Cut-off frequency for curvilinear acoustic black holes

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ABSTRACT

Acoustic black hole (ABH) is a thin wedge-shaped structure whose thickness profile is tapered according to the power-law with power greater than or equal to two. Due to its geometry, the ABH slows down the group speed of incident flexural waves, thereby focusing the wave energy on its tip. Recently, a new design of the ABH with baseline in curvilinear shape was proposed in order to enhance the space efficiency while maintaining the wave attenuation performance. By means of numerical simulations, it was shown that the damping performance of the ABH was maintained above a certain critical frequency, the 'cut-off frequency'. Although the proposed curvilinear ABH is a compact and effective solution to vibration damping in thin beams and plates, it is important to avoid using the curvilinear ABH below the cut-off frequency since the damping performance of the curvilinear ABH is not the same as that of the straight conventional ABH. In this study, by modeling the wave motion within the curvilinear ABH mathematically, we investigate the cut-off frequency for curvilinear ABHs. Specifically, we investigate the effect of geometrical parameters on the cut-off frequency for the arc ABHs and the spiral ABHs. This study ultimately aims at designing space-efficient curvilinear ABHs suitable for various industrial applications.

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1. INTRODUCTION

Effective attenuation of structural vibration in thin structures such as plates or beams by using lightweight techniques is an important consideration in designing a system. A decade ago, a passive technique to dampen the vibration of the thin structures named Acoustic Black Hole (ABH) was established [1,2], and many researchers have been actively investigating the ABH to apply its remarkable feature to practice [3-6]. Basically, the ABH is a thin structure with thickness tailored in the form of the power-law function of power greater than or equal to two whose tip region is covered with a viscoelastic damping layer. Due to the combined effect of the wedge of power-law profile and the damping layer, the ABH effectively attenuates the vibration energy within the thin structure by slowing and absorbing the flexural waves incident upon it.

Although the vibration attenuation performance of the ABH can be enhanced by increasing its length, [2] the space available for the ABH is limited in most applications, which makes it less practical to use the ABH with a long length. To resolve this, revently, Lee and Jeon [7,8] proposed a new design of the ABH with its baseline curved in Archimedean spiral shape to attenuate the flexural vibration in beams compactly. By means of computational simulations, they showed that it is possible to effectively attenuate the structural vibrations in beams using the compact spiral ABH. To understand this remarkable and non-intuitive feature of the curvilinear ABH, they also investigated the frequency response functions of ABHs with baselines in circular arc shape, which have constant curvatures. It was shown that above a certain critical frequency, the 'cut-off frequency', the frequency and magnitude of the frequency reponse functions are not severely changed even though their shapes are distinct from each other.

In our ongoing research, we attempt to investigate the extraordinary features of the curvilinear ABH such as the 'cut-off frequency' of the circular arc ABH analytically. For the investigation, a mathematical model of wave motion in elastic structures with varying thickness and arbitrary curvature is required. In this paper, the mathematical model is proposed and verified with computational simulations.

2. MATHEMATICAL MODEL OF WAVE MOTION IN CURVILINEAR ABH

In this section, the governing equation that governs the elastic waves in curvilinear ABH is established mathematically. The underlying assumptions and the form of the derived governing equations are given without detailed mathematical procedures for the sake of brevity.

We consider a thin elastic structure whose thickness and curvatrue vary along its curved centerline, denoted as s-axis. To derive the governing equation for the thin structure, we employ the Kirchhoff's hypothesis and the linear elastic Hooke's law as constitutive law. The Kirchhoff hypothesis is an assumption which states that the cross-section perpendicular to the neutral surface of a thin elastic structure remains perpendicular to the surface even in the process of deformation.

We derive the explicit expression for the kinetic and potential energies for a small volume of the thin structure with respect to the displacements for the centerline: transverse normal displacement w and the longitudinal tangential displacement u. The governing equations can then be obtained through Hamilton's principle. Derived equations are written in Equations (1) and (2):

$$\frac{d}{ds}\left\{EA\left(\frac{du}{ds}+\frac{w}{R}\right)\right\}+\rho\omega^2 Au=0, \quad (1)$$

$$\frac{EA}{R}\left(\frac{du}{ds}+\frac{w}{R}\right)+EAC\left(\frac{d^2w}{ds^2}+\frac{w}{R^2}\right)-\rho\omega^2 Aw+\frac{d^2}{ds^2}\left\{EACR^2\left(\frac{d^2w}{ds^2}+\frac{w}{R^2}\right)\right\}=0. \quad (2)$$

Here, E is the Young's modulus, ρ is the volume density of mass, and ω is the angular frequency. R denotes the radius of curvature, A denotes the cross-sectional area and C is a non-dimensional constant which is a function of the ration between the thickness and the radius of curvature.

