

## **Energy reduction estimation of dissipative fluid-structure coupled multilayer system based on SEA**

Yifu, GUO<sup>1</sup>

a. Laboratoire MSME, UMR 8208 CNRS, Université Paris-Est, France

b. Centre scientifique et technique du bâtiment, Grenoble, France

Christophe, DESCELIERS

Laboratoire MSME, UMR 8208 CNRS, Université Paris-Est, France

Catherine, GUIGOU CARTER

Centre scientifique et technique du bâtiment, Grenoble, France

### **ABSTRACT**

For several decades, the Statistical Energy Analysis (SEA) is used to predict vibro-acoustic performances in high frequency domain. However, this method needs the determination of some important coefficients such as the Coupling Power Proportionality (CPP), which are not easily calculated. Numerous works have been devoted to calculate analytically or experimentally these coefficients. During the last 40 years, some extensions or alternative models of SEA have also been proposed. Nevertheless, only very few of them are particularly concerned with dissipative materials insulating layers. The objective of this article is to propose an energy based method for the analysis of the medium frequency range of vibrations by using the results from the SEA and by introducing an Equivalent CPP (ECPP) that allows dissipative problems in limited frequency band to be analyzed. This proposed approach is based on a modal decomposition adapted to the vibro-acoustic problem under consideration. In this paper, the case of the fluid-structure coupled multilayer systems is considered in details. A numerical application is proposed to evaluate the mean spectral mechanical energy of each layer. This proposed approach, which is called SEA-ECPP hereinafter, presents the advantage of allowing a bridge between the Finite Element Analysis (FEA) in low frequency range and the traditional SEA in the high frequency range.

**Keywords:** Vibrations, Medium frequency range, Energy based formulation, SEA, Transmission Loss, Multilayer, Dissipative

**I-INCE Classification of Subject Number:** 76

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<sup>1</sup>yifu.guo@cstb.fr

## 1. INTRODUCTION

For the simulation of acoustical performance of a mechanical system, one of the major problem is to construct a solution in a large frequency range that includes the low frequency and the medium frequency ranges of vibrations. For each frequency range, dedicated computational methods have been developed (see for instance [1] for the low and medium frequency ranges). The Statistical Energy Analysis (see for instance [2–6]) has been developed for allowing the construction of numerical computational models that are suitable for analyzing mechanical systems in the high frequency range of vibrations. In the four last decades, some extensions of SEA have been proposed in the literature [6–18]. Nevertheless, only very few are particularly concerned with dissipative structure made up of dissipative materials. Fahy [19] and Sun [20] have proposed some results for non-conservative couplings between two oscillators. The extension of their results to a system of  $N > 2$  oscillators is not straightforward and has never been proposed. We propose a similar method that is adapted for the systems of  $N > 2$  oscillators. It is constructed as an extension of the classical SEA in the framework of weakly damped oscillators system [21]. Additional coefficients to the Coupling Power Proportionality (CPP) are introduced and are denoted as Equivalent CPP (E CPP). The weakly dissipative couplings is also added to the fundamental hypothesis of the SEA. It should be noted that with this extension we do no longer assume that the couplings are conservative, which is one of the fundamental assumption in regular SEA methods. Indeed, the limitation of conservative couplings have been circumvented by modifying the final expression of the CPP coefficients. A numerical example is proposed in order to illustrate the method. It is a double-partition wall system between two acoustical rooms, which are an emission room and a receiving room. The double-partition wall system is a multilayer vibro-acoustical system that is a dissipative acoustical fluid medium sandwiched between two solid elastic media. An analysis of the numerical result is also presented. It is observed that the proposed method can correctly and fast enough predict the sound insulation with small computing costs.

## 2. COMPUTATIONAL MODELS

Hereinafter, we present the construction of a computational model for a tri-dimensional double-partition wall system between two acoustical rooms (see Figure 1b). The subsystems (1) and (5) are the 2 acoustical cavities for which the acoustical fluid medium is air. Subsystem (1) is the emission room in which external forces are applied and are modeled as uncorrelated white noises. Subsystem (5) is the receiving room. The mean total energy in both emission and receiving rooms are the solutions of the problem. We are also interested in calculating the Transmission Loss (TL) between the mean total energy of the two acoustical rooms. The materials of the two panels (2) and (4) are different. Solid elastic layer (2) is modeled as a steel plate and solid elastic layer (4) is modeled as a plaster board. The insulation layer (3) is an acoustic fluid medium with a high damping ratio  $\xi$  or damping loss factor ( $\eta = 2\xi$ ). It should be noted that the elastic and acoustic coefficients of this computational model are hereinafter assumed to be frequency-independent that is a non-correct approximation for realistic materials. Nevertheless, such an approximation has been introduced in order to simplify the presentation of our new approach and its extension to frequency-dependent coefficients is straightforward.

