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A Rainbow Metamaterial For Broadband Multi-frequency Vibration Suppression

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ABSTRACT

In this study, we propose a rainbow metamaterial to achieve multi-frequency broadband vibration suppression. A U-shaped beam is partitioned into subspace by parallel baffle plates. Cantilever-mass microstructures are then attached to the each subspace of the composite beam to tune its vibration. Instead of being uniform, these vibration absorbers are rainbow-shaped for the purpose of suppressing vibration within broad frequency band. An analytical model is first developed to estimate the frequency response function of the composite beam. The interaction forces between cantilevers and the beam are calculated by solving the displacement of the mass absorbers. The baffle plates are considered as attached mass with both vibration and rotation considered. Subsequently, the analytical model is validated by comparison with finite element models and experimental results. On the basis of the analytical model, numerical study is conducted to explore the influences of mass distributions on the frequency response property of the composite beam. Results show that composite beam with rainbow-shaped mass posses broad stopband than that with uniform mass. Multi-frequency range vibration suppression can be also achieved for beams with rainbow-shaped mass on both the left and right sides.

Keywords: Metamaterial, Rainbow, Vibration suppression, Multi-frequency
I-INCE Classification of Subject Number: 46

1. INTRODUCTION

Metamaterials are a new class of artificial composites engineered to have transcendental properties that cannot found from natural materials. In the past decades, metamaterials have attracted much attention in many research fields. Metamaterials are originally introduced to tailor the electromagnetic optical waves [1-4]. Nowadays, the

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concept of metamaterial has expanded to the areas of acoustic/elastic metamaterials. Unusual properties such as negative magnetic permeability, electric permittivity and negative refractive index are achieved by electromagnetic metamaterials. Similarly, negative mass [5] and negative bulk modulus [6] can be seen from elastic/ acoustic metamaterials.

One of the most impressive features of elastic/acoustic metamaterials is the existence of bandgaps within which no waves can propagate. These bandgaps are mainly caused by two phenomena, Bragg scattering and local resonance. Bragg scattering happens when the wavelength of propagating wave is of the same magnitude as the unit cell constants. In contrast, locally resonant bandgap relies on the resonance of internal resonators. As a result, the local resonance bandgap is at frequencies lower than that of Bragg scattering bandgap. Many local resonators are proposed to construct locally resonant metamaterials, such as inclusions coated with soft rubber [5], Helmholtz resonator [7], cantilever beam resonator [8] and membrane with attached mass [9].

Although locally resonant metamaterials are applicable for manipulating wave propagation and providing low-frequency vibration attention, broad bandgaps are hard to be achieved by metamaterials. Few researchers sought to metamaterials with spatially varying resonators as a method of enlarging the width of bandgap. Sun *et al*. [10] and Pai [11] investigated elastic metamaterials composed of beams or bars with spatially varying mass-spring-damper subsystems. These investigations proved that metamaterials with properly designed spatially varying local resonators can achieve broader bandgap than that with uniform resonators.

In order to enlarge the bandgap of locally resonant metamaterials, an elastic metamaterial that constructed by U-shaped beams with rainbow-shaped cantilever-mass resonators is developed in the present paper. An analytical model is proposed to solve the frequency response function of the rainbow metamaterial. It is found that the bandwidth of rainbow metamaterials is broader than that of uniform beams. Furthermore, the two resonators in each subspace of the U-shaped beams are non-symmetric, so that multi-frequency bandgaps are obtained.

2. COMPOSITE BEAM WITH RAINBOW-SHAPED RESONATORS

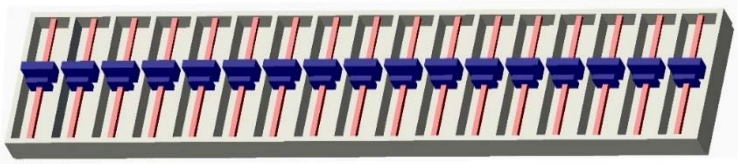

Fig 1. Schematic of composite beam with rainbow-shaped resonators

Figure 1 shows the schematic of the composite beam with rainbow resonators separated by baffle plates. A U-shaped beam is partitioned into subspaces by periodic parallel plates. Two cantilever-mass resonators are attached to each subspace for the purpose of vibration suspension. Instead of uniform resonators, the resonators are non-uniformly distributed resonators along the length of the beam, namely rainbow-shaped resonators. Moreover, the two resonators in each subspace are non-symmetric, which will introduce multi bandgaps. An analytical model is set up in this section to figure out the receptance function of the rainbow metamaterial.

