

Applications and Limitations of a Four-Microphone Impedance Tube in the Liquid Media in the Low-Frequency Range

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ABSTRACT

A four-microphone impedance tube is generally used for the absorption measurements of an acoustic material in the gaseous medium, as presented in the standard ASTM E2611. Corresponding dimensional requirements for the experimental setup can be adapted for the application in a liquid medium. However, the standard does not account for the waveguide-related wavefront curvature in the impedance tube with an elastic wall. In this paper a four-microphone impedance tube is validated for the low-frequency transmission-loss measurements in liquids, complying with the restriction of plane-wave sound propagation. Furthermore, the transmission-loss calculation requires knowledge of the acoustic properties of the propagation medium. Therefore, methods for on-site measurements of the group velocity and the complex wavenumber, which includes phase velocity and attenuation constant, were investigated. Existing methods, applicable to the liquid-filled impedance tube, were compared to the new approach of problem-specific methods. Tube termination related limitations were observed and analysed.

Keywords: impedance tube, sound velocity, attenuation
I-INCE Classification of Subject Number: 26

1. INTRODUCTION

In this paper an impedance tube in configuration with four pressure sensors will be discussed. Scope of use includes measurements of the sound transmission through absorption materials, predominantly in a gaseous medium as stated in the standard ASTM

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E2611 [1]. On the contrary, implementation of the impedance tube into a liquid medium, especially in the low-frequency range, is to a large extent still unexplored. The natural modes of the tube, which is an elastic waveguide, interact with propagating pressure wave by way of the fluid-structure interaction. This can cause a significant radial particle oscillation and a wavefront curvature, as opposed to the required plane wave propagation, which is the fundamental requirement of all methods for evaluating acoustic properties in the four-microphone impedance tube [2–7]. Waveguide effects in the water-filled tube were investigated by Wilson *et al.* [8], based on axi-symmetric modes obtained by Del Grosso [9]. Therefore, an acoustically rigid impedance tube was designed based on their findings.

Transmission loss analysis also requires the knowledge of acoustic properties of the medium, especially group sound velocity and complex wavenumber. In present study the impedance tube is submerged into a transformer oil with unknown acoustic parameters. Thus, the main objective of this paper is investigation and development of methods for on-site measurements of sound velocity and wavenumber in liquids. Foremost, methods should be directly applicable to the impedance tube without any changes to the standard experimental setup.

In Section 3 group velocity is discussed, including two general methods of time difference and cross-correlation analysis, and the minima-frequencies-difference approach adapted accordingly from Wang *et al.* [10]. Wavenumber extraction methods are inspected in Section 4, with emphasis on analytical formulation from the work of Wilson *et al.* [8], broadband spectroscopy by Peters *et al.* [11] and amplitude-matching method introduces in previous work [12]. Most accurate results are produced by cross-correlation and amplitude-matching analysis, with certain restrictions regarding tube's termination.

Furthermore, in Section 4.1 potential waveguide-related distortions of plane wave propagation are investigated. Tube's design is validated on the basis of measured phase and group velocity.

2. FOUR-MICROPHONE IMPEDANCE TUBE

Standardized four-microphone impedance tube consists of a acoustically rigid tube, with pressure transducer on one end and arbitrary termination on the other (Figure 1). Under assumption of plane wave propagation, pressure field on each side of absorption material of thickness d is defined as:

$$p(x < 0) = A e^{-jkx} + B e^{+jkx}, \quad (1)$$

$$p(x > d) = C e^{-jkx} + D e^{+jkx}, \quad (2)$$

where k represents complex wavenumber. Pressure amplitudes A , B , C , D can then be extracted from transfer functions $H_{n,\text{ref}}$ of pressure signal from sensor n at location x_n

with the following equations [1]:

$$A = j \frac{H_{1,\text{ref}} e^{+jkx_2} - H_{2,\text{ref}} e^{+jkx_1}}{2 \sin [k(x_1 - x_2)]}, \quad (3)$$

$$B = j \frac{H_{2,\text{ref}} e^{-jkx_1} - H_{1,\text{ref}} e^{-jkx_2}}{2 \sin [k(x_1 - x_2)]}, \quad (4)$$

$$C = j \frac{H_{3,\text{ref}} e^{+jkx_4} - H_{4,\text{ref}} e^{+jkx_3}}{2 \sin [k(x_3 - x_4)]}, \quad (5)$$

$$D = j \frac{H_{4,\text{ref}} e^{-jkx_3} - H_{3,\text{ref}} e^{-jkx_4}}{2 \sin [k(x_3 - x_4)]}. \quad (6)$$