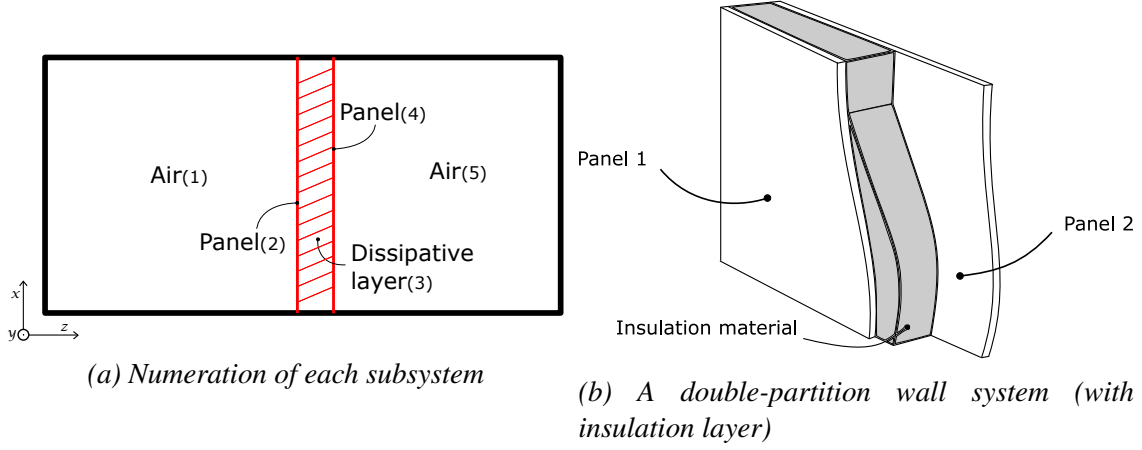


Figure 1: A finite 3D 5-layer-system

## 2.1. Dynamical system and Reduced-Order Model

The computational models of each coupled subsystems are constructed using the Finite Element Method. Then, generalized eigenvalue problems are solved into a large frequency band of analysis for each associated uncoupled conservative computational models. Generalized Reduced-Order Model (ROM) are constructed in projecting the matrices of the computational models on the eigenvectors obtained from the generalize eigenvalue problems. The ROM for subsystems ( $i$ ) with  $i = 1, \dots, 5$  involves generalized reduced-order dynamical stiffness matrices  $[\mathcal{A}^{(i)}(\omega)]$  such as

$$[\mathcal{A}^{(i)}(\omega)] = -\omega^2[\mathcal{M}^{(i)}] + i\omega[\mathcal{D}^{(i)}] + [\mathcal{K}^{(i)}], \quad (1)$$

where  $[\mathcal{M}^{(i)}]$ ,  $[\mathcal{D}^{(i)}]$  and  $[\mathcal{K}^{(i)}]$  are the generalized reduced-order mass, damping and stiffness matrix, respectively. The coupled ROM can be written as

$$\begin{bmatrix} [\mathcal{A}^{(1)}(\omega)] & -i\omega[\mathcal{C}^{(12)}] \\ i\omega[\mathcal{C}^{(12)}]^T & [\mathcal{A}^{(2)}(\omega)] & i\omega[\mathcal{C}^{(23)}] \\ & -i\omega[\mathcal{C}^{(23)}]^T & [\mathcal{A}^{(3)}(\omega)] & -i\omega[\mathcal{C}^{(34)}] \\ & & i\omega[\mathcal{C}^{(34)}]^T & [\mathcal{A}^{(4)}(\omega)] & i\omega[\mathcal{C}^{(45)}] \\ & & & -i\omega[\mathcal{C}^{(45)}]^T & [\mathcal{A}^{(5)}(\omega)] \end{bmatrix} \begin{Bmatrix} \mathbf{q}^{(1)} \\ \mathbf{q}^{(2)} \\ \mathbf{q}^{(3)} \\ \mathbf{q}^{(4)} \\ \mathbf{q}^{(5)} \end{Bmatrix} = \begin{Bmatrix} \widehat{\mathbf{F}}^{(1)} \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}, \quad (2)$$

