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Nonlinear structural-acoustic analysis of orthogonally stiffened composite cylindrical shells with piecewise isolators

Qu, Yegao[1]; Xie, Fangtao; Zhang, Wenming; Peng, Zhike; Meng, Guang

State Key Laboratory of Mechanical System and Vibration
Shanghai Jiao Tong University
No.800 Dongchuan R.D., Minhang District, Shanghai, China

ABSTRACT

This paper is concerned with the numerical analysis of the structural and acoustic responses of a stiffened composite cylindrical shell attached with piecewise isolators. The system is immersed in an infinite acoustic fluid. The cylindrical shell is reinforced by a series of circumferential rings and longitudinal stringers. The isolators in the cylindrical shell contain motion-limiting stops, the restoring forces of which are expressed as bi-linear functions of the isolator deformation. A modified variational method is adopted to establish the nonlinear dynamics model of the structural system, and the acoustic fluid is modelled by a time-domain boundary element method. The contribution of the grouped circumferential wave modes of the shell to the nonlinear vibration and radiated sound responses of the coupled structural system is examined.

Keywords: Nonlinear vibration, acoustic responses, isolation system
I-INCE Classification of Subject Number: 54

1. INTRODUCTION

Vibrating machines are commonly mounted on a stiffened composite cylindrical shell via a series of vibration isolators in underwater vehicles. These isolators may be designed with motion-limiting stops in order to prevent excessive displacement between the machine and the shell, when the system is subjected to large external loads. The motion-limiting stops lead to an abrupt change in the stiffness characteristics of the isolators, accounting for the inherent piecewise non-linearity of the system. The practical importance of a stiffened cylindrical shell with piecewise isolators in an underwater vehicle necessitates a comprehensive understanding of the structural and acoustic behaviors of the system.

The vibration and sound radiation of a coupled shell and isolation system immersed in a compressible fluid have been investigated by few researchers. Most of the previous investigations are limited to linear structural-acoustic problems based on harmonic frequency-domain approaches. Guo [1] investigated the sound scattering of a coupled

[1] Corresponding author: quyegao@sjtu.edu.cn

cylindrical shell carrying a linear spring-mass system. Achenbach et al. [2] studied the structural and acoustic responses of a cylindrical shell attached with an internal mass-spring oscillator. Choi et al. [3] analyzed the vibration and acoustic behaviors of submerged shells carrying a dense array of oscillators. Rebillard et al. [4] investigated the acoustic behaviors of a finite cylindrical shell containing a spring-mass system. Caresta and Kessissoglou [5,6] developed a semi-analytical method in order to predict the radiated sound from a submarine pressure hull with attached passive and active isolation systems. Based on a ray technique, Ho [7] analyzed the sound radiation of a coupled cylindrical shell and mass-spring oscillator system. Titovich and Norris [8] investigated the sound scattering of a cylindrical shell carrying linear isolators. Qu et al. [9] presented a semi-analytical method for analyzing the vibration and acoustic responses of a coupled submarine hull and propeller-shafting system, in which the elastic shaft is connected to the hull through a series of linear springs.

The aim of this paper is to derive a formulation for predicting the nonlinear vibration and acoustic behaviors of a coupled composite cylindrical shell and piecewise isolation system. The shell is orthogonally reinforced by a series of circumferential rings and longitudinal stringers. The nonlinear dynamics model of the structural system is established by a modified variational method for the shell combined with a discrete element method for the stiffeners, and the acoustic fluid is formulated using a time-domain boundary element method. The effects of the circumferential wave mode of the cylindrical shell on the nonlinear structural and acoustic behaviors of the coupled system are discussed.

