

Active control of sound radiation from rib stiffened plate by using decentralized system

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ABSTRACT

The control of low frequency sound radiation from rib stiffened plate has great practical significance since it is widely applied as aircraft or ship fuselage shells. This paper presents an investigation on the performance of active control of sound radiation from the ribbed plate by using decentralized control system. The theoretical model of the active control system with the decentralized control approach is firstly established. Then, the influences of the number and the arrangement of the decentralized unit on the control performance are specially investigated as an important part. And the influence of the rib on the control effects of the decentralized system is also analyzed. Results obtained demonstrate that, similar with that of the unribbed plate case, appropriately choosing the number of the unit and their feedback gains can achieve good control results. Too many units or very high feedback gains will not further bring more noise reduction. The decentralized feedback loops should also been appropriately arranged so as to achieve good control performance.

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1. INTRODUCTION

Thin plates stiffened by a set of beams form a class of structural elements that are widely applied in various engineering applications [1]. Usually the rib stiffened structure is most commonly applied in aerospace, aircraft industry, and ground vehicles, and so on. Such structure naturally has predominant sound insulation performance in mid and high frequency range. However, the performance in the low frequency range declines remarkably. Traditional passive methods of noise reduction require excessive heavy damping material for the low frequency control, and can not always gain remarkable effects. These insufficient aspects have prompted the research into applying active control techniques [2].

In order to explore the physical nature of the active control of sound radiation from ribbed structure, Ma [3] investigated the mechanism of active control of sound radiation from the single ribbed plate using the single point force as the control source. And then, in order to further explore the physical nature of active control in complex double panel case, Ma [4] presents an analytical investigation on actively controlling sound transmission through single rib stiffened double-panel system using cavity control approach. Results demonstrate that active control can also effectively improve the low frequency sound insulation of such structure. And due to the coupling effects of the ribs, the active control mechanism presents additional prominent differences, which offer guidance for practical application of this technic. However, when this is used in practice for constructing the active sound insulation structure, it will usually need to add multisecondary control forces to actively control the structural vibration of the large area ribbed plate. This can guarantee highly and effectively improving the low frequency sound insulation. Accordingly, for the multi-input error signal multi-output control signal (MIMO), the control system is usually the centralized multichannel system for controlling the sound insulation of the practical structure with large area. However, due to the large number of coupling secondary path between the control source and the error sensing devices, the computational loads of such multichannel system is large, which makes the system complicated and even hard for practical implementations. Its control effects and the stability are also hard to guarantee. Another key problem encountered when implementing the active rib stiffened double-panel structure is error sensing strategy. Theoretically the radiated sound power of the radiating ribbed plate is the optimal cost function which can achieve maximal noise reduction. In order to sense this error signal or the signal highly correlated with this, suitable error sensing strategy should be established [5-6], which may be also a challenging work and will also imped the wide practical application of this technic.

On the contrary, the decentralized control system which is consisted of many decentralized unit is simple and easy to implement. The decentralized unit is the single-input and single-output feedback loop. Usually the decentralized, static gain control is the simplest form of feedback control [7]. If it is applied in a stable system where the sensors and actuators are collocated and dual, then stability is guaranteed [7]. And the velocity sensed by the sensor is directly used as the error signal to control. The error sensing strategy is no need further to establish. In a practical situation, it can have the extra advantage that no connections are required between different control locations. And a central processing unit with the actuator, sensor and controller could be produced as identical modular units. The control performance of the decentralized feedback control system has been extensively investigated [8-13]. Elliott [7] compares the

performance of the velocity feedback loop of collocated and dual force actuators and velocity sensors and piezoelectric actuators and velocity sensors. Gardonio [8-10] presents a theoretical and experimental investigation to develop a prototype smart panel with 16 decentralized vibration control units for the reduction of sound radiation/transmission of the plate. The feedback loops consist of piezoceramic patches as actuators and accelerometers as sensors. The sensor and actuator are not dual and the stability of the feedback loop is limited. In order to further set up relevance between the research work and the practical application, Aoki [11] designs the decentralized velocity feedback loop with triangularly shaped piezoceramic patch actuators and accelerometer sensors. D faz [12] and Baumann [13] use the small scale proof mass electrodynamic actuator and electrodynamic inertial actuator to construct the feedback loop. The control performance and stability of the system are analyzed. All the results obtained in the above investigations will offer help for designing and applying this technic.