where  $[\mathcal{C}^{(ij)}]$  are the generalized reduced-order vibroacoustic coupling matrices and  $\mathbf{q}^{(i)}$  are the vectors of the generalized coordinates. Let  $[\mathcal{A}(\omega)]$  be the global generalized reduced-order dynamic stiffness matrix in the left-hand side of Equation 2. Let  $[\mathcal{T}(\omega)] = [\mathcal{A}(\omega)]^{-1}$  be the matrix-valued global generalized reduced-order frequency response function.  $[\mathcal{T}(\omega)]$  can be decomposed into 25 blocs  $[\mathcal{T}^{(ij)}(\omega)]$  with  $i, j = 1, \dots, 5$ . Generalized vector of external forces  $\widehat{\mathbf{F}}^{(1)}(\omega), \omega \in \mathbb{R}$  is a vector-valued random process that is the Fourier Transform of a vector-valued random processes  $\{\mathbf{F}^{(1)}(t), t \in \mathbb{R}\}$  whose components  $F_\alpha$  are a set of uncorrelated white noises with the cross power spectral density  $S_{F_\alpha F_\beta}^{(1)}(\omega) = S_0 \delta_{\alpha\beta}$  where  $S_0$  is a given constant. If  $\langle \cdot \rangle$  denotes the mathematical expectation operator,  $\langle \mathcal{E}_B^{(i)} \rangle$  denotes the mean spectral mechanical energy over the frequency band  $B$  in the angular frequency domain of subsystem ( $i$ ). We then have

$$\langle \mathcal{E}_B^{(i)} \rangle = \sum_{\alpha=1} \langle \mathcal{E}_{\alpha,B}^{(i)} \rangle, \quad \langle \mathcal{E}_{\alpha,B}^{(i)} \rangle = \frac{1}{2\pi} \int_B e_\alpha^{(i)}(\omega) d\omega, \quad (3)$$

where  $e_\alpha^{(i)}(\omega)$  is the modal spectral mechanical energy density of the  $\alpha$ -th mode of subsystem ( $i$ ) written as

$$e_\alpha^{(i)}(\omega) = [\mathcal{M}^{(i)}]_{\alpha\alpha} S_{\dot{q}_\alpha \dot{q}_\alpha}^{(i)}(\omega) \quad , \quad S_{\dot{q}_\alpha \dot{q}_\alpha}^{(i)}(\omega) = \omega^2 S_0 \sum_{\beta=1} \overline{[\mathcal{T}^{(i)}(\omega)]_{\alpha\beta}} [\mathcal{T}^{(i)}(\omega)]_{\alpha\beta} . \quad (4)$$

The ratio of the mean spectral mechanical energy between the emission room and the receiving room, which is also called the TL, is given by

$$\text{TL} = 10 \log_{10} \frac{\langle \mathcal{E}_B^{(1)} \rangle}{\langle \mathcal{E}_B^{(5)} \rangle} . \quad (5)$$

## 2.2. Proposed ROM (PROM) in limited frequency band with condensations

For narrow frequency band of analysis  $B$ , the computational cost of such computational model is decreased if only the calculating of the eigenmodes associated with frequencies in  $B$  (the so called resonant eigenmodes) are needed. Nevertheless, in many works [13, 18, 22, 23], authors have pointed out the non-negligible influence of panel's non-resonant eigenmodes (that is to say, the eigenmodes that are not associated with the frequencies in  $B$ ). These non-resonant eigenmodes are actually involved into the coupling between the generalized coordinates associated with the resonant eigenmodes of different subsystems (see Figure 2). In using a similar approach than in [18], the generalized

