MADRID
inter.noise 2019
June 16 - 19
NOISE CONTROL FOR A BETTER ENVIRONMENT

Vibration reduction of a panel structure by controlling a modal group having the same direction of wave number vector

Yuuta Shimizu[1]
Graduate School of Gifu University
1-1 Yanagido, Gifu-shi, Gifu 501-1193, Japan

Yuichi Matsumura[2]
Gifu University
1-1 Yanagido, Gifu-shi, Gifu 501-1193, Japan

Kohei Furuya[3]
Gifu University
1-1 Yanagido, Gifu-shi, Gifu 501-1193, Japan

Ryouichi Ishikawa[4]
Gifu University
1-1 Yanagido, Gifu-shi, Gifu 501-1193, Japan

ABSTRACT

In the structure composed of frames and panels, the panels often have lighter mass density and rigidity than frames and are easily excited from the frame side. The vibration of a panel causes acoustic radiation because its surface area is large. Hence it is important to evaluate vibration from the initial design stage to prevent large vibration. In this study, attention is paid to the wave number vector that causes an eigenmode of a panel structure. It is known that there is a strong relationship between the wave number vector and eigenmode formation. In this paper, we address a vibration reduction method of a panel structure which controls a modal group having the same direction of the wave number vector. As a numerical example, we confirmed that the proposed method can control a modal group having the same direction of wave number vector and can reduce the vibration of a panel structure efficiently.

Keywords: Noise, Vibration, Wave
I-INCE Classification of Subject Number: 42

1 w3912009@edu.gifu-u.ac.jp
2 y_matsu@gifu-u.ac.jp
3 furuya@gifu-u.ac.jp
4 u3031003@edu.gifu-u.ac.jp

1. INTRODUCTION

In the field of vibration and noise in the automotive field, vibration reduction design at mid frequency is an important technical issue. The middle frequency is a band of 250 to 500 Hz, which frequency band is difficult to design for vibration reduction. This is because modal density is too high for modal analysis and is too low in SEA (Statistical Energy Analysis). In other words, middle frequency has a complex problem of both low frequency and high frequency. As another vibration analysis method, there is a method using wave analysis. Waves are fundamental to cause vibration. Therefore, there is a possibility to understand the vibration mechanism by the viewpoint of wave.

There have been many studies in two-dimensional panels using wave analysis. The representative one is the Image Source Method [1, 2]. By using the Image Source Method, it is possible to identify the wave path and reflection boundary which contribute greatly to the response of the evaluation point. However, when the number of reflections in the system is large, there are difficulties in the analysis.

This paper shows an analytical method using wave number vector and shows how to devise structural modification for vibration reduction. The proposed method can be applied from low frequency to relatively high frequency. Firstly, as an introduction, it is shown that the vibration mode of thin plates have specific angles of wave number vectors [3]. Secondly, phase closure principle is applied to the waves of specific angles to reduce vibration of thin plates. Finally, this research proposes a method to devise structural modification for vibration reduction using phase closure principle. A specific example simulates a panel supported by a frame. To clarify the relation with the theory here, the model of the FEM is a rectangular plate that simply supports. When focusing on the target vibration mode, it was confirmed that ERP(Equivalent Radiated Power) [4] at that frequency can be reduced.

2. PPLICATION OF THE PHASE CLOSURE PRINCIPLE IN TWO DIMENSIONAL PANELS

2.1 Eigen modes of a simple supported rectangular plate

The vibration characteristics of a two-dimensional panel are approximated by using the wave equation according to Kirchhoff's assumption [3]. The wave equation of a microelement when the bending displacement of the panel spreading on the plane is expressed by w (x, y, t) as follows,

$$\rho h \frac{\partial^2 w}{\partial t^2} + D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) = 0, \tag{1}$$

where, ρ is density and h is plate thickness. D represents flexural rigidity of the plate can be written as

$$D = \frac{Eh^3}{12(1-\nu^2)}, \tag{2}$$

where E is Young’s modulus and ν is Poisson's ratio. As shown in Fig. 1, for a plate simply supported on all the sides, the boundary conditions to be satisfied are

$$\begin{cases} u = 0, \quad EI\dfrac{\partial^2 u}{\partial x^2} = 0 \ \ (x = 0,\ L_x) \\ u = 0, \quad EI\dfrac{\partial^2 u}{\partial y^2} = 0 \ \ \left(y = 0,\ L_y\right). \end{cases} \tag{3}$$