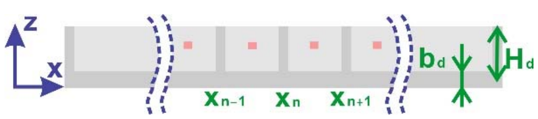

Fig. 2 Side view of U-shaped beam with rainbow resonators partitioned by baffle plates

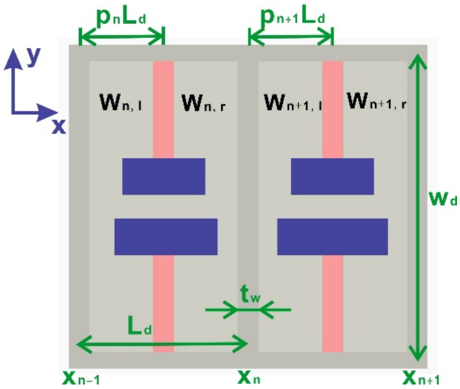

Fig. 3 Top view of composite beam with rainbow resonators

As a Euler-Bernoulli beam, displacements of the U-shaped beam inside the nth segment before and after the cantilever-mass can be written as,

$$w_{n,l} = \alpha_{n,l} e^{-ik(x-x_{n-1})} + \beta_{n,l} e^{-k(x-x_{n-1})} + \chi_{n,l} e^{ik(x-x_{n-1})} + \varepsilon_{n,l} e^{k(x-x_{n-1})} \tag{1}$$

$$w_{n,r} = \alpha_{n,r} e^{-ik(x-(x_{n-1}+p_nL_d))} + \beta_{n,r} e^{-k(x-(x_{n-1}+p_nL_d))} + \chi_{n,r} e^{ik(x-(x_{n-1}+p_nL_d))} + \varepsilon_{n,r} e^{k(x-(x_{n-1}+p_nL_d))} \tag{2}$$

where $k = (\rho A / EI_z)^{1/4} \sqrt{\omega}$, $A = w_d b_d + 2t_d H_d$ represents the cross section area of the U-shaped beam. I_z is the cross section moment of inertia of the U-shaped beam about its centroidal axis, given as

$$I_z \approx \frac{H_d^4 t_d^2 + 2t_d H_d^3 w_d b_d}{3A} \tag{3}$$

The relation between the two displacements is derived as

$$\begin{bmatrix} \alpha_{n,r} \\ \beta_{n,r} \\ \chi_{n,r} \\ \varepsilon_{n,r} \end{bmatrix} = \mathbf{R}_c^{-1} \mathbf{R}_n \mathbf{\Lambda}_{n,l} \begin{bmatrix} \alpha_{n,l} \\ \beta_{n,l} \\ \chi_{n,l} \\ \varepsilon_{n,l} \end{bmatrix} \tag{4}$$

where the matrices $\mathbf{R}_c$, $\mathbf{R}_n$ and $\mathbf{\Lambda}_{n,l}$ are given as

$$\mathbf{R}_c = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -i & -1 & i & 1 \\ -1 & 1 & -1 & 1 \\ iEI_z k^3 & -EI_z k^3 & -iEI_z k^3 & EI_z k^3 \end{bmatrix}$$

$$\mathbf{R}_n = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -i & -1 & i & 1 \\ -1 & 1 & -1 & 1 \\ (iEI_z k^3 + N_{s,n}) & (-EI_z k^3 + N_{s,n}) & (-iEI_z k^3 + N_{s,n}) & (EI_z k^3 + N_{s,n}) \end{bmatrix} \tag{5}$$

$$\mathbf{\Lambda}_{n,l} = \mathrm{diag}\left(e^{-ikp_nL_d}, e^{-kp_nL_d}, e^{ikp_nL_d}, e^{kp_nL_d} \right)$$

Displacements of the U-shaped beam inside $(n+1)$th unit cell are written as,

$$w_{n+1,l} = \alpha_{n+1,l} e^{-ik(x-x_n)} + \beta_{n+1,l} e^{-k(x-x_n)} + \chi_{n+1,l} e^{ik(x-x_n)} + \varepsilon_{n+1,l} e^{k(x-x_n)} \tag{6}$$

