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Acoustic sensitivity analysis using isogeometric BEM with wideband fast multipole method

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ABSTRACT

According to the concept of isogeometric analysis (IGA), Non-Uniform Rational B-splines (NURBS) are used to describe the geometry and approximate the physical fields. This treatment circumvents the requirement to generate a mesh, which is a significant progress in reducing the gap between engineering design and analysis. Some researchers have applied the IGA to the boundary element method (BEM), forming the IGA BEM. In this work, we introduce the wideband fast multipole method (WFMM) to the IGA BEM of wideband acoustic problems to improve its numerical efficiency. The Burton-Miller method is adopted to conquer the fictitious eigen-frequency problem in solving exterior acoustic problems. Furthermore, we apply the WFMM IGA BEM to acoustic sensitivity analysis by using the direct differentiation method (DDM) and the adjoint variable method (AVM). Compared with the DDM, the AVM is more suitable for problems with a large number of design variables. An example of scattering by an infinite rigid cylinder is presented to demonstrate the acceleration and the improved accuracy of WFMM IGA BEM. Finally, we verify the efficiency of the developed sensitivity algorithms through a more engineering problem and demonstrate their potential in solving large-scale engineering problems.

Keywords: Isogeometric analysis, Boundary element method, Wideband fast multipole method, Acoustic sensitivity analysis
I-INCE Classification of Subject Number: 76

1. INTRODUCTION

Reducing the sound radiation of structures is of great concern in engineering problems, and shape optimization has been considered to be an effective approach [1]. IGA BEM could achieve a higher accuracy than the conventional BEM in solving acoustic problems, as it uses control points to describe the geometry. Therefore, it is

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more convenient to carry out the shape optimization by IGA BEM. However, the IGA BEM demands more computational costs than the conventional BEM, and the sensitivity analysis even requires more efforts. This made the efficient analysis techniques indispensable. Thus, in addition to the acoustic field analysis, the main contribution of this paper is to present a wideband FMM IGA BEM for acoustic sensitivity analysis.

2. WIDEBAND FMM FOR IGA BEM IN 2D ACOUSTICS

For the IGA BEM in 2D acoustics, a series of efforts have been made by [1, 2].

Using the Burton-Miller method to conquer the fictitious eigen-frequency problem, we get the conventional BIE and outward BIE as follows:

$$\begin{aligned} c(x)p(x)+\int_S F(x,y)p(y)dS(y) &= \int_S G(x,y)q(y)dS(y) \\ c(x)q(x)+\int_S F^1(x,y)p(y)dS(y) &= \int_S G^1(x,y)q(y)dS(y) \end{aligned} \tag{1}$$

where $G(x,y)$、 $F(x,y)$、 $G^1(x,y)$、 $F^1(x,y)$ are the kernel functions.

For IGA BEM, we have the NURBS interpolation [1] as follows:

$$\begin{aligned} x(\xi) &= \sum_{i=1}^{n} R^g_{i,p_g}(\xi)X_i \\ p(\xi) &= \sum_{i=1}^{n_f} R^f_{i,p_f}(\xi)p_i \\ q(\xi) &= \sum_{i=1}^{n_f} R^f_{i,p_f}(\xi)q_i \end{aligned} \tag{2}$$

Substituting eqn (2) to eqn (1), and performing linear combination, the following linear algebraic equations in matrix form is obtained：

$$\boldsymbol{H}p = \boldsymbol{G}q \tag{3}$$

Finally we obtained the unknown boundary values by solving eqn (3).

Here the wideband FMM approach[3] is introduced to accelerate the matrix-coefficient calculation of IGA BEM in eqn (3). We divide the FMM into two parts, one is the low frequency FMM and the other is the high frequency FMM.

For the low frequency FMM, the Greens function in BIE can be expanded into the following series:

$$G(x,y) = \frac{\mathrm{i}}{4}\sum_{n=-\infty}^{+\infty} O_n(\overrightarrow{y_c x})I_{-n}(\overrightarrow{y_c y}) \tag{4}$$

Introducing the multipole expansion and translations as follows:

$$\begin{aligned} &Multipole: M_n(y_c) = \frac{\mathrm{i}}{4}\int [I_{-n}(\overrightarrow{y_c y})q(y) - D_n(\overrightarrow{y_c y})p(y)]\mathrm{d}S(y) \\ &M2M: M_n(y_c^1) = \sum_{m=-\infty}^{+\infty} I_{-n+m}(\overrightarrow{y_c^1 y_c})M_m(y_c) \\ &M2L: L_n(x_l) = \sum_{m=-\infty}^{+\infty} (-1)^n O_{n-m}(\overrightarrow{y_c^1 x_l})M_{-m}(y_c^1) \\ &L2L: L_n(x_l^1) = \sum_{m=-\infty}^{+\infty} I_{n-m}(\overrightarrow{x_l x_l^1})L_m(x_l) \end{aligned} \tag{5}$$