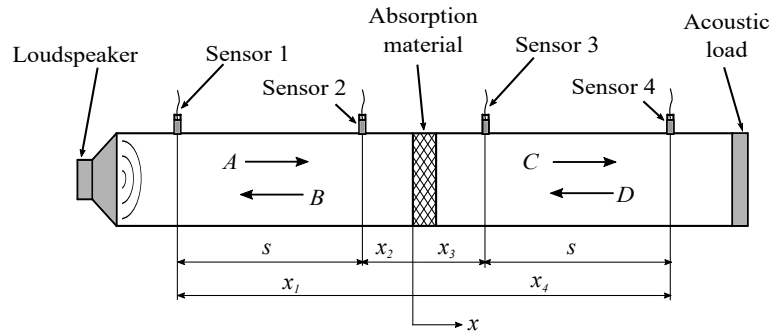


Figure 1: The four-microphone impedance tube based on ASTM E2611.

A liquid-filled impedance tube intended for the use in the low-frequency range is associated with problems related to the tube design and waveguide effects. Firstly, the size of the structure should allow sufficient distance between the pressure sensors s as to achieve necessary accuracy of the measurements. According to Bodén and Åbom [13] the pressure transfer functions are the least susceptible to the bias error when s equals a quarter of a wavelength. Peng *et al.* [14] reported that error increase is insignificant in the wider interval of $0.05c < fs < 0.4c$, while ASTM E2611 [1] expands the lower limit to $0.01c < fs$.

Furthermore, wall of the impedance tube must be acoustically rigid in order to minimize elastic waveguide effects, such as wavefront curvature and radial particle oscillation. Del Grosso [9] and Baik *et al.* [15] obtained analytical formulation for the axi-symmetric modes in the liquid-filled cylinders with elastic wall of finite thickness. Del Grosso [9] observed less than 1% dispersion of sound velocity, when wall thickness was equal to inner tube radius. Wilson *et al.* [8] confirmed this with measurements in water-filled impedance tube.

Considering these guidelines, an impedance tube was designed from a thick-walled steel tube, as shown in Figure 2. Dimensions of the tube are: inner diameter 79 mm, wall thickness 40 mm, length 3 m and distance between sensors $s = 1$ m, limiting frequency range to 70-560 Hz. Tube termination is executed as a detachable 40-mm-thick plate, which allows variable boundary condition in form of decompression slot of thickness δ . For the purposes of easy handling of the samples, the entire assembly was submerged into the liquid medium, i.e., Nynas Nytro 10XN transformer oil.

2.2.1. Experimental setup

Impedance-tube test station and the measurement procedure are presented in Figure 2. During measurements of acoustic properties of the transformer oil no absorption material was present in the tube. The excitation signal was generated in the LabVIEW programming environment and amplified with Renkforce PA MP-2000 RMS. Underwater loudspeaker DNH Aqua-30 acted as the transducer, producing sound pressure waves. The pressure response was obtained with hermetically sealed PCB 106B52 pressure sensors at four measuring points along the tube. The sensors have a steel membrane to enforce the tube's acoustic rigidity to avoid the perturbation of the pressure field. Pressure signals were transferred to the computer through the NI 9234 data acquisition module for further post-processing. The oil temperature was monitored with a resistance thermometer (RTD) and NI 9219 input module.