In this condition, the state of the phase is given by phase closure principle in the *x* direction and *y* direction respectively can be written as

$$\begin{aligned} k_x L_x &= n\pi \quad\ (n = 1, 2, \cdots) \\ k_y L_y &= m\pi \quad (m = 1, 2, \cdots), \end{aligned} \tag{4}$$

where L_x and L_y represent the length of the side in each direction, and k_x and k_y represent wave numbers in the respective directions. The natural frequency in this case is

$$\omega_{nm} = \pi^2 \left(\frac{n^2}{{L_x}^2} + \frac{m^2}{{L_y}^2}\right)\sqrt{\frac{D}{\rho h}} \ \ (n, m = 1, 2, \cdots). \tag{5}$$

The case of a plate simply supported on all edges is mode shape easily obtainable. Figure 1 shows *m*, *n* modes that is the mode shape existing in the plate.

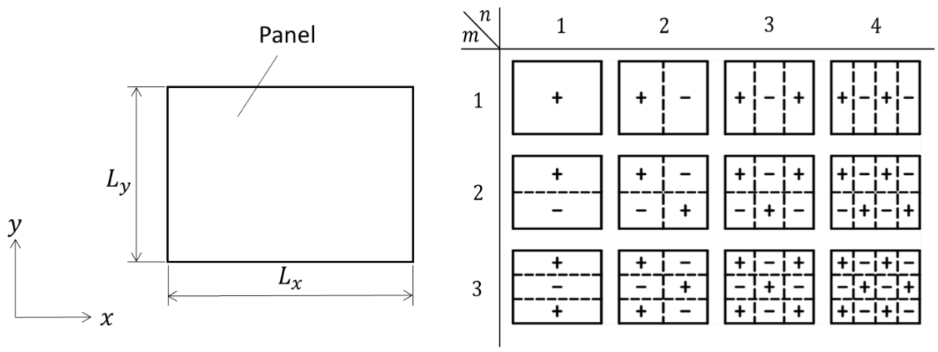


Figure 1 Normal mode of the simply supported plate

2.2 Wavenumber at eigen modes of a simple supported rectangular plate

In simply supported on all edges, the eigen modes are described by ω_{mn}. Then the general solution of Eq. 1 is

$$W_{mn}(x, y, t) = w_{mn}(x, y)e^{j\omega_{mn}t}. \tag{6}$$

Substituting Eq. 6 into the wave equation of Eq. 1 gives

$$\rho h(j\omega_{mn})^2 w_{mn} + D\left(\frac{\partial^4 w_{mn}}{\partial x^4} + 2\frac{\partial^4 w_{mn}}{\partial x^2 \partial y^2} + \frac{\partial^4 w_{mn}}{\partial y^4}\right) = 0, \tag{7}$$

where, $w_{mn}(x, y)$ in Eq. 6 is a function of position, and since the wave is a periodic function, it can be rewritten as $w_{mn}(x, y) = \sin k_x x \sin k_y y$. Here k_x and k_y are wave numbers of waves propagating in the *x* and *y* directions, respectively. Substituting Eq. 6 into Eq.7 gives

$$\rho h(j\omega_{mn})^2 + D\left(k_x^4 + 2k_x^2k_y^2 + k_y^4\right) = 0. \tag{8}$$

Solving for k_x and k_y, the relationship between wavenumber and frequency ω_{mn} is

$$\sqrt{{k_x}^2 + {k_y}^2} = \sqrt[4]{\frac{\rho h}{D}}\sqrt{\omega_{mn}}\,. \tag{9}$$

Generally, the wave number of the thin plate is

$$k = \sqrt[4]{\frac{\rho h}{D}}\sqrt{\omega}. \tag{10}$$

By comparing Eq. 9 and Eq. 10, it can say that wavenumber is a combination of wavenumbers in the *x* and *y* directions.

2.3 Relationship between eigen modes and wavenumber vectors

The wave number when satisfying the condition of Eq. 5 is defined as k_{xy}. At this condition, using Eq. 5, 9 and 10, k_{xy} gives

$$k_{xy} = \sqrt{{k_x}^2 + {k_y}^2} = \sqrt{\left(\frac{n\pi}{L_x}\right)^2 + \left(\frac{m\pi}{L_y}\right)^2}\,. \tag{11}$$

On the other hand, the angle of the wavenumber vectors are

$$\tan^{-1}\theta = \frac{k_y}{k_x} = \frac{mL_x}{nL_y}\,. \tag{12}$$

The above equation is shown in the schematic diagram, the relationship between the wavenumber vector in each direction and its angle can be considered as shown in Fig. 2.