The derived equations are coupled ordinary differential equations with 4th order in terms of w and 2nd order in terms of u. The equation is suitable for studying the wave behavior in the curvilinear ABH since A and R are inside the differential operators. Also, note that the normal motion w is coupled with the tangential motion u. This implies that the flexural motion inside the curvilinear ABH is closely coupled with the longitudinal motion due to the presence of the curvature.

3. MODEL VERIFICATION VIA FINITE ELEMENT SIMULATION

In this section, we verify the model presented in Section 2 by comparing the frequency response function obtained from the mathematical model with that from a numerical simulation.

The commercial finite element software COMSOL Multiphysics 5.2a was used as the tool for the comparison. The governing equation for the numerical simulation is the two-dimensional Navier's equation for linear elastic solids. Since the structure under consideration is assumed to be thin, the plane stress assumption was employed. The size of the elements were made sufficiently small compared to the wavelength at the location especially near the tip region of the ABH.

Figure 1 (a) shows the schematic of the curvilinear ABH that was used in the comparison. Important geometrical dimensions for the curvilinear ABH such as the thickness profile $h(s)$ and the arc length L are written in Figure 1 (a) as well. Free boundary conditions were imposed on all the edges of the structure, and a harmonic point force was applied at the mid-point of the attached beam. The driving point mobility of the curvilinear ABH with the beam is shown in Figure 1 (b) for the theoretical model (blue solid line) and the finite element simulation result (black dashed line).

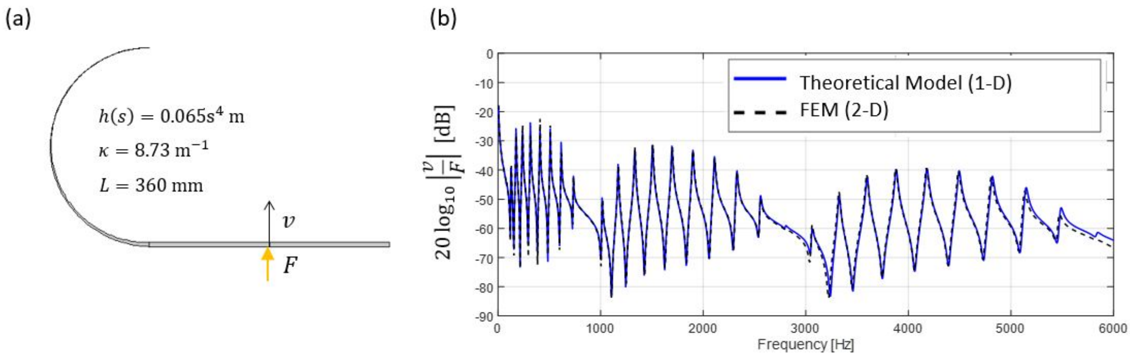

Figure 1: (a) Schematic of the circular arc ABH with curvature as 8.41 m^{-1}. (b) Comparison between the results of the mathematical model presented in Section 2 and the two-dimensional numerical simulation.

As shown in Figure 1 (b), both the magnitude and the frequency of the peaks in the frequency response function obtained from the theoretical model are in good agreement with the that obtained from a two-dimensional finite element simulation up to 6000 Hz.

4. CONCLUSIONS

The concept of the curvilinear ABH was proposed in a recent paper by Lee and Jeon. To utilize the extraordinary features of compact and effective vibration absorber, a systematic approach should be established in order to design the compact curvilinear ABH suitable for various applications. The cut-off frequency of the circular arc ABH could be one important measure in designing the curvilinear ABH. In doing so, a mathematical model that describes the wave motion in elastic structures with varying thickness and arbitrary curvature with accuracy should be fomulated. In this coference proceeding, the mathematical model was derived and was verified via computational simulation result for a circular arc ABH.

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