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THE COMPLEX TECHNIQUE FOR MODELLING VIBROACOUSTIC CHARACTERISTICS OF PIPELINE SYSTEM WITH DAMPING VIBRATION PROTECTION DEVICE

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ABSTRACT

The present paper describes a numeric technique based on the solving non-stationary differential equation system of interaction between solid and oscillating fluid in a pipeline. The finite element model of pipe base on used space-time joint type elements. The joint type of finite element is used for modeling vibroacoustical interaction between solid and oscillating fluid. Time response of the pipeline vibration are resulted from this technique. The boundary conditions for fluid - parameters combination of complex pressure oscillation amplitude of pipeline inlet section, complex pressure oscillation amplitude of pipeline outlet section, complex velocity oscillation amplitude of pipeline inlet section, complex velocity oscillation amplitude of pipeline outlet section, load impedance, input impedance. The boundary conditions for solid are pipeline supports. The pipeline vibration insulator with MR material damper consider like supporting for pipeline and modeling by differential equation system. This equation system is substituted in the weighted residuals and Galerkin methods approximating equations like a boundary conditions. The developed technique allows calculate vibroacoustical characteristics of complex configuration pipeline system with the different type of units.

Keywords: Pulsation, Vibration, Finite Element Model
I-INCE Classification of Subject Number: 43

1. INTRODUCTION

Pipelines are widely used in the different types of system. There are power plants, equipment's, mobile machines, process pipes and other. Pressure pulsation and vibration has a great impact on reliability, durability, efficiency and other operation parameters of pipe system.

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The pipe vibration must be determined before it can be dealt with effectively.

Such studies has a great importance to the different industries where pipes have to withstand high pressure [1-3].

Flow-induced vibration analysis of pipes conveying fluid has been one of the attractive subjects in structural dynamics [4]. The analytical and computational models are used for describing the dynamics of a pipeline system under the force excitation by oscillating fluid flow [5]. Most of them developed for the straight pipes.

Numerical method is widely used for calculation of vibroacoustical characteristics of complex configuration pipe system [6-11].

Extensive studies in this area subject to different boundary conditions and loadings [12-15]. In most cases, the corresponding ordinary differential motion equations of fluid conveyed pipes are deduced using Galerkin's method in Lagrange system. Then many numerical methods, such as transfer matrix method, finite element method, perturbation method, Runge-Kutta method, and differential quadrature method, are applied to solve these equations. One of the major factors affecting the dynamic behavior of pipes is the boundary conditions [16].

In this paper considers two different types the boundary conditions for pipe: rigid supports of the ends of pipe and MR damper installed in the right end of pipe and the rigid support on the left end. These boundary conditions are used for calculating differential equation system has been derived from the mathematical model of the complex configuration piping system dynamics [17]. The developed technique allows calculating a complex configuration pipe system. The technique is developed for pipeline diameter much smaller than acoustic wave-length in a fluid.

2. BASIC GOVERNING EQUATIONS

The technique is based on the solving differential equation system of interaction between solid and oscillating fluid in the pipeline. The differential equation system was derived from the mathematical model of the complex configuration piping system dynamics [18].

The assumption of the model is that the piping system is split into finite length parts with the constant inner and outer piping radiuses, normal and binormal curvature radiuses within their limits. A differential equation system is derived for each part to describe its vibroacoustic parameters. The equation system includes: the equilibrium condition for a curvelinear pipeline part, elastic displacements dependency from forces and moments, acting bulk force equations, and the expression for fluid motion in elastic pipe. The pipeline is considered like a beam. Thus, the flexural vibrations are considered to be prevailing, and the radial cross-section deformations are neglected.