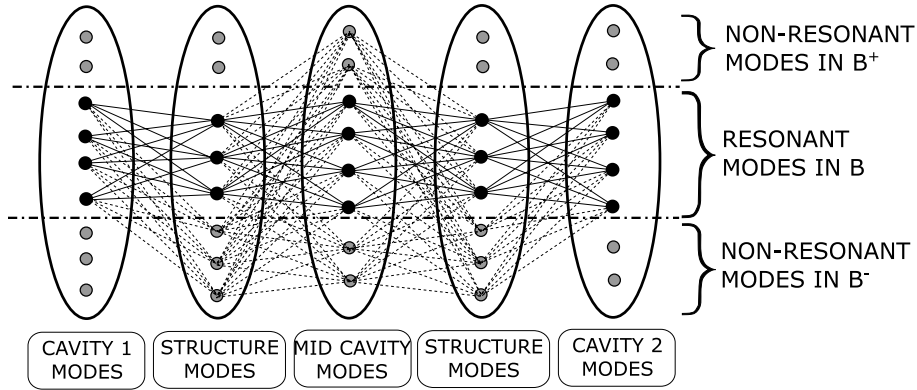


Figure 2: Modal coupling of a 5-layer system

coordinates  $\mathbf{q}_{B^+}$  and  $\mathbf{q}_{B^-}$ , for non-resonant eigenmodes that are respectively associated to angular-frequencies  $\omega^+ \in B^+$  and  $\omega^- \in B^-$  with  $\omega^+ > \omega > \omega^-$  for all  $\omega \in B$ , are eliminated from Equation 2 by using a condensation method. Note that these  $\mathbf{q}_{B^-}$  and  $\mathbf{q}_{B^+}$  are involved into the coupling of subsystems. Consequently, rewritten only in term of generalized coordinates  $\mathbf{q}_B$  associated with resonant eigenmodes, additional matrices must be introduced in order to take into account these couplings, since  $\mathbf{q}_{B^-}$  and  $\mathbf{q}_{B^+}$  have been eliminated. We then obtain a Proposed Reduced-Order Model (PROM) that is written as

$$\begin{bmatrix} [\mathbf{A}_B^{(1)}(\omega)] & [\mathbf{W}_B^{(12)}(\omega)] & [\mathbf{W}_B^{(13)}(\omega)] & [\mathbf{W}_B^{(14)}(\omega)] & [\mathbf{W}_B^{(15)}(\omega)] \\ [\mathbf{W}_B^{(21)}(\omega)] & [\mathbf{A}_B^{(2)}(\omega)] & [\mathbf{W}_B^{(23)}(\omega)] & [\mathbf{W}_B^{(24)}(\omega)] & [\mathbf{W}_B^{(25)}(\omega)] \\ [\mathbf{W}_B^{(31)}(\omega)] & [\mathbf{W}_B^{(32)}(\omega)] & [\mathbf{A}_B^{(3)}(\omega)] & [\mathbf{W}_B^{(34)}(\omega)] & [\mathbf{W}_B^{(35)}(\omega)] \\ [\mathbf{W}_B^{(41)}(\omega)] & [\mathbf{W}_B^{(42)}(\omega)] & [\mathbf{W}_B^{(43)}(\omega)] & [\mathbf{A}_B^{(4)}(\omega)] & [\mathbf{W}_B^{(45)}(\omega)] \\ [\mathbf{W}_B^{(51)}(\omega)] & [\mathbf{W}_B^{(52)}(\omega)] & [\mathbf{W}_B^{(53)}(\omega)] & [\mathbf{W}_B^{(54)}(\omega)] & [\mathbf{A}_B^{(5)}(\omega)] \end{bmatrix} \begin{Bmatrix} \mathbf{q}_B^{(1)} \\ \mathbf{q}_B^{(2)} \\ \mathbf{q}_B^{(3)} \\ \mathbf{q}_B^{(4)} \\ \mathbf{q}_B^{(5)} \end{Bmatrix} = \begin{Bmatrix} \widehat{\mathbf{F}}_B^{(1)} \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} , \quad (6)$$

where  $\mathbf{q}_B = (\mathbf{q}_B^{(1)}, \dots, \mathbf{q}_B^{(5)})$  is decomposed into blocks of generalized coordinates  $\mathbf{q}_B^{(i)}$  associated with the resonant eigenmodes of subsystem ( $i$ ). Note that the construction of these matrices still requires the computation of the non-resonant eigenmodes and, consequently, the initial generalized eigenvalue problem must still be solved with an *ad hoc* computational model for all  $\omega$  belonging to  $B^-$ ,  $B$  and  $B^+$ . Let  $[\mathbf{A}_B(\omega)]$  be the global equivalent reduced-order dynamic stiffness matrix in the left-hand side of Equation 6. Let  $[\mathbf{T}_B(\omega)] = [\mathbf{A}_B(\omega)]^{-1}$  be the matrix-valued global equivalent reduced-order frequency response function for the PROM. An approximation is then used in order to calculate  $\langle \mathcal{E}_B^{(i)} \rangle$  for the PROM based on Equations 3.