It can be found that most of the existed research works use a simple plate as the control object, which allows the attention been concentrated on explore the control performance and physical nature of the decentralized control system. And as for the more practical structure, such as the ribbed plate widely used in engineering, there is little research on using this structure as the control object. Hence, in order to further improve the relevance of the research work with the practical engineering, this paper use the rib stiffened plate as the control object, and particularly investigates the control performance of the active rib stiffened plate using the decentralized control system. The specific research works carried out are focused on the influence of the number of the feedback loops and their arrangement on the control performance. And the influence of the rib on the control effects of the decentralized control system.

The remainder parts are organized as follows. The model of the active rib stiffened double-panel system with feedback loops is established in Sec.2. Control results and discussions are presented in Sec.3. Finally, conclusions are summarized in Sec.4.

2. THEORETICAL MODELING

2.1 Vibrating response of the ribbed plate with point control force

Figure 1 (a) shows the sketch of the system. For both simplifying the theoretical model and also suitably representing the practical complicated ribbed structure, an orthogonally rib stiffened plate is used in the model. The orthogonally rib stiffened plate is constituted with a simply supported base plate and two simply supported ribs with one rib along the horizontal and another along the vertical directions. An oblique incident plane wave is applied as the primary excitation. The beam/plate interface is considered as a nonslip line connection in which a pair of coupling force *F* and moment M_m is considered to simulate the beam/plate coupling effects [14], as shown in Fig 1 (b). The vertical rib locates at $x = x_a$ and the horizontal rib locates at $y = y_a$. A controllable point force $f_s(\mathbf{r}_s, t)$ is added on the ribbed plate to actively control the noise radiation from the ribbed plate. When the decentralized control system with *S* feedback loops is used for controlling the sound radiation, the control force can also be expressed as

$$f_s = \sum_{i=1}^{3} f_i \delta(x - x_i, y - y_i).$$

Under the excitation of an oblique incident plane wave, the base plate is subjected to the coupling force and moment of the ribs, the sound pressure, and the control force. The transverse bending displacement *w* satisfies the motion of equation:



Fig.1. Systemic model (a) Active orthogonally rib stiffened plate; (b) the coupling effects between the base plate and the rib

$$D\nabla^{4}w + \rho h \frac{\partial^{2}w}{\partial t^{2}} = f_{p}(\mathbf{r},t) + F_{v}\delta(x-x_{a}) + M_{m,v}\delta'(x-x_{a})$$
$$F_{h}\delta(y-y_{a}) + M_{m,h}\delta'(y-y_{a}) + f_{s}(\mathbf{r}_{s},t), \qquad (1)$$

where ∇^4 denotes the Laplacian, *D* is the flexural rigidity of the base plates, $D = Eh^3/12(1-\sigma^2)$. *E*, ρ , *h* and σ are the Young's modulus, density, thickness and Poisson's ratio of the base plate. $f_p(\mathbf{r}, t)$ is the primary excitation of the oblique incident plane wave, which can be referenced in Ref [15].

The rib is assumed as a uniform beam element with a rectangular cross section. Further assumed that the beam is symmetrical with respect to its neutral plane so that the flexural and torsional vibration is uncoupled. The governing equations of the flexural and torsional displacements (U, θ_r) of the ribs can be expressed as [14] (using a vertical rib for example),

$$EI\frac{\partial^4 U}{\partial y^4} + \rho A\frac{\partial^2 U}{\partial t^2} = -F_v, \qquad (2)$$

$$EI_{r,w}\frac{\partial^4\theta_r}{\partial y^4} - GJ\frac{\partial^2\theta_r}{\partial y^2} + \rho I_0\frac{\partial^2\theta_r}{\partial t^2} = -M_{m,v}, \qquad (3)$$