The mathematical model is expressed as equation system:

$$
\begin{aligned}
&\frac{\partial^2 \overline{u}}{\partial \tau^2} = \frac{\partial \overline{Q}'}{\partial \varepsilon} + q + \bar{f} + \overline{\gamma}_1, \quad \frac{\partial p}{\partial \tau} = -B_{mp}\frac{\partial w}{\partial \varepsilon},\\
&\overline{M} = A(\overline{\chi} - \overline{\chi}_0) + \overline{M}_0, \frac{\tilde{\partial} \overline{\omega}}{\partial \varepsilon} - \frac{\tilde{\partial} \overline{\chi}}{\partial \tau} = \overline{\omega} \times \overline{\chi}, \frac{\partial \overline{u}}{\partial \varepsilon} = e_1 - e_{10},\\
&\frac{\partial \overline{M}}{\partial \varepsilon} + e_1 \times \overline{Q}_б' + \overline{\mu} + \sum_{i=1}^{m} \overline{M}_{ci}\delta(\varepsilon - \varepsilon_i) = 0, \quad \bar{f} = f_1 \overline{e}_1 + f_2 \overline{e}_2 + f_3 \overline{e}_3,\\
&n(\frac{\partial^2 \overline{u}}{\partial \tau^2} + w\frac{\partial^2 \overline{u}}{\partial \tau \partial \varepsilon} + \frac{\partial w}{\partial \tau}\left(\overline{e}_1 + \left(\frac{\partial \overline{u}}{\partial \varepsilon}\right)_2 \overline{e}_2 + \left(\frac{\partial \overline{u}}{\partial \varepsilon}\right)_3 \overline{e}_3\right) + w\frac{\partial w}{\partial \varepsilon}\left(\overline{e}_1 + \left(\frac{\partial \overline{u}}{\partial \varepsilon}\right)_2 \overline{e}_2 + \left(\frac{\partial \overline{u}}{\partial \varepsilon}\right)_3 \overline{e}_3\right)) = -\frac{\partial (p\overline{e}_1)}{\partial \varepsilon} + \overline{\gamma}_2 - \bar{f}
\end{aligned}
\tag{1}
$$

w_0 and Δw is the constant and variable parts of the fluid velocity; p and Δp is the constant and variable parts of the fluid pressure; ε is a coordinate, measured along the centroidal line of the pipeline sections from zero point to the arbitrary cross-section; τ

is the non-dimensional time; $e_1(\varepsilon,\tau)$ is a unitary vector, binormal to the piping center line; $e_2(\varepsilon,\tau)$ is a unitary vector, normal to the piping center line; $e_3(\varepsilon,\tau)$ is a unitary vector, binormal to the piping center line; u_i is the vibration displacement in $e_i(\varepsilon,\tau)$ direction; χ is a nonplanar center line curvature vector.

As an example, consider solving differential equation system for pipe with the axial line lying in one plane under force excitation by oscillating fluid flow. There is technique used new 7-node finite element [19].

The differential equation system was derived from the mathematical model of the complex configuration piping system dynamics [20].

The mathematical model is expressed as equation system:

$$\left.\begin{aligned}
&u_2 = \frac{1}{\chi_{30}}\frac{\partial u_1}{\partial \varepsilon},\\
&\chi_{30}^2\frac{\partial^2 u_1}{\partial \tau^2} - \frac{\partial^4 u_1}{\partial \varepsilon^2 \partial \tau^2} - nw\frac{\partial^4 u_1}{\partial \varepsilon^3 \partial \tau} - nw\chi_3^2\frac{\partial^2 u_1}{\partial \varepsilon \partial \tau} - H\chi_3^2\frac{\partial^5 u_1}{\partial \tau^4 \partial \varepsilon} = \frac{\partial^6 u_1}{\partial \varepsilon^6} + \left(\left(p + nw^2\right) + 2\chi_3^2\right)\frac{\partial^4 u_1}{\partial \varepsilon^4} +\\
&+\frac{\partial^2 u_1}{\partial \varepsilon^2}\left(\left(p + nw^2\right) + \chi_{30}^2\right)\chi_{30}^2 + nw\chi_{30}^2\frac{\partial w}{\partial \varepsilon} - n\chi_{30}^2\frac{\partial w}{\partial \tau},\\
&\varphi_3 = \frac{1}{\chi_3}\frac{\partial^2 u_1}{\partial \varepsilon^2} + \chi_3 u_1,
\end{aligned}\right\}, \quad (2)$$