$$\langle \mathcal{E}_B^{(i)} \rangle = \sum_{\alpha=1} \langle \mathcal{E}_{\alpha,B}^{(i)} \rangle \quad , \quad \langle \mathcal{E}_{\alpha,B}^{(i)} \rangle \approx \frac{1}{2\pi} \int_B e_{\alpha,B}^{(i)}(\omega) d\omega \quad , \quad (7)$$

where  $e_{\alpha,B}^{(i)}(\omega)$  is the approximated modal spectral mechanical energy density of the  $\alpha$ -th mode of subsystem ( $i$ ) defined as

$$e_{\alpha,B}^{(i)}(\omega) = [\mathbf{M}_B^{(i)}]_{\alpha\alpha} \mathbf{S}_{q_\alpha q_\alpha}^{(i)}(\omega) \quad , \quad \mathbf{S}_{q_\alpha q_\alpha}^{(i)}(\omega) = \omega^2 S_0 \sum_{\beta=1} \overline{[\mathbf{T}_B^{(i)}(\omega)]_{\alpha\beta}} [\mathbf{T}_B^{(i)}(\omega)]_{\alpha\beta} \quad , \quad (8)$$

where  $[\mathbf{M}_B^{(i)}]$  is the mass matrix associated with the dynamic stiffness matrix  $[\mathbf{A}_B^{(i)}(\omega)]$  in the block decomposition of  $[\mathbf{A}_B(\omega)]$  (see Equation 6). It should be noted that  $[\mathbf{M}_B^{(i)}]$  is not the original uncoupled modal mass of subsystem ( $i$ ) since it has been modified by added mass during the condensation step and it is no longer a diagonal matrix.

### 3. PROPOSED SEA-EQUIVALENT COUPLING POWER PROPORTIONALITY (ECPP) APPROACH

One of the reasons that might explain why SEA can be tricky to be used in many engineering applications, it is related to its fundamental hypotheses that can be very restrictive. These hypotheses are clearly classified by authors [21, 24], and the first hypothesis is that "couplings are conservative", which means no dissipative coupling is allowed in SEA. In this section, we propose to rewrite the usual equations of Coupling Power Proportionality (CPP) in order to circumvent this limitation and to extend the theory to non-conservative couplings. This new approach is hereinafter referred by Equivalent Coupling Power Proportionality (ECPP) for which new ECPP  $\zeta$  and  $\chi$  are introduced for the construction of the ECPP. For the sake of simplicity, detailed explanations are not presented in this paper. Let us consider a N-oscillators second-order system

$$[\mathbf{M}] \ddot{\mathbf{q}} + ([\mathbf{D}] + [\mathbf{G}]) \dot{\mathbf{q}} + ([\mathbf{K}] + [\mathbf{R}]) \mathbf{q} = \mathbf{F} \quad . \quad (9)$$

where  $[\mathbf{M}]$ ,  $[\mathbf{D}]$ ,  $[\mathbf{K}]$  are symmetric positive-definite and matrices  $[\mathbf{G}]$  and  $[\mathbf{R}]$  are skew-symmetric matrices. The non-usual matrix  $[\mathbf{R}]$  can represent the additional matrices added into the PROM after the condensation of the non-resonant eigenmodes. It is assumed that the couplings between the components  $q_\alpha$  of the solution  $\mathbf{q}$  are weak, which means that  $[K]_{\alpha\beta} \ll [K]_{\alpha\alpha}$ ,  $[R]_{\alpha\beta} \ll [K]_{\alpha\alpha}$ ,  $[M]_{\alpha\beta} \ll [M]_{\alpha\alpha}$ ,  $[G]_{\alpha\beta} \ll [D]_{\alpha\alpha}$  and  $[D]_{\alpha\beta} \ll [D]_{\alpha\alpha}$  for all  $\alpha \neq \beta$ . The construction of the solution of Equation 9 is quite similar to the construction proposed in [5, 21]. The main difference is due to the additional dissipative couplings

$[D]_{\alpha\beta} \neq 0$  and the additional couplings  $[R]_{\alpha\beta}$ , for  $\alpha \neq \beta$ . The mean modal power balance equation is written as

$$\langle \mathcal{P}_\alpha \rangle_{\text{in}} = \langle \mathcal{P}_\alpha \rangle_{\text{diss}} + \sum_{\beta \neq \alpha} \langle \mathcal{P}_{\alpha\beta} \rangle_{\text{out}}, \quad (10)$$

where  $\langle \mathcal{P}_\alpha \rangle_{\text{in}}$  is the mean modal input power,  $\langle \mathcal{P}_\alpha \rangle_{\text{diss}}$  is the mean modal dissipated power and  $\langle \mathcal{P}_{\alpha\beta} \rangle_{\text{out}}$  is the mean modal output power including mean modal exchanged power and mean modal exchanged dissipated power that are such that