2. Theory

The structural system considered here consists of an orthogonally stiffened composite cylindrical shell and a rigid body, which are connected to each other through a number of vibration isolators, as shown in Fig.1. The acoustic fluid exterior to the structural system is assumed to be light, inviscid and compressible with density ρ_f and speed of sound c_f. The cylindrical shell is assumed to be thin and composed of an arbitrary number of orthotropic material layers with length L, mean radius R and total thickness H. A cylindrical coordinate system (x, θ, z) located on the mid-surface of the shell is introduced, in which the axial, circumferential and normal displacements at an arbitrary material point of the shell are denoted by u, v and w, respectively. The reinforcements, including the circumferential rings and the longitudinal stringers, in the cylindrical shell may be few or many in number, non-uniform or uniform in size, and arbitrarily distributed in space. Without loss of generality, the i th ring stiffener is assumed to be attached to the cylindrical shell at x_i^{rg} measured from the left end of the shell, and the i th longitudinal stringer is located along a single attachment line at θ_i^{st} of the shell. The isolators are represented by piecewise linear springs and viscous dampers, which may be mounted inside of the cylindrical shell at arbitrary positions. The connecting point of the i th isolator with the cylindrical shell is defined as: $\mathbf{r}_i = \mathbf{r}(x_i, \theta_i)$. In the present analysis, the nonlinear coupled system is excited by external forces applied either on the shell or on the rigid body.

The rigid body simplified as a lumped mass is mounted on a series of vibration isolators. Each isolator is represented by two stage suspension springs and a viscous damper. The primary suspension spring is always effective. The secondary spring is used to prevent excessive relative displacement between the rigid body and the shell when the relative displacement exceeds a certain value Δ, i.e., the gap of the two suspension springs. The restoring force of the isolator is expressed as:

$$\mathcal{F}(\eta)=k_1\eta+\mathcal{H}(\eta+\Delta)k_2(\eta+\Delta) \tag{1}$$

where k_1 and k_2 are the stiffness coefficients of the primary and secondary suspension springs, respectively. η is the relative displacement between the rigid body and the shell. $\mathcal{H}(\eta+\Delta)$ is the Heaviside step function.

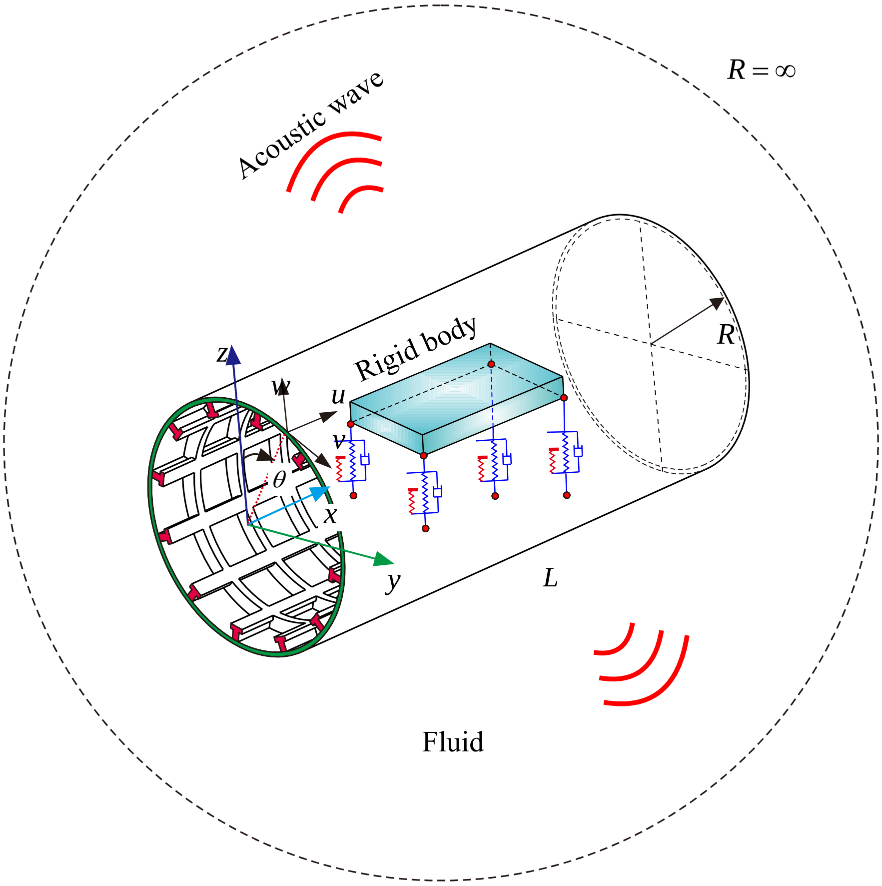

Fig.1 Coupled stiffened composite cylindrical shell and isolation system in fluid