where ε is a nondimensional coordinate, measured along the centroidal line of the pipeline sections from zero point to the arbitrary cross-section; τ is the nondimensional time; $n = \frac{m_2}{m_1 + m_2}$; $m_1(s)$ stands for the pipe mass per meter; $m_2(s)$ is the fluid mass per meter; χ_3 is the piping center line flexure in a plane, normal to e_3; l is the pipeline length; w is a vector of a nondimensial working fluid flow velocity; p is the nondimensional pressure; u_1 is the vibration displacement in $e_1(s,t)$ direction; u_2 is the vibration displacement in $e_1(s,t)$ direction; $H\frac{\partial^5 u_1}{\partial \varepsilon^4 \partial \tau}$ is the friction model where the deformation component, directly proportional to the deformation speed, or the rate of intrisctic restoration force; $H = hEI$; I inertia moment.

Since the Equation 2 has only one variable, explicitly differentiable with respect to time - u_1. It would be appropriate to solve it for this argument. All other variables can be specified from the solution, found for u_1.

3. GALERKIN FINITE ELEMENT METHOD

Solution algorithm for the second equation in the Equation 2 used partial discretization and weighted residual methods [15, 17-19]

$$u_1 \approx \hat{u}_1 = \sum_{m=0}^{M-1} a_m(\tau) N_m(\varepsilon) \quad (3)$$

where $N_m(\varepsilon)$ are the basis functions.

The weighted residuals method approximating equation:

$$\int_\Omega W_l R_\Omega d\Omega + \int_\Gamma \overline{W}_l R_\Gamma d\Gamma = 0 \quad (4)$$

where $R_{\Omega} = Lu_1 + p - -\alpha \dfrac{du_1}{d\tau} - \beta \dfrac{d^2 u_1}{d\tau^2}$ is an approximation residual for the domain; $R_{\Gamma} = Mu_1 + r$ is an approximation residual for the boundary conditions; $W_l, \overline{W_l}$ are the linearly independent weights.

The Lagrange polymonial of 6th order was used as the basis functions, in order to achieve the accurate solution (Figure 1, 2).

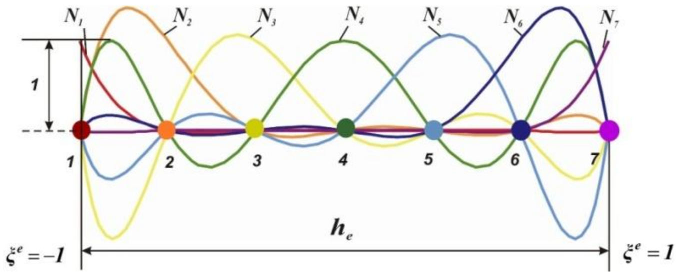


Figure 1. One-dimensional element and the Lagrange basis functions of 6th order.