$$\langle \mathcal{P}_\alpha \rangle_{\text{diss}} = \left( [D]_{\alpha\alpha} + \sum_{\beta \neq \alpha} [D]_{\alpha\beta} \right) \langle \dot{q}_\alpha^2 \rangle, \quad \langle \mathcal{P}_{\alpha\beta} \rangle_{\text{out}} = \zeta_{\alpha\beta} \langle \mathcal{E}_\alpha \rangle - \chi_{\alpha\beta} \langle \mathcal{E}_\beta \rangle, \quad (11)$$

where  $\zeta_{\alpha\beta}$  and  $\chi_{\alpha\beta}$  are new ECPP defined as

$$\zeta_{\alpha\beta} = \frac{\int_{-\infty}^{\infty} i\omega \left( ([K]_{\alpha\beta} + \omega^2 [M]_{\alpha\beta})^2 + \omega^2 [G]_{\alpha\beta}^2 - \omega^2 [D]_{\alpha\beta}^2 - [R]_{\alpha\beta}^2 \right) T_\beta(\omega) |T_\alpha(\omega)|^2 d\omega}{\int_{-\infty}^{\infty} [M]_{\alpha\alpha} \omega^2 |T_\alpha(\omega)|^2 d\omega} - \frac{\int_{-\infty}^{\infty} 2\omega^2 ([K]_{\alpha\beta} [D]_{\alpha\beta} + \omega^2 [M]_{\alpha\beta} [D]_{\alpha\beta} - [G]_{\alpha\beta} [R]_{\alpha\beta}) T_\beta(\omega) |T_\alpha(\omega)|^2 d\omega}{\int_{-\infty}^{\infty} [M]_{\alpha\alpha} \omega^2 |T_\alpha(\omega)|^2 d\omega}, \quad (12)$$

$$\chi_{\alpha\beta} = \frac{\int_{-\infty}^{\infty} i\omega \left( ([K]_{\alpha\beta} + \omega^2 [M]_{\alpha\beta})^2 + \omega^2 ([G]_{\alpha\beta} - [D]_{\alpha\beta})^2 + [R]_{\alpha\beta}^2 \right) T_\alpha(\omega) |T_\beta(\omega)|^2 d\omega}{\int_{-\infty}^{\infty} [M]_{\beta\beta} \omega^2 |T_\beta(\omega)|^2 d\omega} - \frac{\int_{-\infty}^{\infty} 2i\omega (\omega^2 [M]_{\alpha\beta} [R]_{\alpha\beta} + [K]_{\alpha\beta} [R]_{\alpha\beta}) T_\alpha(\omega) |T_\beta(\omega)|^2 d\omega}{\int_{-\infty}^{\infty} [M]_{\beta\beta} \omega^2 |T_\beta(\omega)|^2 d\omega}, \quad (13)$$

where  $T_\alpha(\omega) = \left( -\omega^2 [M]_{\alpha\alpha} + i\omega [D]_{\alpha\alpha} + [K]_{\alpha\alpha} \right)^{-1}$ . It should be noted that, unlike the usual SEA, there is no reciprocity for the mean modal output power, that is to say  $\langle \mathcal{P}_{\alpha\beta} \rangle_{\text{out}} \neq -\langle \mathcal{P}_{\beta\alpha} \rangle_{\text{out}}$ . In addition, if  $[R]_{\alpha\beta} = 0$  for all  $\alpha$  and  $\beta$ , and if  $[D]_{\alpha\beta} = 0$  for all  $\alpha \neq \beta$ , then ECPP are both equals to the CPP and we obtain the usual relation of the SEA. We can then consider ECPP as an extension of usual CPP that broadens its validity to non zero matrix  $[R]$  and non zero dissipative couplings  $[D]_{\alpha\beta}$ .

#### 4. NUMERICAL EXAMPLE

In this section, two numerical examples are presented that carry out the construction of the PROM (see section 2.2) and the SEA-ECPP formulation (see section 3). A first numerical example is presented for which the TL is calculated for a relatively small design system whose parameters are presented in Table 1. The dimension are small in order to limit the computational costs of the ROM and PROM for being able to validate the predictability of the SEA-ECPP. Once again, such a small design system is not chosen in regard to the efficiency of the SEA-ECPP but in regard to the computational cost induced by using ROM and PROM. Consequently, calculations are performed only for the frequencies [300, 3000] Hz in one-third octave band which will allow the validation of the SEA-ECPP to be asserted. It is shown in Fig. 3 that both the solutions calculated by the SEA-ECPP and the PROM are very close to the reference solution calculated by using the ROM.