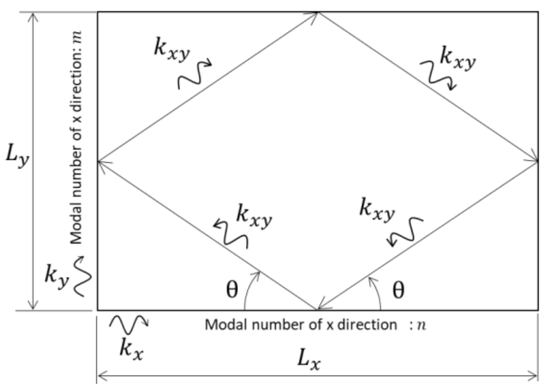


Figure 2 Relationship between eigenmode and wave vector

2.4 Reflection phase of waves at boundary conditions

The phase change when reflected by the boundary differs depending on the boundary conditions. Figure 3 shows a schematic diagram of the wave reflection at the simple support boundary. The reflection coefficient is defined as the ratio of the amplitude of the reflected wave to the incident wave. Also, the complex number represents the phase angle. r_{PP} is the bending wave and r_{PN} is the reflection coefficient of the evanescent waves as follows,

$$\begin{cases} r_{PP} = \dfrac{b}{a} \\ r_{PN} = \dfrac{b_N}{a}\ , \end{cases} \tag{13}$$

where, a is incident wave, b is reflected wave and b_N is the evanescent wave generated by the reflection of the incident wave. As the general solution of the wave equation, applying $\mathrm{W}(x, 0, \mathrm{t}) = \partial^2 W(x, 0, t)/\, \partial x^2$ from the simple support boundary condition, the reflection coefficients r_{PP} and r_{PN} are obtained as follows

$$\begin{cases} \dfrac{b}{a} = -1 = r_{PP} \\ \dfrac{b_N}{a} = 0 = r_{PN}\ . \end{cases} \tag{14}$$

As a result, the reflection phase angle of the bending wave is π in simple support boundary, meaning that the evanescent wave does not exist as a reflected wave.

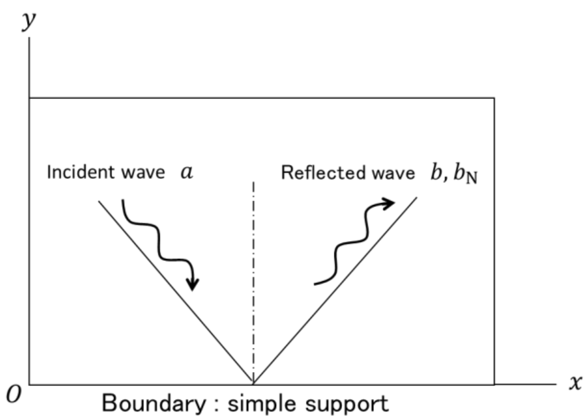


Figure 3 Relationship of wave reflection

2.5 Phase closure principle

Phase closure principle [5, 6] is to describe the phase of the wave reflected at the boundary by an equation. And this equation clarifies the overlap of waves propagating in the same direction at the same position. The condition of resonance and cancellation of the calculated phase is as follows,

$$2kL + \phi_L + \phi_R = \begin{cases} 2(d+1)\pi \quad (d = 1,\ 2, \cdots)\ , \text{Resonance} \\ (2d+1)\pi \quad (d = 1,\ 2, \cdots)\ , \text{Cancellation.} \end{cases} \tag{15}$$

where, L is the distance between the boundaries, ϕ_L is the reflection phase of the left boundary, and ϕ_R is the reflection phase of the right boundary. In case of simple support, ϕ_L and ϕ_R are π as shown in the previous section.