where *E* and *I* are the Young's modulus and sectional moment of inertia of the beam. ρ and *A* are the density and sectional area of the beam. *EI* is the flexural rigidity and ρA is the mass per unit length. $I_{r,w}$ is the warping moment of inertia related to the torsion, $EI_{r,w}$ is the warping rigidity. *G* is the shear modulus and its relation with the Young's

modulus *E* can be expressed as $G = E/2(1 + \sigma)$. *J* is the Saint-Venant torsional constant, and *GJ* is the torsional rigidity. ρI_0 is the mass moment of inertia per unit length. The warping rigidity $EI_{r,w}$ is much smaller than the torsional rigidity *GJ*, and the differential term $\partial^4 \theta_r / \partial y^4$ in Eq. (3) can be neglected for simplicity.

According to the modal expansion method, the bending displacement w of the base plate, the flexural and torsional displacements of two beams (U, θ_r) can be decomposed

over their mode shape functions as
$$w = \sum_{m=1}^{M} \sum_{n=1}^{N} q_{mn}(t) s_{mn}(x, y)$$
, $U = \sum_{n=1}^{N} u_n(t) \varphi_n(y)$,

 $\theta_r = \sum_{n=1}^{N} \theta_{r,n}(t) \varphi_n(y) \cdot s_{mn}(x, y) = \phi_m(x) \varphi_n(y) \text{ is the mode shape function of the simply}$

supported base plate. $q_{nn}(t)$, $u_n(t)$, $\theta_{r,n}(t)$ are the modal amplitudes with respect to each subsystem. $M \times N$ is the upper limit number of modes. As for the ribbed plate, combining Eq. (1)~Eq. (3), applying the orthogonality of the mode shape functions and two compatibility conditions at the beam/plate interface [the displacement and rotation continuity conditions [14]: $U_v(y) = w(x_a, y)$, $U_h(x) = w(x, y_a)$, $\theta_{r,v}(y) = \frac{\partial w}{\partial x(x_a, y)}$ and $\theta_{r,h}(x) = \frac{\partial w}{\partial y(x, y_a)}$] can yield the equation that the modal amplitude $q_{mn}(\omega)$ of the base plate satisfies:

$$(k_{mn} - \omega^{2} \mu_{mn})q_{mn}(\omega) + \sum_{p=1}^{M} [\phi_{m}(x_{a})(k_{Bn} - \omega^{2} \mu_{Bn})\phi_{p}(x_{a}) - \phi_{m}^{'}(x_{a})(k_{Tn} - \omega^{2} \mu_{Tn})\phi_{p}^{'}(x_{a})]$$

$$\cdot q_{pn}(\omega) + \sum_{q=1}^{N} [\phi_{n}(y_{a})(k_{Bm} - \omega^{2} \mu_{Bm})\phi_{q}(y_{a}) - \phi_{n}^{'}(y_{a})(k_{Tm} - \omega^{2} \mu_{Tm})\phi_{q}^{'}(y_{a})]q_{mq}$$

$$= Q_{p,mn} + F_{s}Q_{s,mn}. \quad (4)$$

The variables k_{mn} , μ_{mn} , k_{Bn} , μ_{Bn} , k_{Tn} , μ_{Tn} , k_{Bm} , μ_{Bm} , k_{Tm} and μ_{Tm} can be referenced in Ref. [4]. $Q_{p,mn}$ and $Q_{s,mn}$ are the generalized primary and secondary modal force, $Q_{p,mn} = \int_{S} f_{p}(\mathbf{r}, \omega)\phi_{m}(x)\phi_{n}(y)ds$, $Q_{s,mn} = \phi_{m}(x_{s})\phi_{n}(y_{s})$. Then, the $M \times N$ -length modal amplitude vector \mathbf{q} of the base plate satisfies the following matrix equation,

$$(\mathbf{K} - \omega^2 \mathbf{M})\mathbf{q} = \mathbf{Q}_p + F_s \mathbf{Q}_s.$$
 (5)

Where **K** and **M** are the $(M \times N) \times (M \times N)$ stiffness and mass matrices of the base plate. These can be referenced in Ref. [4], where the coupling items of the horizontal rib should also be added into their respective expressions. $\mathbf{Q}_p = [Q_{p,11}, Q_{p,12}, ..., Q_{p,MN}]^T$ and $\mathbf{Q}_s = [Q_{s,11}, Q_{s,12}, ..., Q_{s,MN}]^T$ are the generalized modal force vectors. $\mathbf{q} = [q_{11}, q_{12}, ..., q_{MN}]^T$ is the modal amplitude vector of the base plate. F_s is the amplitude of the control force.