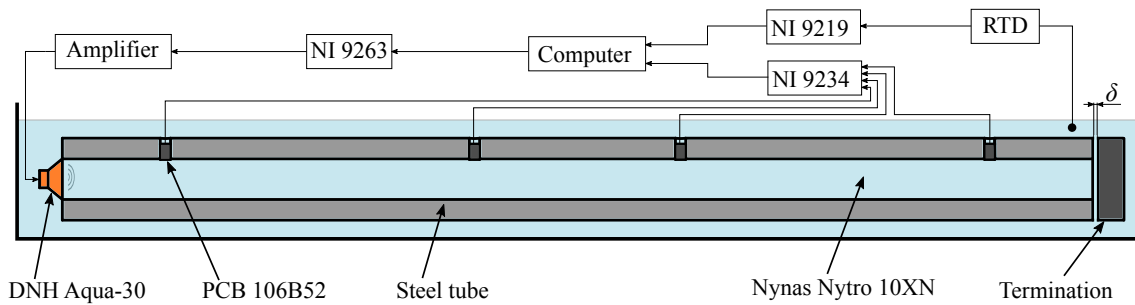


Figure 2: Experimental setup with the impedance tube submerged in the transformer oil.

3. GROUP VELOCITY

Group sound velocity is fundamental acoustic parameter of sound propagation medium, describing the velocity at which the signal is being transmitted. Transformer oil is dispersive medium, meaning group velocity c_1 must be distinguished from frequency dependent phase velocity c_0 , defined as the real component of $\frac{\omega}{k}$. Although group velocity is not directly used for derivation of the pressure-field amplitudes (Equations 3-6), it is required for the study of waveguide effects. It might also be used for calculation of the tube attenuation with a theoretical formulation.

The group sound velocity can be obtained from the pressure response of the four-microphone impedance tube by multiple different approaches. Three of them are analysed in this paper, as they allow implementation without any further changes to the existing measurement system. These methods involve extraction of the time difference between the signals, a cross-correlation of the signals and the minima-frequencies-difference method by Wang *et al.* [10].

The straight-forward approach of the time difference is based on calculation of the needed amount of time for the impulse signal to travel the distance between two sensors. Example of an impulse signal propagating through the impedance tube is shown in Figure 3. Time difference between different pairs of pressure sensors can be easily obtained by observing the time progress of the impulse signal before the first reflection from the tube termination. Resulting group velocities corresponding to each of sensor pairs are presented in Table 1. High sampling frequency is required in order to achieve acceptable accuracy of the results. Presented signals were sampled with frequency of 25.6 kHz and calculated group velocities deviate by as much as 20 m/s (Figure 6).

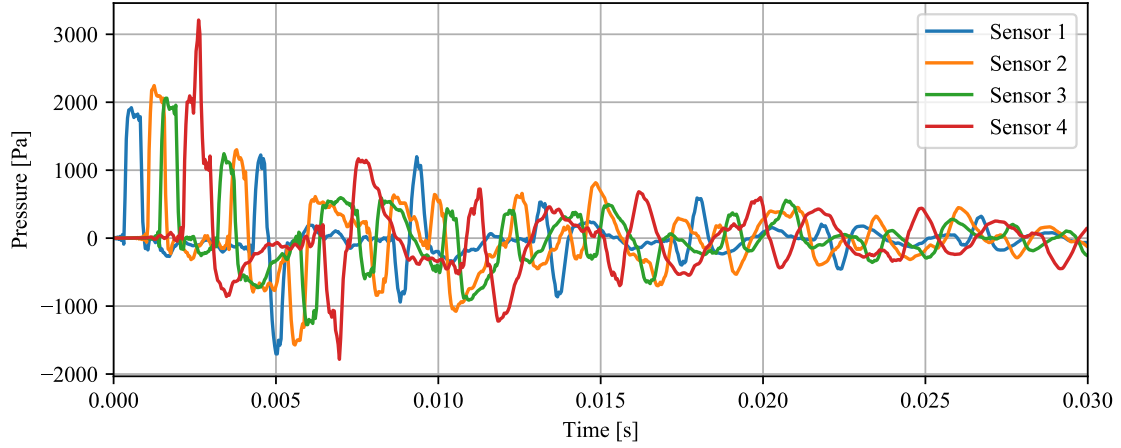


Figure 3: The impulse signal obtained with pressure sensors in the impedance tube.