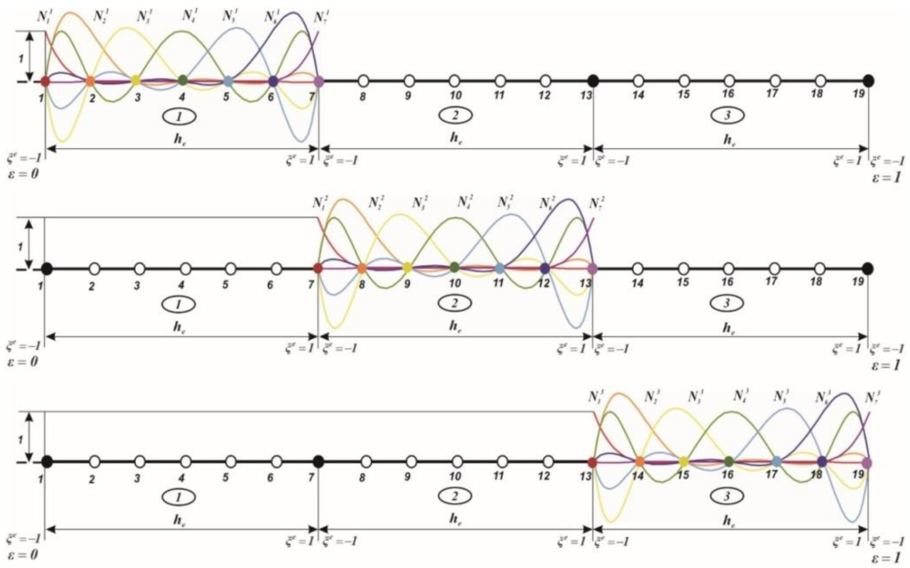


Figure 2. One-dimensional 7-node elements at the spatial domain.

The differential equation system in vectorial form:

$$
\begin{aligned}
&[M]\left[\frac{d^2 u_1}{d\tau^2}\right] + [C]\left[\frac{du_1}{d\tau}\right] + [K][u_1] = [f] \\
&M_{lm} = \int_0^1 \left(\frac{d^2 N_m}{d\varepsilon^2} - \chi_{30}^2 N_m \right) N_l \, d\varepsilon \\
&C_{lm} = \int_0^1 nw \left(\frac{d^3 N_m}{d\varepsilon^3} + \chi_{30}^2 \frac{dN_m}{d\varepsilon} \right) N_l \, d\varepsilon - \int_{-1}^{1} H \chi_{30}^2 N_m N_l \, d\varepsilon \\
&K_{lm} = \int_0^1 \left(\frac{d^6 N_m}{d\varepsilon^6} + (2\chi_{30}^2 + p + nw^2) \frac{d^4 N_m}{d\varepsilon^4} + (\chi_{30}^2 + p + nw^2) \chi_{30}^2 \frac{d^2 N_m}{d\varepsilon^2} \right) N_l \, d\varepsilon \\
&f_l = \int_0^1 n \chi_{30}^2 \left(\frac{\partial w}{\partial \tau} - w \frac{\partial w}{\partial \varepsilon} \right) N_l \, d\varepsilon
\end{aligned}
\tag{5}
$$

The Equation 5 has been solved by the method of basis functions in time domain using the Crank-Nicolson scheme [15].

$$\left[M+\gamma\Delta t_n C(\Delta t_n)+\beta\Delta t_n^2 K(\Delta t_n)\right]\mu_1^{2n+2}+\left[-2M+(1-2\gamma)\Delta t_n C(\Delta t_n)+(1/2-2\beta+\gamma)\Delta t_n^2 K(\Delta t_n)\right]\mu_1^{2n+1}+ \\ +\left[M-(1-\gamma)\Delta t_n C(\Delta t_n)+(1/2+\beta-\gamma)\Delta t_n^2 K(\Delta t_n)\right]\mu_1^{2n}=f^n\Delta t_n^2 \quad (6)$$

4. BOUNDARY CONDITIONS

For the calculating flow-induced vibration was defined pressure pulsation along pipe and pulse transit time. For the steady-state condition, the boundary conditions for the hydraulic subsystem are defined by parameters: fluid oscillation frequency f, pressure pulsation amplitude at the inlet section of the pipe, load impedance *Zload*.

The instantaneous values of fluid pressure and velocity was defined from amplitude-phase-frequency response of pipe with unmatched load and complex load impedance $Z_{load}=\propto$ [20]

$$\frac{p(j\omega,s)}{p(j\omega,o)}=\frac{1}{\frac{Z_{wave}}{Z_{load}}sh[(\delta+j\varepsilon)s]+ch[(\delta+j\varepsilon)s]}, \quad (7)$$

δ - damp coefficient; ε - phase coefficient; ω - wave circular frequency; Z_{wave} - wave impedance of pipe.