Finally we get the following formulations to compute the integrals as

$$
\begin{aligned}
&\int [G(x,y)q(y) - F(x,y)p(y)]\mathrm{d}S(y) = \sum_{n=-\infty}^{+\infty} I_{-n}(\overrightarrow{x_l^1 x})L_n(x_l^1) \\
&\int [G^1(x,y)q(y) - F^1(x,y)p(y)]\mathrm{d}S(y) = \sum_{n=-\infty}^{+\infty} D_n(\overrightarrow{x_l^1 x})L_n(x_l^1)
\end{aligned}
\tag{6}
$$

For the high frequency FMM part, we expand $G(x,y)$ into the following series:

$$
G(x,y) = \frac{\mathrm{i}}{8\pi}\oint e^{ik\hat{\alpha}\bullet\overrightarrow{x_l x}} T(\theta, \overrightarrow{y_c x_l}) e^{-ik\hat{\alpha}\bullet\overrightarrow{y_c y}} d\theta \tag{7}
$$

Also the multipole expansion and translations as

$$
\begin{aligned}
&Multipole: B(\theta, y_c) = \int_S \left[e^{-ik\hat{\alpha}\bullet\overrightarrow{y_c y}} q(y) - E(\overrightarrow{y_c y}) p(y) \right] dS(y) \\
&B2B: B(\theta, y_c^1) = e^{-ik\hat{\alpha}\bullet\overrightarrow{y_c^1 y_c}} * B(\theta, y_c) \\
&B2H: H(\theta, x_l) = T(\theta, \overrightarrow{y_c^1 x_l}) * B(\theta, y_c^1) \\
&H2H: H(\theta, x_l^1) = e^{ik\hat{\alpha}\bullet\overrightarrow{x_l x_l^1}} * H(\theta, x_l)
\end{aligned}
\tag{8}
$$

Finally the integrals in eqn (3) can be computed as

$$
\begin{aligned}
&\int [G(x,y)q(y) - F(x,y)p(y)]\mathrm{d}S(y) = \frac{i}{8\pi}\oint e^{ik\hat{\alpha}\bullet\overrightarrow{x_l^1 x}} H(\theta, x_l^1) d\theta \\
&\int [G^1(x,y)q(y) - F^1(x,y)p(y)]\mathrm{d}S(y) = \frac{i}{8\pi}\oint \frac{\partial e^{ik\hat{\alpha}\bullet\overrightarrow{x_l^1 x}}}{\partial n(x)} H(\theta, x_l^1) d\theta
\end{aligned}
\tag{9}
$$

In wideband FMM, we choose a parameter ν. If $kd \le \nu$, we compute the far-field integrals by low frequency FMM; Otherwise high frequency FMM is used, where k is the wave number and d is the size of square in the tree structure of wideband FMM. At the same time, we have the transformation of multipole expansions between low frequency FMM and high frequency FMM as follows:

$$
\begin{aligned}
&M2B: B(\theta, y_c) = -4i \sum_{n=-\infty}^{n=+\infty} e^{in\theta} M_n(y_c) \\
&H2L: L_n(x_l) = \frac{i}{8\pi}(-1)^n \oint e^{in\theta} H(\theta, x_l) d\theta
\end{aligned}
\tag{10}
$$

3. ACOUSTIC SENSITIVITY ANALYSIS

Sensitivity analysis plays a great role in the shape optimization. Usually the control points in IGA can be chosen as the design variables. Some works about acoustic sensitivity analysis using DDM [1-3, 5-7] have been developed.

By using the DDM, eqn (1) is differentiated with respect to the design variables as follows:

$$
\begin{aligned}
&c(x)\dot{p}(x) + \int_S F(x,y)\dot{p}(y)dS + \int_S \dot{F}(x,y)p(y)dS + \int_S F(x,y)p(y)d\dot{S} \\
&= \int_S G(x,y)\dot{q}(x)dS + \int_S \dot{G}(x,y)q(y)dS + \int_S G(x,y)q(y)d\dot{S} \\
&c(x)\dot{q}(x) + \int_S F^1(x,y)\dot{p}(y)dS + \int_S \dot{F}^1(x,y)p(y)dS + \int_S F^1(x,y)p(y)d\dot{S} \\
&= \int_S G^1(x,y)\dot{q}(x)dS + \int_S \dot{G}^1(x,y)q(y)dS + \int_S G^1(x,y)q(y)d\dot{S}
\end{aligned}
\tag{11}
$$

where the upper dot $(\dot{\ })$ denotes the differentiation with respect to the design variables.