Time difference of signal between two sensors can be more accurately obtained with cross-correlation analysis. The cross-correlation function is defined as:

$$R_{21}(\tau) = \int_0^{t_{\max}} p_2(t) p_1(t + \tau) dt \quad (7)$$

where τ represents time delay between pressure signals p_1 and p_2 . Peak of the function corresponds to the sought time difference. Only part of the signals before sound wave is reflected from the tube termination can be taken into account. Reflected wave can be observed in Figure 3 as an increased pressure at location of the sensor 4. Therefore, relevant signal lengths are shown in Figure 4a. Corresponding correlation functions are presented in Figure 4b and group velocities in Table 1. Signal pairs containing sensor 4 are excluded from the analysis because impulse shape is distorted due to the reflected wave.

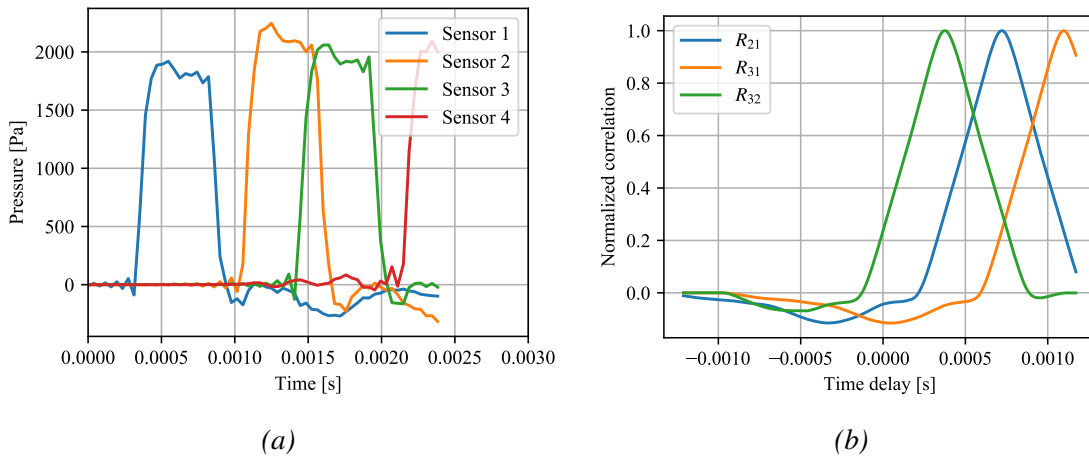


Figure 4: Cross-correlation method; a) Pressure signal segments used in analysis, b) Cross-correlation functions.

Another method applicable to the impedance tube is minima-frequencies-difference method, adapted from the work of Wang *et al.* [10], where parameter of interest Δf indicates the frequency difference between two successive minima of the pressure

amplitudes' ratio $\left| \frac{P_1}{P_2} \right|$ on logarithmic scale (see Figure 5). When sensor 1 and 2 are positioned at distance l_1 and l_2 from the tube termination, and $l_1 > l_2$, then sound velocity is defined as $c_1 = 2 l_1 \Delta f$. Since velocity value is susceptible to minima position errors, it seems most convenient to perform measurement in a minimally damped impedance tube, such as tube with hard termination, with corresponding distinctive peaks. Furthermore, amplitude and phase mismatch of sensors can be taken into account by using calibrated transfer functions instead of Fourier transform of the pressure, considering $\left| \frac{P_1}{P_2} \right| = \left| \frac{H_{1,\text{ref}}}{H_{2,\text{ref}}} \right|$. Calibration procedure is described in ASTM E2611 [1].