Displacement recursion formulas between nth and $(n+1)$th unit cell is given as,

$$\begin{bmatrix} \alpha_{n+1,l} \\ \beta_{n+1,l} \\ \chi_{n+1,l} \\ \varepsilon_{n+1,l} \end{bmatrix} = \mathbf{R}^{-1}\mathbf{U}\mathbf{\Lambda}_{n,r} \begin{bmatrix} \alpha_{n,r} \\ \beta_{n,r} \\ \chi_{n,r} \\ \varepsilon_{n,r} \end{bmatrix} \tag{7}$$

where $\mathbf{R}$, $\mathbf{U}$ are

$$\mathbf{\Lambda}_{n,r} = \mathrm{diag}\left(e^{-ik(1-p_n)L_d}, e^{-k(1-p_n)L_d}, e^{ik(1-p_n)L_d}, e^{k(1-p_n)L_d}\right)$$

$$\mathbf{R} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -i & -1 & i & 1 \\ -EI_z k^2 & EI_z k^2 & -EI_z k^2 & EI_z k^2 \\ iEI_z k^3 & -EI_z k^3 & -iEI_z k^3 & EI_z k^3 \end{bmatrix} \tag{8}$$

$$\mathbf{U} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -i & -1 & i & 1 \\ \left(-EI_z k^2 + iJ_f\omega^2 k\right) & \left(EI_z k^2 + J_f\omega^2 k\right) & \left(-EI_z k^2 - iJ_f\omega^2 k\right) & \left(EI_z k^2 - J_f\omega^2 k\right) \\ \left(iEI_z k^3 + m_f\omega^2\right) & \left(-EI_z k^3 + m_f\omega^2\right) & \left(-iEI_z k^3 + m_f\omega^2\right) & \left(EI_z k^3 + m_f\omega^2\right) \end{bmatrix}$$

Assuming the finite composite beam is of free-free boundary and subjected to an excitation F at one end, given the equilibrium conditions, governing equations at the two ends are,

$$F + m_f\omega^2 w_{1,l}\big|_{x=x_0} = EI_z \left.\frac{\partial^3 w_{1,l}}{\partial x^3}\right|_{x=x_0} \tag{9}$$

$$-J_f\omega^2 w'_{1,l}\big|_{x=x_0} = EI_z \left.\frac{\partial^2 w_{1,l}}{\partial x^2}\right|_{x=x_0} \tag{10}$$

$$EI_z \left.\frac{\partial^3 w_{m,r}}{\partial x^3}\right|_{x=L} + m_f\omega^2 w_{m,r}\big|_{x=L} = 0 \tag{11}$$

$$EI_z \left.\frac{\partial^2 w_{m,r}}{\partial x^2}\right|_{x=L} - J_f\omega^2 w'_{m,r}\big|_{x=L} = 0 \tag{12}$$

The receptance function of the composite beam with absorbers is defined as,

$$R_{ec} = 20\log_{10}\left|\frac{w_{m,r}\big|_{x=L}}{F}\right| \tag{13}$$

3. EXPERIMENT

The analytical model is verified by comparing with the experimental results. The tested samples are fabricated by additive manufacturing method. The receptance function of printed samples are measured by a mechanical shaker.

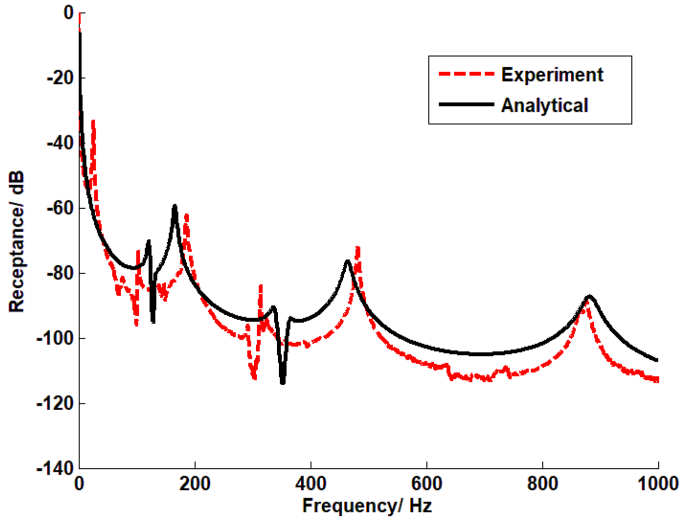

Fig. 4 Comparison between experimental and analytical results

4. NUMERICAL STUDY

Numerical study is conducted in this section to explore the influence of rainbow cantilever-mass resonators on the receptance functions of the composite beams.