Substituting eqn (2) to eqn (11), and applying the Burton–Miller method, we get linear algebraic equations in matrix form as：

$$\dot{\boldsymbol{H}}\, p + \boldsymbol{H}\, \dot{p} = \dot{\boldsymbol{G}}\, q + \boldsymbol{G}\, \dot{q} \tag{12}$$

where the matrix-vector products can be performed by the wideband FMM. By swapping the unknowns, we can obtain the unknown sensitivity values of boundary by the following equation:

$$\boldsymbol{A}\dot{\psi} = \boldsymbol{B}\dot{\phi} - \dot{\boldsymbol{H}}\, p + \dot{\boldsymbol{G}}\, q \tag{13}$$

where $\dot{\psi}$ is the unknown sensitivity values on boundary.

So we obtain the sensitivity value of inner points as

$$\dot{p}_{inner}(x) = \{u\}^T \dot{\psi} + \{k\}^T \dot{\phi} + \{\dot{e}\}^T q - \{\dot{b}\}^T p \tag{14}$$

However for AVM, if we only consider inner points' sensitivity values, according to eqn (13), we have the formulation as follow:

$$\dot{\psi} = \boldsymbol{A}^{-1}\{\boldsymbol{B}\dot{\phi} - \dot{\boldsymbol{H}}\, p + \dot{\boldsymbol{G}}\, q\} \tag{15}$$

Introducing the adjoint equations as follows:

$$\begin{aligned} \boldsymbol{A}^T \eta &= u \\ \{\eta\}^T &= \{u\}^T \boldsymbol{A}^{-1} \end{aligned} \tag{16}$$

Substituting eqns (15) and (16) into eqn (14), the new formulation can be obtained as

$$\dot{p}_{inner}(x) = \{\eta\}^T \{\boldsymbol{B}\dot{\phi} - \dot{\boldsymbol{H}}\, p + \dot{\boldsymbol{G}}\, q\} + \{k\}^T \dot{\phi} + \{\dot{e}\}^T q - \{\dot{b}\}^T p \tag{17}$$

As we can see, eqn (16) is independent with design variables. This means we only need to compute it once even if there are many design variables. And this will significantly improve the efficiency of sensitivity analysis and optimization process.

4. NUMERICAL TESTS

To validate the correctness and efficiency of the proposed approach, some numerical examples are carried out. We consider an infinite rigid cylinder excited by plane wave (wave speed c=340m/s) as shown in Fig. 1.

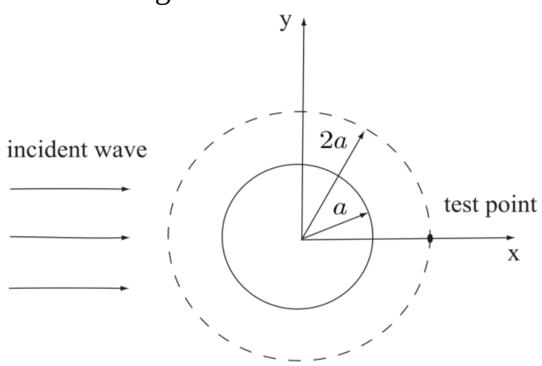

Fig. 1 Scattering from an infinite rigid cylinder

Firstly we compare IGA BEM with the conventional BEM using Lagrange basis functions. As Fig. 2 shows, IGA BEM achieves a higher accuracy compared with constant, discontinuous linear and discontinuous quadratic elements. Then we investigate the acceleration of wideband FMM in Fig. 3. Obviously, by choosing suitable parameter, wideband FMM could achieve higher efficiency with the same accuracy compared with the low frequency FMM. Finally, Fig .4 shows the correctness of the sensitivity analysis. We compare the two analytical methods, i.e., the DDM and AVM, with finite difference method (FDM), and they all get the same sensitivity results with the change of design variables.