According to the discrete elemental approach [8], by dividing the base plate into N_e elements, the power output can be expressed as:

 $W = \mathbf{V}^{\mathsf{H}} \mathbf{R} \mathbf{V} = (\mathbf{\Phi} \mathbf{q})^{\mathsf{H}} \mathbf{R} (\mathbf{\Phi} \mathbf{q})$

$$= [\mathbf{\Phi}(\mathbf{K} - \omega^2 \mathbf{M})^{-1} (\mathbf{Q}_p + F_s \mathbf{Q}_s)]^{\mathrm{H}} \mathbf{R} [\mathbf{\Phi}(\mathbf{K} - \omega^2 \mathbf{M})^{-1} (\mathbf{Q}_p + F_s \mathbf{Q}_s)]$$
(6)

Where $\mathbf{R} = \Delta S \operatorname{Re}(\mathbf{Z})/2$, in which \mathbf{Z} is the $N_e \times N_e$ transfer impedance matrix. \mathbf{V} is the N_e -length velocity vector of the ribbed plate. $\mathbf{\Phi}$ is the value of the mode shape function on the center of the discrete element. Set $F_s = 0$ in Eq. (6), the radiated sound power of the ribbed plate before control can be calculated. And when the optimal control force f_s of the feedback loops is obtained, the radiated sound power after control can be calculated accordingly.

2.2 Decentralized control system with feedback loops

Decentralized control system with *S* feedback loops is introduced to control the vibration and sound radiation of the orthogonally rib stiffened plate. The sketch of the decentralized system is shown in Fig.2. For each feedback loop, the velocity v_i sensed by the sensor is feed back to the controller with control gain h_i . Then the control output $u_i(j\omega)$ is added to the ribbed plate to control the vibration of this sensing point. For the whole system, the plant and controller responses are square matrices, $\mathbf{G}(j\omega)$ and $\mathbf{H}(j\omega)$, as illustrated by the block diagram in Fig.3.



Fig.2. The sketch of the decentralized system



Fig.3. The block diagram of the closed-loop system

Provided the control system is stable, the vector the residual signals at the sensor outputs $\mathbf{y}(j\omega)$, such as the velocity vector $\mathbf{v}(j\omega)$, is related to the primary disturbance $\mathbf{d}(j\omega)$ by,

$$\mathbf{y}(j\omega) = [\mathbf{I} + \mathbf{G}(j\omega)\mathbf{H}(j\omega)]^{-1}\mathbf{d}(j\omega).$$
(7)

Then the vector of control inputs to the actuators $\mathbf{u}(j\omega)$ can be expressed as,

$$\mathbf{u}(j\omega) = \mathbf{H}(j\omega)[\mathbf{I} + \mathbf{G}(j\omega)\mathbf{H}(j\omega)]^{-1}\mathbf{d}(j\omega).$$
(8)

In the case under consideration here, $\mathbf{G}(j\omega)$ is the fully populated matrix of input and transfer responses between the actuators and sensors on the panel. $\mathbf{H}(j\omega)$ is a diagonal matrix. It contains the control gains of the feedback loops, which we assume to have constant gains on each channel so that $\mathbf{H}(j\omega) = h\mathbf{I}$. Thus, given a set of panel responses $\mathbf{G}(j\omega)$ and the feedback gain *h*, the control input $\mathbf{u}(j\omega)$ can be calculated. Then, the response of the ribbed panel before and after control can be obtained. In this research, the velocity feedback loop is used, i.e., the ideal collocated and dual point force and velocity.