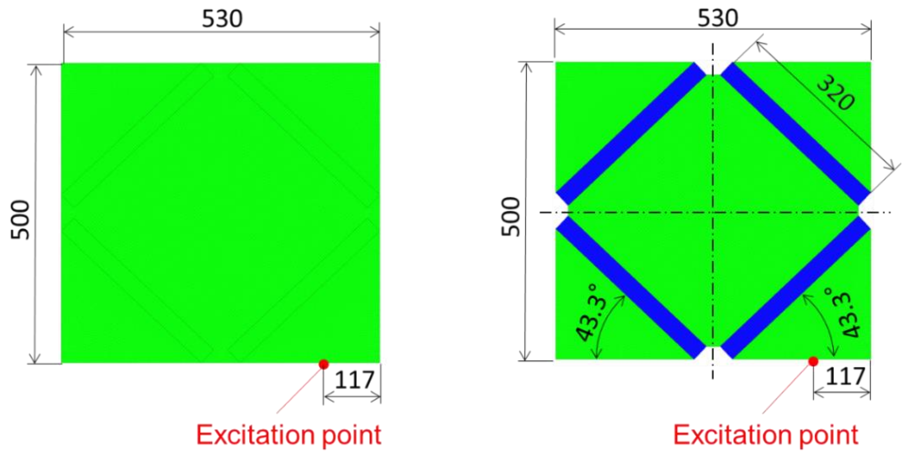
3. VIBRATION REDUCTION DESIGN USING THE PHASE CLOSURE PRINCIPLE

3.1 Proposal of structural change

Section 2.3 show that the wave vectors of normal modes having n and m antinodes in the x and y directions can be calculated. Creating the cancellation condition shown in Eq. 15 for these wavenumber vectors. For example, as shown (b) in the Fig. 4, boundaries are set perpendicular to the direction of the wavenumber vector so that the waveguide of distance L satisfies the cancellation condition of Eq. 15. It can be considered that within these boundaries, the phase closure principle is satisfied, and the vibration response is reduced.

3.2 Confirmation of validity of structure change

Evaluate ERP(Equivalent Radiated Power) [4] when structure is modified for vibration reduction by frequency response analysis by finite element method. The object model is a thin steel material of 530 × 500 × 1 mm as shown in Fig. 4, and specifications are shown in Table 1. Modify the structure as shown in (a) and (b) in Fig. 4. Structural changes are four parts in the middle of boundary. The target modes for vibration reduction are 4×4, 6×6, and 8×8 mode shapes. Based on the idea of phase closure principle, (b) in Fig. 4 has waveguides of L=320 mm in length parallel to the wavenumber vector direction. The angle is determined by Eq. 12 and cancellation phase is determined by Eq. 15. This boundary is intended to cancel the wave that forms the mode shapes of target.



(a) Initial model (b) Modified model

Figure 4 Structural modification

Table 1 Conditions of FEA

Number of shell elements	Initial model	17555
	Modified model	17320
Number of nodes	Initial model	17508

	Modified model	17309
Solver	Altair HyperWorks (OptiStruct)	
Analysis type	Frequency Response (Modal) and Normal Modal	
Structural damping coefficient	0.001	
L_x	530 mm	
L_y	500 mm	
L_{xy}	320 mm	
θ	43.3°	

3.2 Analysis result

Figure 5 shows the respective mode shapes in normal modal analysis. For comparison, Initial model and Modified model are arranged in 4×4, 6×6, and 8×8 mode shapes. Comparing before and after the modified, it can be confirmed from the contour diagram that the response between the structural changed boundary in the panel is lower than the surroundings. This tendency can be expected to be similarly low in response to frequency response analysis.

Finally, Fig. 6 compares the ERP(Equivalent Radiated Power) of the whole panel obtained by frequency response analysis in respective normal modes. Since the frequency of the corresponding mode changes before and after the structural change, comparison was made by tracing the same mode shape. Changes are indicated by arrows in the graph in Fig. 6. As a result, it can be confirmed that the ERP is reduced by two dB or more in the target modes. These indicate that operation is possible as a mode group, and the effectiveness of the proposed method was confirmed.