Subsystems	$L_x$ (m)	$L_y$ (m)	$L_z$ (m)	$\rho$ (kg/m <sup>3</sup> )	$C$ (m/s)	$\xi/\eta$
(1)	0.8	0.6	0.8	1.29	340	0.005 / 0.01
(3)	0.8	0.6	0.045	3	200	0.5 / 1
(5)	0.8	0.6	0.7	1.29	340	0.005 / 0.01

Subsystems	$L_x$ (m)	$L_y$ (m)	$L_z$ (m)	$\rho$ (kg/m <sup>3</sup> )	$E$ (Pa)	$\nu$	$\xi/\eta$
(2)	0.8	0.6	0.001	7800	$2 \times 10^{11}$	0.3	0.005 / 0.01
(4)	0.8	0.6	0.0125	736	$2.7 \times 10^9$	0.1	0.015 / 0.03

Table 1: Properties of each layer for the first numerical example

Furthermore, for a calculation of TL in this frequency range, the elapsed computational time for the calculation of the reference solution in using the ROM and the PROM is respectively 12 hour and 1350 seconds in using parallel computing with a computational cluster with 30 cores. The elapsed computational time is 30 seconds with the SEA-ECPP approach and without using any computational cluster. It should be noted that the elapsed computational times do not include the computing of the eigenvalues and eigenvectors. Figure 4 shows the comparison of the mean spectral mechanical energy of subsystems (1)

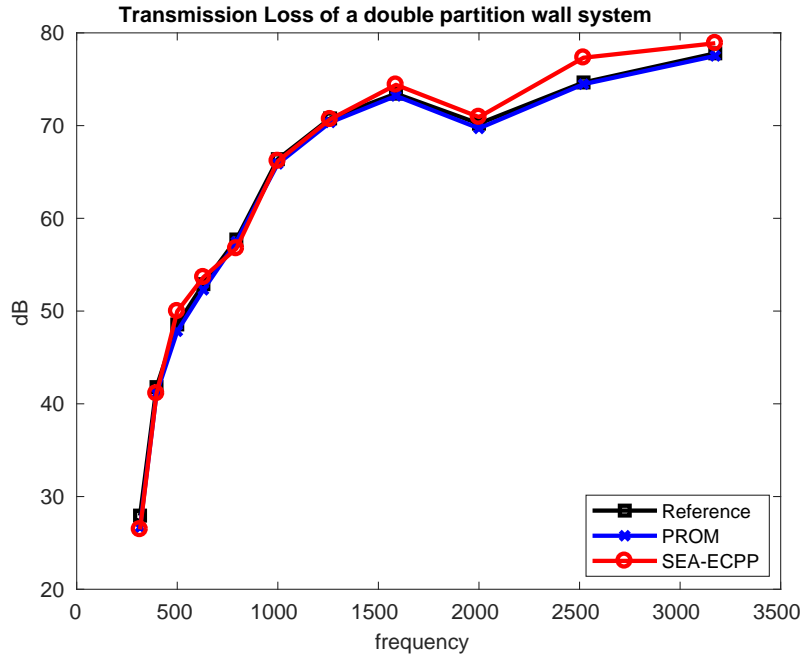
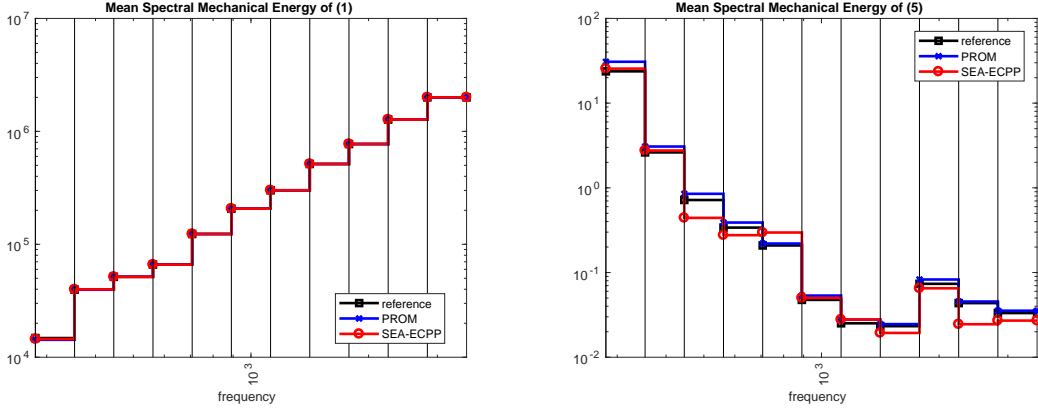


Figure 3: Comparison between reference, PROM and SEA-ECPP for a highly dissipative system

and (5) in the same frequency bands calculated in using the SEA-ECPP, the PROM and the ROM. It can be seen that the results are again very satisfying.