A modified variational method is utilized here to establish the nonlinear dynamics model of the coupled cylindrical shell and piecewise isolation system. Based on the method, the equations of motion of the coupled shell and piecewise isolation system can be obtained by setting the following formulation to zero, given as:

$$\delta\mathbb{P}_{\text{Total}}=\delta\mathbb{P}^{\text{s}}+\delta\mathbb{P}^{\text{rg}}+\delta\mathbb{P}^{\text{st}}+\delta\mathbb{P}^{\text{iso}} \tag{2}$$

where δ is the variational operator. $\mathbb{P}_{\text{Total}}$ is the total energy of the coupled structural system. $\mathbb{P}^{\text{s}}$ is the energy of the cylindrical shell. $\mathbb{P}^{\text{rg}}$ and $\mathbb{P}^{\text{st}}$ are the total energies contributed by the circumferential rings and longitudinal stringers in the shell, respectively. $\mathbb{P}^{\text{iso}}$ is the energy of the isolation system. The detailed expressions of $\mathbb{P}^{\text{s}}$, $\mathbb{P}^{\text{rg}}$ and $\mathbb{P}^{\text{st}}$ are referred to Ref. [9].

The rigid body is assumed to move in the vertical direction. The variation of the energy of the isolation system is expressed as:

$$\delta\mathbb{P}^{\text{iso}} = \delta\hat{u}\left(-m\ddot{\hat{u}}\right) - \sum_{l=1}^{N_{\text{iso}}} \mathcal{F}_l\left(\eta_l\right)\delta\eta_l + \sum_{l=1}^{N_{\text{iso}}} \delta\eta_l\left(-c_l\dot{\eta}_l\right) + F\delta\hat{u} \tag{3}$$

where $\hat{u}$ and $\ddot{\hat{u}}$ are the displacement and acceleration of the rigid body, respectively. m is the mass of the rigid body. N_{iso} is the number of isolators. η_l and $\dot{\eta}_l$ are the deformation and velocity of the lth isolator, respectively. c_l is the damping coefficient of the l th isolator. F is the external force applied on the rigid body.

The deformation of the l th isolator is equal to the relative displacement of the rigid body and the shell (see Fig.2), written by:

$$\eta_l = \hat{u} - \left[w\left(\mathbf{r},t\right)\cos\theta_l\right]_{\mathbf{r}=\mathbf{r}_l(x_l,\theta_l)} + \left[v\left(\mathbf{r},t\right)\sin\theta_l\right]_{\mathbf{r}=\mathbf{r}_l(x_l,\theta_l)} \tag{4}$$

where the circumferential displacement $v(\mathbf{r},t)$ and the normal displacement $w(\mathbf{r},t)$ of the shell are measured at the connection point of the lth isolator. Once η_l is obtained, the relative velocity $\dot{\eta}_l$ can be computed directly.