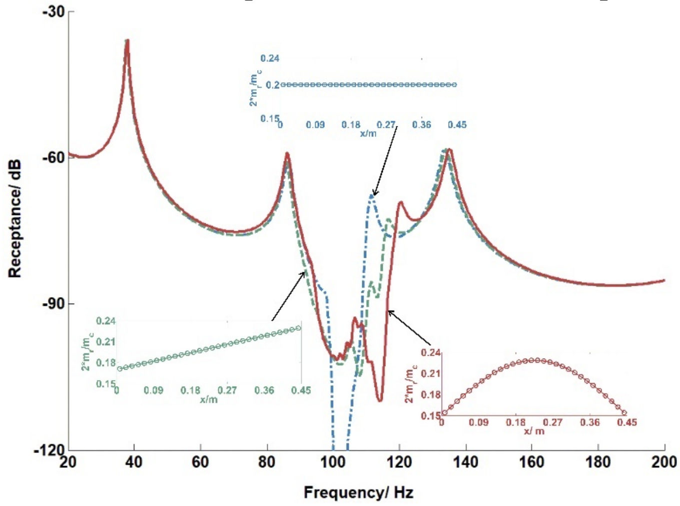

Fig. 5 Receptance function of composite beams with uniform, linearly varying and sinusoidally varying cantilever-mass resonators.

For composite beams with spatially varying resonators, their receptance functions are compared with that of composite beam with uniform resonators in Fig. 5. Two typical non-uniform distributions, linearly and sinusoidally varying mass are considered. The two resonators in each subspace are symmetrical for simplicity. The total mass of resonators in the three beams are identical. It can be seen from Fig. 5 that compared with composite beam with uniform mass, the beams with non-uniform mass have bigger bandwidth.

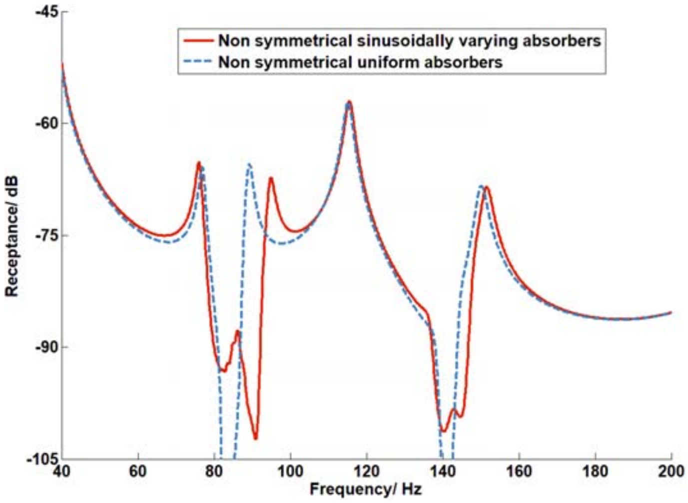

Fig. 6 Receptance functions of composite beams with non-symmetric sinusoidally varying and uniform cantilever-mass resonators.

Figure 6 compares receptance function of composite beams with non-symmetric sinusoidally varying and non-symmetric periodic cantilever-mass resonators. Total mass of the resonators in the two beams is identical. It can be seen from Fig. 6 that the composite beams with non-symmetric cantilever-mass resonators has two bandgaps. In addition, Fig. 6 shows that the bandwidth is enlarged with sinusoidally varying absorbers. Broadband multi-frequency range attenuation can be achieved with beams with non-symmetrical rainbow-shaped cantilever-mass resonators.

4. CONCLUSIONS

A rainbow metamaterial to proposed in the present paper to improve the width of bandgap caused by local resonators. The metamaterial is composed of a U-shaped beam with rainbow-shaped cantilever-mass resonators partitioned by baffle plates. An analytical model as well as experimental measurement is proposed to calculate the receptance function of the rainbow metamaterial. It is found that the rainbow shaped resonators can enlarge the bandgap than uniform resonators.

5. ACKNOWLEDGEMENTS

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