve $\overline{BP_2}$.

Using the cosines law $c = \sqrt{a^2 + b^2 - 2ab \cos \gamma}$, the result is $P_2B = 513.12 \text{ m}$.

Now it is possible to complete the calculation of the Δx values as follows:

$$\Delta x_3 = 720.16 + 24.61 - 726.28 = 18.49 \text{ m}, \quad \Delta x_4 = 500 + 41.59 - 513.12 = 28.47 \text{ m}$$

For calculation of the insertion loss (IL) of each triangle, equation (6) can be used, which for $\lambda=1 \text{ m}$ and becomes:

$$L_b - L_f = IL = +10 \text{ Lg}_{10} \sum_{i=1}^{i=3} \left(\frac{1}{3 + 40 \Delta x_i} \right) \text{ dBA}$$

This includes the top of the barrier and both vertical sides of the barrier. For example, assume for the control point P_2 , that Δx at the top of the barrier is 0.8 m (fictitious), while the calculated values, due to the vertical side walls, are 0.91 m and 18.49 m, then the IL of the semi-infinite wall is -15.44 dB and it deteriorates due to the finiteness of the wall to $IL = -12.57 \text{ dB}$.

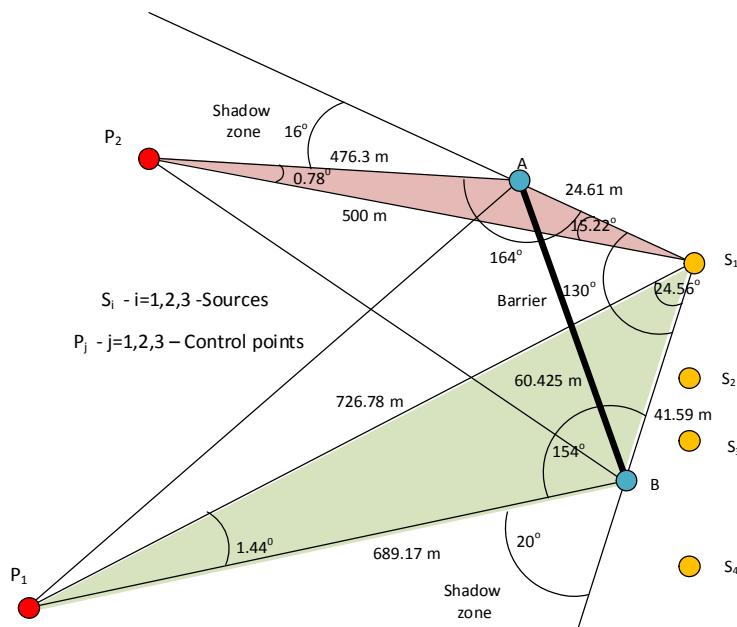


Figure 8. A barrier, sources and control points.

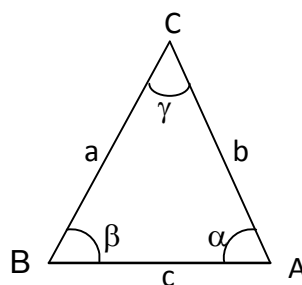


Figure 9. Notations used during calculations done in section 5

6. CONCLUSIONS

Motto: "Simplicity is the ultimate sophistication"

Attributed to Leonardo da Vinci because of texts in his Note Book

Formulae to calculate noise reduction by semi-infinite and finite sound barrier are available. In such cases the horizontal view from the observer to the wall is blocked by an angle of 180 degrees. Concerning finite barriers, the angle is smaller than 180 degrees, and it becomes much less effective than the infinite one in getting noise reduction.

It is possible to estimate the effect of the angle of vision of the finite barrier on its noise reduction ability. The angle can easily be calculated by using relevant linear lengths. This method is useful for calculating the protection against traffic noise.

However, for point sources it is much better to use the formulae for finding the dependence of barriers insertion loss on Fresnel number. The results can be used to optimize the finite sound barrier. Many cases of finite sound barriers near many noise sources and many control points to be protected need a large amount of calculations, especially when optimization is required. Because of these reasons and since the problem involves geometry that can combine generalized triangles and quadrilaterals, the use of the simplex theory and method seems to be effective.

The simplex theory and computerized algorithms are highly mathematical but for the analysis presented here its complication is reasonable.

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