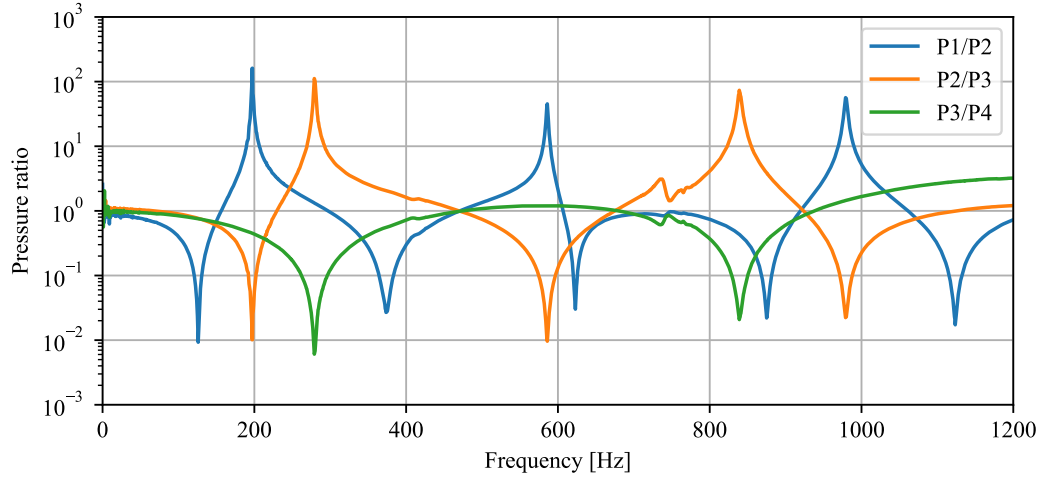


Figure 5: Ratio of pressure amplitudes for determination of minima-frequencies difference.

Comparison of presented methods in Figure 6 highlights the advantage of using cross-correlation method with minimal scattering. Based on it, group velocity at 21.4°C equals 1385.7 m/s.

Table 1: Calculated group velocities using different approaches for multiple pairs of pressure sensors.

Sensor pair	Time difference* [m/s]	Cross-correlation* [m/s]	Minima difference* [m/s]
1-2	1401.6	1386.3	1379.6
1-3	1388.5	1385.7	/
1-4	1390.8	/	/
2-3	1363.9	1385.3	1379.3
2-4	1383.7	/	/
3-4	1394.3	/	1391.8

*: at 21.1°C

4. COMPLEX WAVENUMBER

The real component of the complex wavenumber k corresponds to the frequency dependant phase velocity c_0 and the imaginary component represents the tube attenuation

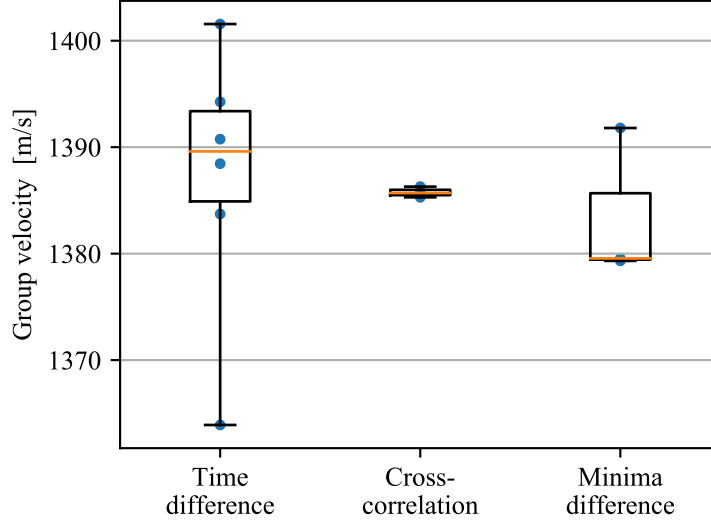


Figure 6: Boxplot of group velocity distribution for different methods. Blue points are the data points, orange band is the median, and ends of whiskers represent minimum and maximum of all data.

constant. Former is caused by dispersion, and latter represents the intrinsic dissipation of the fluid in impedance tube as a result of viscosity losses, thermal dissipation mechanisms and molecular relaxation. In an air-filled impedance tube the attenuation is negligible due to shorter wavelengths and consequently smaller tube's dimensions. On the other hand, attenuation in liquid-filled tube has a significant value and should be included in the wave equation as part of the wavenumber. Wavenumber can be evaluated with a analytical formulation, as given by Wilson *et al.* [8]:

$$k = \frac{\omega}{c_1} + (1 - j) \alpha_w, \quad (8)$$

under assumption that $\frac{\omega}{c_1} \gg \alpha_w$. Parameter α_w represents the viscous losses in the fluid-structure interaction layer at the tube wall with inner radius b :