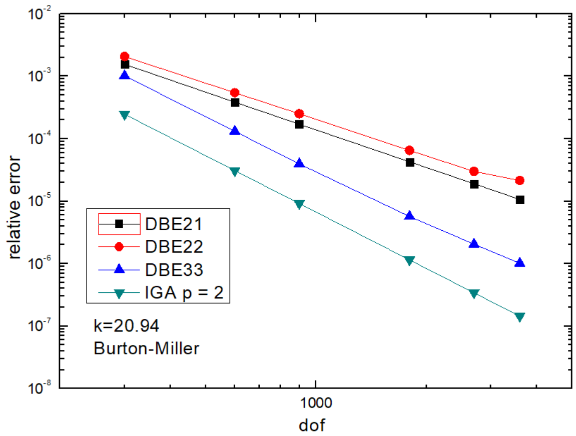

Fig. 2 Comparison of the relative error of discontinued BEM and IGA BEM

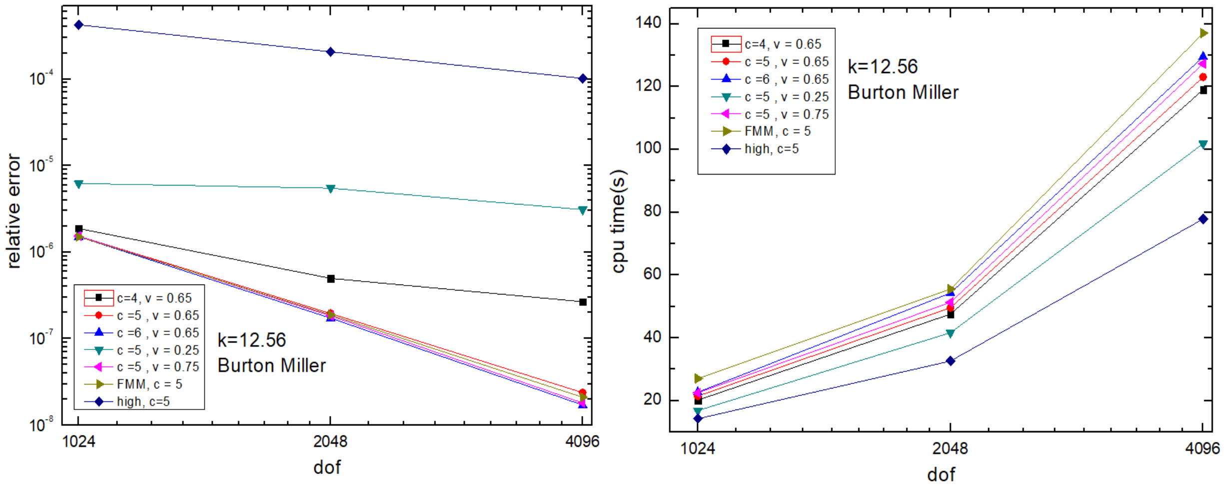

Fig. 3 Comparison of the relative error and CPU time of FMM IGA BEM and wideband FMM IGA BEM

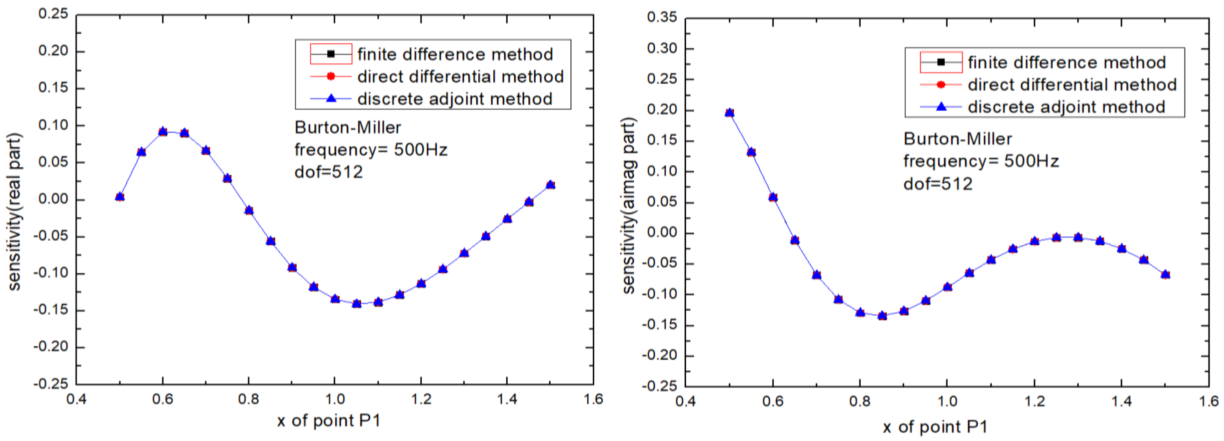

Fig. 4 Sound pressure sensitivities at point (2,0) with respect to the horizontal coordinate of P_0

5. CONCLUSIONS

Numerical tests show that wideband FMM IGA BEM is able to solve acoustic problems with high accuracy and efficiency. And the proposed sensitivity analysis method is able to calculate sensitivity for acoustics efficiently. Although this study only discusses the sensitivity analysis, the optimization work will be carried out in the following research.

6. ACKNOWLEDGEMENTS

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