$$p=\sum_{i=1}^{k}\frac{F_2 l^2 p_{\text{в}xi}}{A_{33}}\frac{cos\left[\frac{2\pi f_i l}{c}(1-\varepsilon)\right]}{cos\frac{2\pi f_i l}{c}}cos\left(2\pi f_i l^2\sqrt{\frac{m_1+m_2}{A_{33}}}\tau+\varphi_i\right); \quad (8)$$

$$w=\sum_{i=1}^{k}\frac{p_{\text{в}xi}l}{\rho c}\sqrt{\frac{m_1+m_2}{A_{33}}}\frac{sin\left[\frac{2\pi f_i l}{c}(1-\varepsilon)\right]}{cos\frac{2\pi f_i l}{c}}sin\left(2\pi f_i l^2\sqrt{\frac{m_1+m_2}{A_{33}}}\tau+\varphi_i\right); \quad (9)$$

$$\frac{\partial w}{\partial\varepsilon}=-\sum_{i=1}^{k}\frac{2\pi f_i l^2 p_{\text{в}xi}}{\rho c^2}\sqrt{\frac{m_1+m_2}{A_{33}}}\frac{cos\left[\frac{2\pi f_i l}{c}(1-\varepsilon)\right]}{cos\frac{2\pi f_i l}{c}}sin\left(2\pi f_i l^2\sqrt{\frac{m_1+m_2}{A_{33}}}\tau+\varphi_i\right); \quad (10)$$

$$\frac{\partial w}{\partial\tau}=\sum_{i=1}^{k}\frac{2\pi f_i l^3 p_{\text{в}xi}}{\rho c}\frac{m_1+m_2}{A_{33}}\frac{sin\left(\frac{2\pi f_i l}{c}(1-\varepsilon)\right)}{cos\frac{2\pi f_i l}{c}}cos\left(2\pi f_i l^2\sqrt{\frac{m_1+m_2}{A_{33}}}\tau+\varphi_i\right). \quad (11)$$

The boundary conditions for the mechanic (piping) subsystem are defined by differential equations, describing the piping supports.

The left end of a pipe has a rigid support. The right end of pipe has a MR (Metal–Rubber Elastic Porous Material) damper.

The damper is widely used for reduce the vibration in a pipeline systems [21].

The MR material is a two-component heterogeneous system. One component consists of a three-dimensional skeleton made of pieces of wire spirals laid with mutual crossing and pressed in a mold up to the size of the finished product. The second component consists of the free space between the wire, i.e., it is a system of interconnected through pores of various sizes [22,-24]

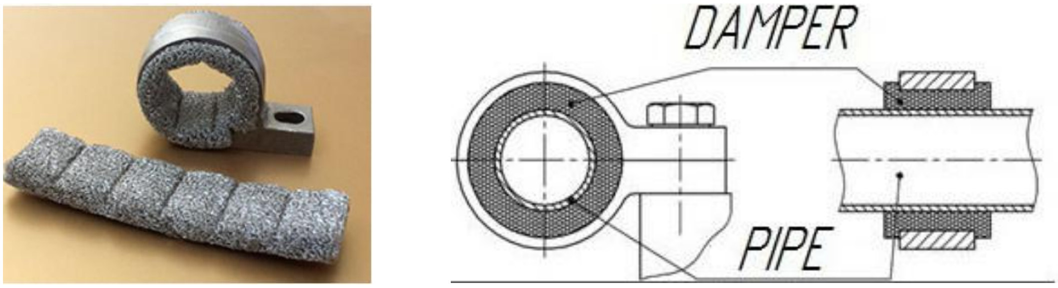


Figure 3. Scheme of the metal rubber(MR) coating

Consider the boundary conditions for the left end of pipe. In this case, at $\varepsilon = 0$ and $\varepsilon = 1$ $u_1 = u_2 = 0$. Then, according to equation system Equation 2, $\frac{\partial u_1}{\partial \varepsilon} = 0$, , $\frac{\partial^2 u_1}{\partial \varepsilon^2} = 0$. Discard the singular index of the u parameter. The boundary conditions for the analyzed finite elements model may be expressed as follows:

$$\begin{aligned} &\sum_{m=0}^{M-1} u_{1m} = 0 \\ &\sum_{m=0}^{M-1} \frac{\partial u_{1m}}{\partial \varepsilon} = 0 \quad \text{at } \varepsilon = 0 \text{ and } \varepsilon = 1 \\ &\sum_{m=0}^{M-1} \frac{\partial^2 u_{1m}}{\partial \varepsilon^2} = 0 \end{aligned} \tag{12}$$