$$\alpha_w = \frac{1}{b} \sqrt{\frac{\mu \omega}{2 \rho c_1^2}}, \quad (9)$$

with μ being dynamic viscosity of fluid. Such approximation neglects bulk viscosity and thermal losses, and also potential effects of wall roughness, local discontinuities in geometry and potential oil impurities. Therefore, it is more reliable to experimentally evaluate the tube's attenuation constant. Once more, methods applicable to the four-microphone impedance tube will be discussed. These include broadband spectroscopy method by Peters *et al.* [11] and a new amplitude-matching method [12]. Two more approaches can be found in Hou *et al.* [16], Han *et al.* [17], but were found to be unfit for the present impedance tube [12].

As opposed to the group velocity evaluation, for the phase velocity a noticeable influence of the tube termination was observed. Therefore, results of two contrasting termination configurations will be compared: rigid termination and partially anechoic termination. Corresponding absorption coefficients are shown in Figure 7.

In first approach by Peters and Petit [11] phase velocity is obtained by taking into account time delay between signals. Observing signals from sensors 1 and 3, the

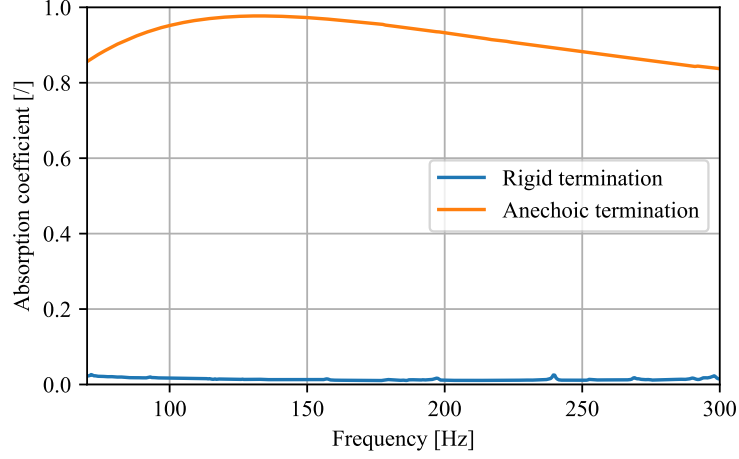


Figure 7: Absorption coefficient of the different impedance tube terminations.

attenuation constant k_i and the phase velocity c_0 are defined as:

$$k_i(f) = -\frac{\log \left| \frac{P_3(f)}{P_1(f)} \right|}{|x_3 - x_1|}, \quad (10)$$

$$c_0(f) = -\frac{2\pi f |x_3 - x_1|}{\text{Arg} \left(\frac{P_3(f) e^{2j\pi f \tau}}{P_1(f)} \right) - 2\pi f \tau}, \quad (11)$$

with P_n and x_n representing the Fourier transform of the pressure signal and position of the sensor n . Equivalent to the cross-correlation (see Section 3) only pulse signal before wave reflection is used calculate time delay τ and perform spectral analysis. Due to shortened signals this method is better suited for broadband analysis of an ultrasound. For the current case of low-frequency sound, the signals were padded with zeroes to keep the frequency resolution of 0.5 Hz. In Figures 8-9 results are compared to the analytical formulation and amplitude-matching method. Evidentially, broadband spectroscopy method cannot predict dispersion in low-frequency range.

The new approach of amplitude-matching was presented in previous work [12]. This method takes advantage of the redundant pair of pressure sensors in an impedance tube without the insert of an absorption material. With four sensors, two separate pressure fields can be obtained for the upstream and downstream part of the tube. Both are actually enclosed in the same volume, yielding pairs of equal amplitudes for incident ($A = C$) and reflected ($B = D$) wave. Implementing Equations 3-6 one can numerically solve for the complex wavenumber with a least-squares algorithm. Results agree with the analytical equation for the configuration with the rigid termination (Figures 8a and 9a). As expected, analytical formulation underestimated attenuation and dispersion of the phase velocity. On the other hand, configuration with partially anechoic termination is not appropriate for amplitude-matching method (Figures 8b and 9b). This is connected to substantial difference between amplitudes of incident and reflected wave, and consequently larger numerical error of iterative solver.