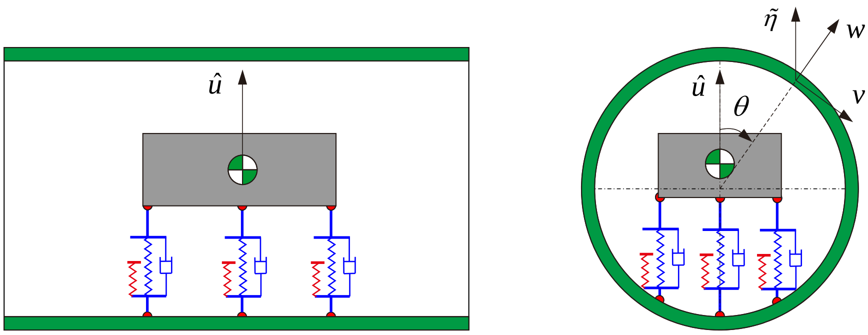

Fig.2 Displacement relation of rigid body and stiffened cylindrical shell

Considering the periodicity of the cylindrical shell in the circumferential direction, the displacement components of each shell segment can be expanded by Fourier series. In doing so, a three-dimensional vibration problem of the shell is transformed into a set of one-dimensional problems, which correspond to the harmonics of the Fourier expansion. The displacement field of each shell segment is expanded by Fourier series for the circumferential coordinate and Chebyshev orthogonal polynomials for the axial coordinate, given as:

$$u\left(x,\theta,t\right) = \sum_{n=0}^{N}\sum_{m=0}^{M} \mathcal{G}_m\left(x\right)\left[\cos\left(n\theta\right)\bar{u}_{nm}\left(t\right) + \sin\left(n\theta\right)\tilde{u}_{nm}\left(t\right)\right] \tag{5a}$$

$$v\left(x,\theta,t\right) = \sum_{n=0}^{N}\sum_{m=0}^{M} \mathcal{G}_m\left(x\right)\left[\cos\left(n\theta\right)\bar{v}_{nm}\left(t\right) + \sin\left(n\theta\right)\tilde{v}_{nm}\left(t\right)\right] \tag{5b}$$

$$w\left(x,\theta,t\right) = \sum_{n=0}^{N}\sum_{m=0}^{M} \mathcal{G}_m\left(x\right)\left[\cos\left(n\theta\right)\bar{w}_{nm}\left(t\right) + \sin\left(n\theta\right)\tilde{w}_{nm}\left(t\right)\right] \tag{5c}$$

where $\mathcal{G}_m(x)$ is the mth order Chebyshev orthogonal polynomials of first kind. M is the highest order of the polynomials truncated in the analysis. n is the circumferential wave mode number of the shell. N is the maximum circumferential wave mode number. $\bar{u}_{nm}$, $\tilde{u}_{nm}$, $\bar{v}_{nm}$, $\tilde{v}_{nm}$, $\bar{w}_{nm}$ and $\tilde{w}_{nm}$ are generalized displacement coefficients.

Based on the modified variational method, the discretized equations of motion of the coupled structural system can be obtained as:

$$\mathbf{M}\ddot{\mathbf{U}}+\mathbf{C}\dot{\mathbf{U}}+\left[\mathbf{K}-\mathbf{K}_{\lambda}+\mathbf{K}_{\kappa}+\mathbf{K}_{\text{fst}}\right]\mathbf{U}=\mathbf{F}_{s}+\mathbf{F}_{\text{snd}}\left(\mathbf{U}\right)+\mathbf{F}_{f} \tag{6}$$

where $\mathbf{U}$, $\dot{\mathbf{U}}$ and $\ddot{\mathbf{U}}$ are the generalized displacement, velocity and acceleration vectors of the coupled structural system, respectively. $\mathbf{M}$ is the generalized mass matrix of the system. $\mathbf{C}$ is the damping matrix contributed by the viscous dampers of the isolators and the structural damping of the stiffened composite shell. $\mathbf{K}_{\lambda}$ and $\mathbf{K}_{\kappa}$ are the generalized interface stiffness matrices. $\mathbf{K}_{\text{fst}}$ is the coupled stiffness matrix due to the connection of the primary suspension springs and the cylindrical shell. $\mathbf{F}_{s}$ is generalized force vectors of the mechanical loads applied on the shell. $\mathbf{F}_{\text{snd}}$ is the vector containing the restoring forces of the secondary suspension springs. $\mathbf{F}_{f}$ is the generalized force vector due to the fluid pressure.