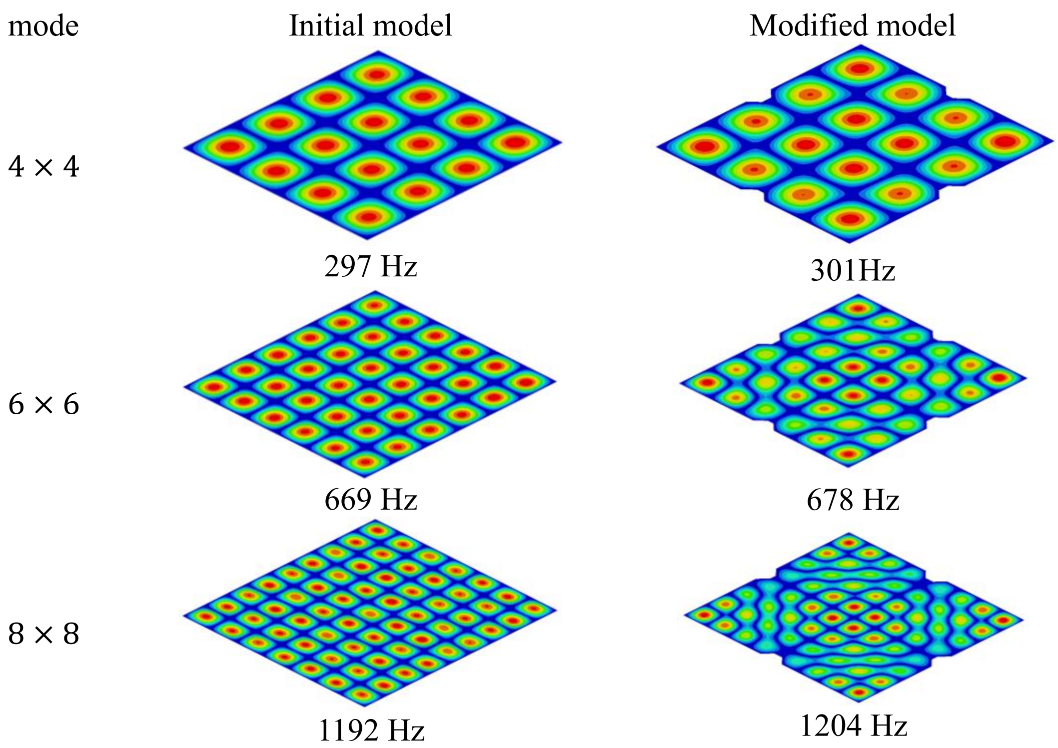


Figure 5 Mode shape in Normal Modal

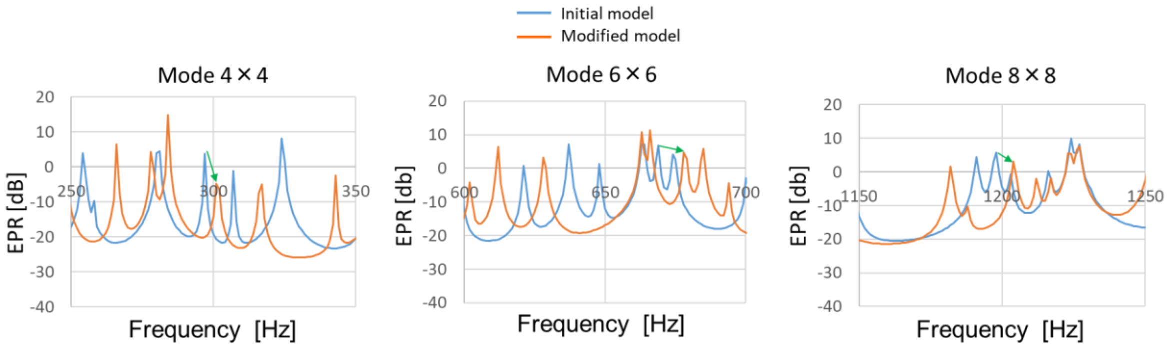


Figure 6 ERP of the entire panel before and after structural modification

4. CONCLUSIONS

The normal modes in the two-dimensional panel can be associated with the wave vector direction, indicating that normal modes can be operated as a group by changing the structure focusing on the wave vector. With this method, the vibrational response can be reduced in the vibration of a panel structure efficiently.

5. REFERENCES

1. Jacques Cuenca, Wave models for the flexural vibrations of thin plates, Doctoral Thesis in Acoustic, Université du Maine, (2009).
2. Vincent Martin, Thomas Guignard, Justification of the image sources in ray-tracing method, 12th International Congress on Sound and Vibration, (2005).
3. Karl F. Graff, Wave Motion in Elastic Solids, Dover Books on Physics(1991)
4. Dynamic Analysis User's Guide, MSC Nastran 2017.1
5. L. Cremer, M. Heckl, E.E. Ungar, Structure-borne Sound, Springer-Verlag, Berlin, 1973
6. Bin Tang, Michael J. Brennan, Elisabetta Manconi, On the use of the phase closure principle to calculate the natural frequencies of a rod or beam with nonlinear boundaries(2018)