4.4.1. Waveguide effects

Phase velocity is directly related to waveguide effects due to fluid-structure interaction, which contributes to the generation of axi-symmetric modes in fluid. They were studied

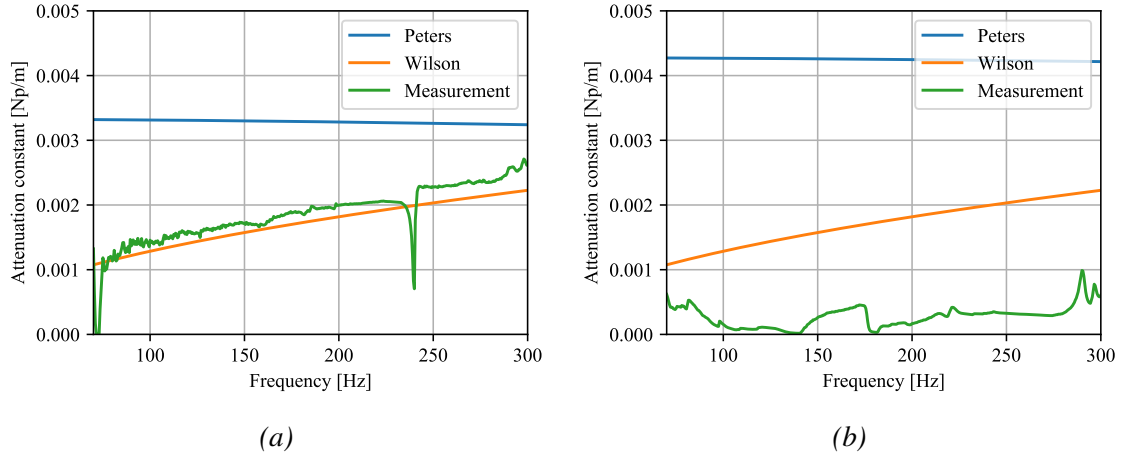


Figure 8: Attenuation constant obtained with different approaches, Peters: broadband spectroscopy method [11], Wilson: analytical equation [8], Measurement: amplitude-matching method [12]; a) Impedance tube with the rigid termination, b) Impedance tube with the partially anechoic termination.

by Del Grosso [9] and Baik *et al.* [15], who obtained characteristic equations for determination of sound velocity dispersion of each mode. While the fundamental mode, denoted ET0, has nearly plane wavefront and negligible radial particle displacement, higher modes exhibit pronounced wavefront curvature and radial oscillation. In order to comply with the requirement of plane wave propagation, higher modes should not be excited. Following derivation by Del Grosso [9], dispersion curves were obtained for the problem-specific data in Table 2. Results in Figure 10 demonstrate that modes ET0 and ET1 are relevant in the low-frequency range, with ET1 propagating through the tube almost four times faster. Calculated group and phase velocities both have similar value of around 1380 m/s, proving that only mode ET0 is present in the impedance tube and overall design of the experiment is adequate.