3. RESULTS AND DISCUSSION

3.1 Parameters of the model

The parameters of the model used in simulation are listed in Table 1. The ribs are uniform beam elements with rectangular cross section. The geometrical size of the two ribs is the same to simplify the model. The primary excitation is the plane wave with amplitude $P_0 = 1$ Pa, incident at $\theta = \pi/4$ and $\alpha = \pi/4$. The structure damping is ignored in the modeling, thus a constant internal loss factor $\eta = 0.01$ is introduced into the complex Young's modulus as $E(1 + i\eta)$ to make the simulation results more practice [14].

Table 1	The	parameters	of the	model
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Geometrical parameters	Value	Material properties	Value
The length and width of the base plate	$l_x = 0.8 \text{ m}$ $l_y = 0.6 \text{ m}$	The material of the base plate and the ribs	aluminum
The thickness of the base plate	$h = 0.004 \mathrm{m}$	The density of aluminum	$\rho = 2790 \mathrm{kg/m^3}$
The locations of the vertical and horizontal ribs	$x_a = 0.6m$ $y_a = 0.45m$	The Young's modulus of aluminum	$E = 7.2 \times 10^{10}$ N/m ²
The size of the rectangular cross section of the ribs(wide×high)	$A = 0.003 \times 0.04 \mathrm{m}^2$	The Poisson's ratio of aluminum	$\sigma = 0.34$
The primary excitation	$P_0 = 1 \operatorname{Pa}$ $\theta = \pi/4$ $\alpha = \pi/4$	The density of air	$\rho_0 = 1.21 \text{kg/m}^3$
		The sound speed of air	$c_0 = 344 \text{ m/s}$

3.2 Active control with velocity feedback loops

In this research, an upper limit of 7 channel decentralized feedback controller is considered. The locations and coordinates of these feedback loops are shown in Fig.4. Firstly, a 4 channel feedback a loop (1#, 2#, 3#, and 4#) is applied to validate the control performance of the decentralized system. The radiated sound power of the ribbed plate before and after control is calculated and plotted in Fig.5. In this case, the

feedback gain h is set as 50, 200, and 5000, respectively. It can be found that the decentralized feedback controller can effectively control the vibration and sound radiation of the orthogonally rib stiffened plate. Like in the case of unribbed plate, as the gains of the feedback loops are increased, the resonances in the response become more heavily damped. However, if the gains of the feedback loops are increased beyond a certain value, the response of the ribbed plate is enhanced at certain frequency and displays new peaks. It is resonant for the controlled dynamic system with this high feedback gain. The resonant frequency of this control spillover is also correlated with the arrangement of the feedback loops. Hence, the feedback gain should be chosen appropriately. Another 4 channel feedback loops with locations (1#, 4#, 6#, and 7#) is also considered to investigate the influence of the arrangement of the feedback loops on the control performance. The results are shown in Fig.6, which illustrates that the active sound reduction is slightly different between these two cases (case1: (1#, 2#, 3#, and 4#), and case2: (1#, 4#, 6#, and 7#)). Though the locations may have little impact on the control results, but it should also be arranged appropriately so as to achieve the optimal control performance.



Fig.4. The locations and coordinates of the feedback loops



Fig.5. Active control results with a 4 channel feedback loops (1#, 2#, 3#, and 4#)



Fig.6. Active control results with a 4 channel feedback loops (1#, 4#, 6#, and 7#)

The decentralized systems with 2, 4, 5, 6, and 7 channel feedback loops are also examined to explore the impact of number of the feedback loops on the control results. The active control results of these cases are calculated and also plotted in Fig.7. As the number of feedback loops increases, the active noise reduction may also slightly increases. However, too many feedback loops with tiny increment of noise reduction is meaningless for practical application. Hence, the number of the feedback loops should also be chosen appropriately.



Fig.7. Active control results with different number of feedback loops

4. CONCLUSIONS

This paper presents an analytical investigation on active control of sound radiation from the ribbed plate using decentralized velocity feedback loops. The control performance of the decentralized system for the complicated ribbed plate is examined. Results obtained demonstrate that appropriately choosing the number of the decentralized unit and the feedback gain of each unit can achieve good control effects. Too many decentralized unit or very high feedback gains will not further effectively improve the active sound reduction. The decentralized units should also been appropriately arranged so as to achieve good control performance.

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