The boundary conditions for the right end of pipe were installed damper can be write:

$$\begin{aligned} &\varphi_3(l,\tau) = 0, \\ &u_2(l,\tau) = D - \chi' \frac{\partial u_2}{\partial t} 0, \\ &\frac{\partial u_2(l,\tau)}{\partial \varepsilon} = 0, \end{aligned} \tag{13}$$

where $D = \frac{\sigma}{E}$, σ stress tensor.

Rewrite Equation 13 with one variable u_1

$$\begin{aligned} &\frac{1}{\chi_{30}} \frac{\partial^2 u_1(l,\tau)}{\partial \varepsilon^2} + \chi_{30} u_1(l,\tau) = 0, \\ &\frac{1}{\chi_{30}} \frac{\partial u_1}{\partial \varepsilon}(l,\tau) = D - \frac{\chi'}{\chi_{30}} \frac{\partial^2 u_1}{\partial \varepsilon \partial t}, \\ &\frac{\partial^2 u_1}{\partial \varepsilon^2} = 0, \end{aligned} \tag{14}$$

Normalized local coordinate ξ more convenient for further calculations. Make up a coordinate ξ that describes positions insted of nondimensional coordinate ε .

$$\xi = \frac{2(\varepsilon - \varepsilon_c^e)}{h^e}, \quad -1 \le \xi \le 1 \tag{15}$$

h^e - element length, ε_c^e - coordinate of element center.