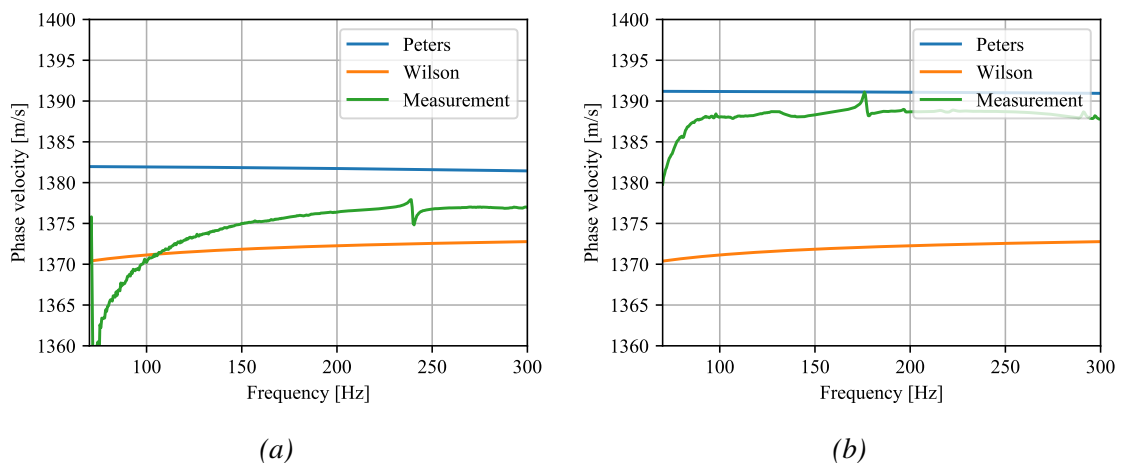


Figure 9: Phase velocity obtained with different approaches, Peters: broadband spectroscopy method [11], Wilson: analytical equation [8], Measurement: amplitude-matching method [12]; a) Impedance tube with the rigid termination, b) Impedance tube with the partially anechoic termination.

Table 2: Material parameters of the transformer oil and the impedance tube: inner tube radius b , outer tube radius d , transformer-oil density ρ_1 , steel density ρ_w , group sound velocity c_1 , longitudinal and shear sound velocity in steel c_1 and c_s , respectively.

Transformer oil	Steel tube
$b = 39.5 \text{ mm}$	$\rho_w = 7850 \text{ kg/m}^3$
$d = 79.5 \text{ mm}$	$c_1 = 5960 \text{ m/s}$
$\rho_1 = 871.96 \text{ kg/m}^3$	$c_s = 3235 \text{ m/s}$
$c_1 = 1385.7 \text{ m/s}$	

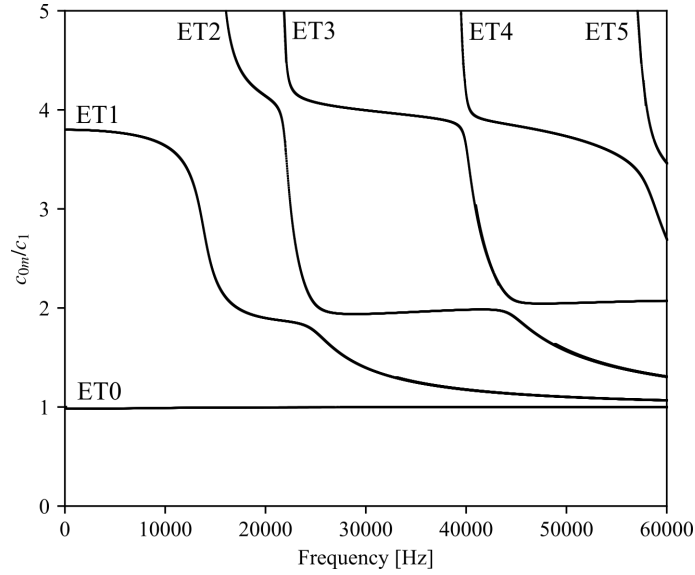


Figure 10: Dispersion curves of modes ET0–ET5.

5. CONCLUSIONS

Measurement techniques for obtaining acoustic characteristic of the experimental setup with a four-microphone impedance tube submerged into transformer oil were investigated. Approaches for measuring group velocity and complex wavenumber, which can be directly applied to the tube setup, were adapted from the literature and compared to the methods introduced in previous paper.

Extended analysis was performed on the methods for calculating group velocity employing all possible sensor pairs. Distribution of the results demonstrates that general method of cross-correlating pressure signals is most reliable with minimal scattering, as opposed to time-difference and minima-difference methods. The minima-difference method was subjected inaccuracy even after modifying it to include sensor-mismatch correction and the time-difference method is least repeatable due to sampling limitations.

Next, the established method broadband spectroscopy for measuring the phase velocity and attenuation in the ultrasound scope was presented. It's adaptation to the low-frequency range was unsuccessful. Therefore, the newly developed amplitude-matching approach is favourable in determination of the complex wavenumber. There are limitations regarding use of partially anechoic termination due to numerical errors of iterative solver.

Finally, influence of the axi-symmetric modes in elastic waveguide on the dispersion

was evaluated. Comparison of measures group and phase velocities confirmed the acoustic stiffness of the impedance tube.

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