A second numerical example is presented for which the TL is calculated for a larger design system whose parameters are given in Table 2. This design system is a 72/48 double-partitions wall, composed of two plasterboard BA13 and an equivalent dissipative layer between 2 large acoustic cavities. The TL is calculated on a broad frequency range of analysis [80, 5000] Hz. The FEA is used for calculating the TL on the low frequency domain [80, 300] Hz. The SEA-ECCP is used for calculating the TL into the medium



(a) Mean spectral mechanical energy of (1) (b) Mean spectral mechanical energy of (5)

Figure 4: Comparison between reference, PROM and SEA-ECPP

Subsystems	$L_x$ (m)	$L_y$ (m)	$L_z$ (m)	$\rho$ (kg/m <sup>3</sup> )	$C$ (m/s)	$\xi/\eta$
(1)	4.2	2.5	3	1.29	340	0.005 / 0.01
(3)	4.2	2.5	0.045	3	200	0.05 / 0.1
(5)	4.2	2.5	3	1.29	340	0.005 / 0.01

Subsystems	$L_x$ (m)	$L_y$ (m)	$L_z$ (m)	$\rho$ (kg/m <sup>3</sup> )	$E$ (Pa)	$\nu$	$\xi/\eta$
(2)	4.2	2.5	0.0125	736	$2.7 \times 10^9$	0.1	0.015 / 0.03
(4)	4.2	2.5	0.0125	736	$2.7 \times 10^9$	0.1	0.015 / 0.03

Table 2: Properties of each layer for the second numerical example

frequency domain [250, 2000] Hz. Finally, the commercial software Acousys is used for carrying out the Transfer Matrix Method (TMM) and for calculating the TL in the high frequency domain [1250, 5000] Hz. Results are presented on Figure 5. It can be shown that these 3 methods are well overlapped, especially the FEA and SEA-ECPP in the overlapping domain [250, 300] Hz.

## 5. CONCLUSIONS

In this paper, we have introduced an energy based method that is an extension of the SEA for a system of coupled oscillators in order to estimate the mean spectral mechanical energy of a multilayer system that contains highly dissipative layer. The proposed SEA-ECPP method allows the restrictions of conservative couplings in classic SEA to be lifted. The only inputs of this method are modal information, which can be obtained by FEA. With these information, associated with a proposed order model (PROM) with condensation of non-resonant eigenmodes, such a method (called SEA-ECPP) allows computing of mean spectral energies to perform very fast in the medium frequency domain. A further work in the optimization of the identification of coupling matrices can be a perspective of this paper.



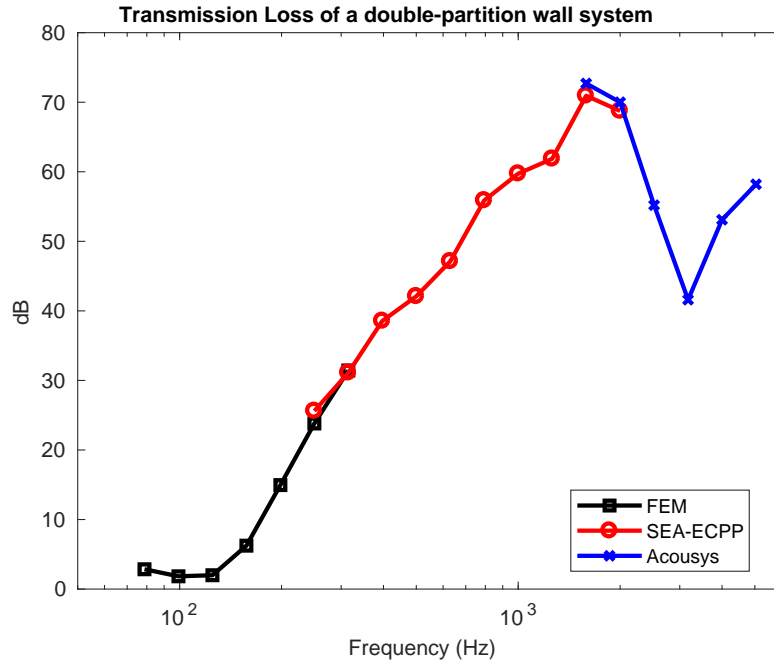


Figure 5: TL simulation with 3 methods

## 6. ACKNOWLEDGEMENTS

This research was supported and financed by CSTB. We thank our colleagues from CSTB and from University of Paris-Est who provided insight and expertise that greatly assisted the research.

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