Substitute Equation 3 into Equations 12 and 14

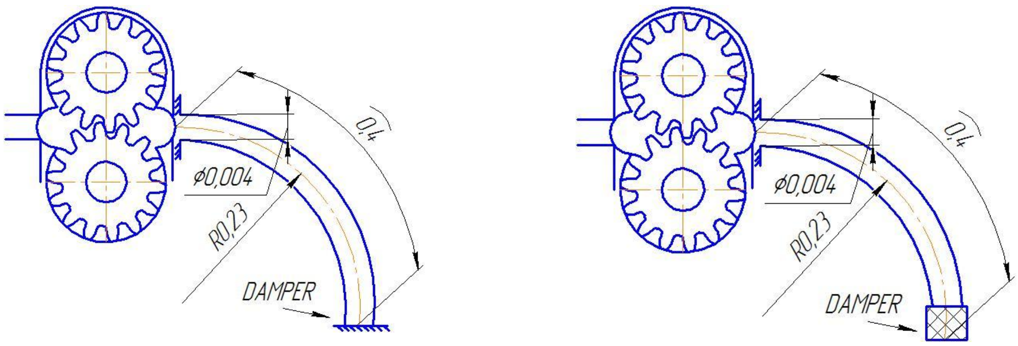
$$
\begin{aligned}
&a_1=0,\\
&a_2\frac{\partial N_2(-1)}{\partial\xi}+a_3\frac{\partial N_3(-1)}{\partial\xi}+a_4\frac{\partial N_4(-1)}{\partial\xi}+a_5\frac{\partial N_5(-1)}{\partial\xi}+a_6\frac{\partial N_6(-1)}{\partial\xi}+a_7\frac{\partial N_7(-1)}{\partial\xi}=0,\\
&a_2\frac{\partial^2 N_2(-1)}{\partial\xi^2}+a_3\frac{\partial^2 N_3(-1)}{\partial\xi^2}+a_4\frac{\partial^2 N_4(-1)}{\partial\xi^2}+a_5\frac{\partial^2 N_5(-1)}{\partial\xi^2}+a_6\frac{\partial^2 N_6(-1)}{\partial\xi^2}+a_7\frac{\partial^2 N_7(-1)}{\partial\xi^2}=0,\\
&\frac{1}{\tilde{\chi}_{30}}\left(a_2\frac{\partial^2 N_2(1)}{\partial\xi^2}+a_3\frac{\partial^2 N_3(1)}{\partial\xi^2}+a_4\frac{\partial^2 N_4(1)}{\partial\xi^2}+a_5\frac{\partial^2 N_5(1)}{\partial\xi^2}+a_6\frac{\partial^2 N_6(1)}{\partial\xi^2}+a_7\frac{\partial^2 N_7(1)}{\partial\xi^2}\right)+\\
&+\chi_{30}\left(a_2N_2(1)+a_3N_3(1)+a_4N_4(1)+a_5N_5(1)+a_6N_6(1)+a_7N_7(1)\right)=0,\\
&\frac{1}{\tilde{\chi}_{30}}\left(a_2\frac{\partial N_2(1)}{\partial\xi}+a_3\frac{\partial N_3(1)}{\partial\xi}+a_4\frac{\partial N_4(1)}{\partial\xi}+a_5\frac{\partial N_5(1)}{\partial\xi}+a_6\frac{\partial N_6(1)}{\partial\xi}+a_7\frac{\partial N_7(1)}{\partial\xi}\right)-D+\\
&+\frac{\chi'}{\chi_{30}}\left(\frac{\partial a_2}{\partial\tau}\frac{\partial N_2(1)}{\partial\xi}+\frac{\partial a_3}{\partial\tau}\frac{\partial N_3(1)}{\partial\xi}+\frac{\partial a_4}{\partial\tau}\frac{\partial N_4(1)}{\partial\xi}+\frac{\partial a_5}{\partial\tau}\frac{\partial N_5(1)}{\partial\xi}+\frac{\partial a_6}{\partial\tau}\frac{\partial N_6(1)}{\partial\xi}+\frac{\partial a_6}{\partial\tau}\frac{\partial N_7(1)}{\partial\xi}\right)=0,\\
&a_2\frac{\partial^2 N_2(1)}{\partial\xi^2}+a_3\frac{\partial N^2{}_3(1)}{\partial\xi^2}+a_4\frac{\partial^2 N_4(1)}{\partial\xi^2}+a_5\frac{\partial^2 N_5(1)}{\partial\xi^2}+a_6\frac{\partial^2 N_6(1)}{\partial\xi^2}+a_7\frac{\partial^2 N_7(1)}{\partial\xi^2}=0
\end{aligned}
\tag{16}
$$

Initial conditions defined only for the mechanical subsystem, because of steady state oscillations. Initial conditions for the mechanical subsystem are the variable u_1 and its derivative at the initial time $a(\tau=0)=a^0=0$, $\frac{da}{dt}(\tau=0)=0$.

Equation 16 are substituted in Equation 5.

5. NUMERICAL RESULTS

As a case in point, take the pipeline with the following parameters: l=0,4 m; d=0,004 m; δ=0,0006 m; ρ=7800 kg/m^3, E=2·10^{11} Pa; R=0,23 m; where d stands for the outside pipe diameter, δ stands for wall thickness; R is the curvature radius.



a) b)

Figure 4. Piping supports a) rigid supports, b) rigid support and MR damper

The pipeline is loaded with the steady-state fluid oscillations with the following parameters: f=250 Hz, рвх1=10^5 Pa. Fluid parameters: ρ_{fluid}=870 kg/m^3, c=1300 m/s.

Let take one seven-node element and a time step $\Delta t=0{,}014$ for the calculations. The initial data, required for the analyzed Crank-Nicolson scheme, is prescribed as follows:

$$u_1(\tau=0)=u_1{}^0=0,\ \frac{du_1}{dt}(\tau=0)=0 \tag{17}$$

The following values are chosen: $\beta=\frac{1}{4}$, $\gamma=\frac{1}{2}$. The scheme is unconditionally stable with these values and does not produce the artificial numeric attenuation.

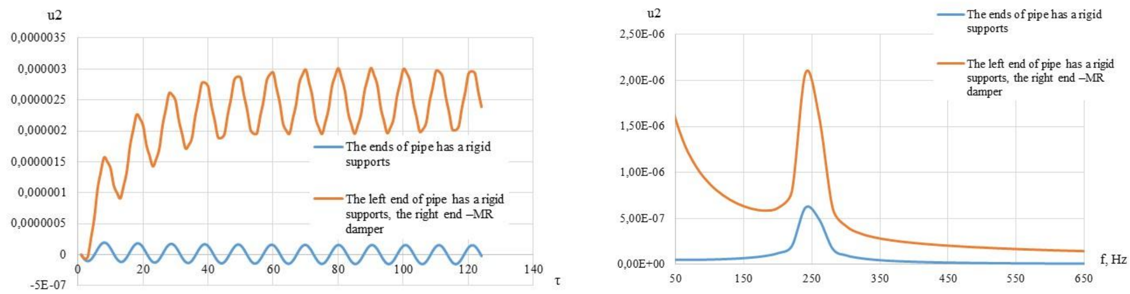


Figure 5. Time response and amplitude spectrum of the non-dimensional normal vibration displacement in the 4 node $\varepsilon = 0.2$ *with the friction parameter H=2*

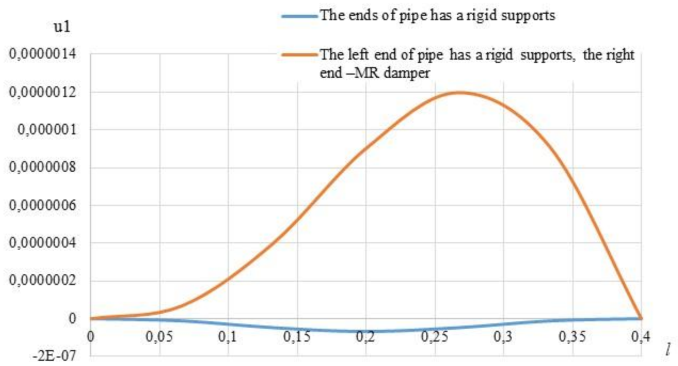


Figure 6. The non-dimensional normal vibration displacement in the 4 node $\varepsilon = 0.2$ *with the friction parameter H=2*

This type of pipe fixation consider as example for show how we can install the MR damper in the pipe system for our finite element model. In the Figure 5 are shown time response and amplitude spectrum of the non-dimensional normal vibration displacement in the 4 node $\varepsilon = 0.2$ with the friction parameter *H*=2. The non-dimensional normal vibration displacement in the 4 node $\varepsilon = 0.2$ with the friction parameter *H*=2 is shown in the Figure 6. Vibration amplitude for the pipe has MR damper on the right end higher than for the pipe with rigid supports because on the right end a pipe has elastic support.

6. CONCLUSIONS

The paper describes a numeric technique based on the solving non-stationary differential equation system of interaction between solid and oscillating fluid in a pipeline. The finite element model of pipe base on used space-time joint type elements.. As an example, consider solving differential equation system for pipe with the axial line lying in one plane under force excitation by oscillating fluid flow. The boundary conditions for fluid – fluid oscillation frequency f, pressure pulsation amplitude at the inlet section of the pipe, load impedance *Zload.* The boundary conditions for solid are different types of pipeline supports.

The paper considers two different types of pipeline supports: rigid supports of the ends of pipe and MR damper installed in the right end of pipe and the rigid support on the left end.

The pipeline vibration insulator with MR material damper modelling by differential equation system. This equation system is substituted in the weighted residuals and Galerkin methods approximating equations like a boundary conditions.

The developed technique allows calculate vibroacoustical characteristics of complex configuration pipeline system with the different type of units.

The pipeline vibration was calculated for the 2 types of supports. Calculating results was comparing each other. Time responses of the pipe vibration are resulted from this technique.

7. ACKNOWLEDGEMENTS

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