The stiffened composite cylindrical shell is assumed to be closed at its two ends with rigid acoustic baffles. The Kirchhoff integral equation equivalent to the transient wave equation and the Sommerfeld radiation condition is adopted here for computing the radiated sound from the shell, given as [10]:

$$c(\mathbf{r})\,p(\mathbf{r},t)=-\frac{1}{4\pi}\iint_{S_f}\left\{\frac{\partial\mathcal{R}_s}{\partial\hat{n}}\left[\frac{p(\mathbf{r}_s,\tau_{ret})}{\mathcal{R}_s^2}+\frac{\dot{p}(\mathbf{r}_s,\tau_{ret})}{c_f\mathcal{R}_s}\right]+\frac{1}{\mathcal{R}_s}\frac{\partial p(\mathbf{r}_s,\tau_{ret})}{\partial\hat{n}}\right\}\mathrm{d}S \tag{7}$$

For the stiffened cylindrical shell with acoustic baffles, a numerical solution of the above equation is imperative. In this paper, Equation 7 is solved by a time-domain BEM, which consists of two basic steps: (1) a discretization of the real time axis into a sequence of equally spaced time intervals with certain variations of sound pressure and flux over each time interval, and (2) a discretization of the fluid-structure interface S_f into a number of flat quadrilateral elements, over which a constant distribution of sound pressure and flux is assumed. On the vibrating surface of the shell, the following interface condition is enforced: $\partial p(\mathbf{r}_s,t)/\partial\hat{n}=-\rho_f\ddot{w}(\mathbf{r}_s,t)$. The linear system of algebraic equations for the acoustic fluid can be achieved by using the collocation method, given as [10]:

$$c(\mathbf{r})\,p(\mathbf{r},t_i)=\sum_{\gamma=1}^{\gamma_{\min}}\mathcal{G}_{\gamma}\mathcal{L}_{i-\gamma+1}+\sum_{\gamma=1}^{\gamma_{\min}}\mathcal{Q}_{\gamma}\left[\gamma\mathbf{p}_{i-\gamma+1}-(\gamma-1)\mathbf{p}_{i-\gamma}\right] \tag{8}$$

The nonlinear dynamics of the structural system is solved using the implicit Newmark integration method. Once $\ddot{\mathbf{U}}$ is known from Equation 6, the sound pressure p at any given time on the structural-acoustic surface can be computed from Equation 8 by seting $c=0.5$ and starting at the first time step and solving recursively until reaching the desired time. Once the sound pressure $\mathbf{p}$ on all boundary elements are determined, the sound pressure at any field point in the fluid can be computed by Equation 8 with $c=1.0$.

3. Results and discussion

The nonlinear vibration behaviors of a coupled composite stiffened cylindrical shell and isolation system are examined. A composite laminated [0°/90°/0°/90°] cylindrical shell stiffened by circumferential rings and longitudinal stringers is considered. The physical properties of the shell are: $L=4$ m, $R=1$ m and $H=0.005$ m; $E_1=50$ GPa, $E_2=2$ GPa, $G_{12}=1.0$ GPa, $\mu_{12}=0.25$ and $\rho=1500$ kg/m^3. Nine ring stiffeners with height $h^{\text{rg}}=0.015$ m and width $d^{\text{rg}}=0.015$ m are equally-spaced along the axial direction of the shell. The ith ring stiffener is located at $x_i^{\text{rg}}=0.4i$ ($i=1,2,\cdots,9$) from the left end of the cylindrical shell. In addition, the cylindrical shell is reinforced by 3 longitudinal stringers of rectangular cross-section with height $h^{st} = 0.015$ m and width $d^{st} = 0.015$ m. The stringers are located at: $\theta_i^{st} = (i+4)\pi/6$ ($i = 1,2,3$). All the ring stiffeners and longitudinal stringers are placed concentrically with respect to the mid-surface of the cylindrical shell. The mass of the rigid body is 12kg. The rigid body is supported by four groups of piecewise isolators, which are mounted on the shell at: $(x,\theta) = (1.6\text{m}, 5\pi/6)$ for Isolator I, $(x,\theta) = (1.6\text{m}, 7\pi/6)$ for Isolator II, $(x,\theta) = (2.4\text{m}, 5\pi/6)$ for Isolator III, and $(x,\theta) = (2.4\text{m}, 7\pi/6)$ for Isolator IV. The two ends of the cylindrical shell are clamped. The density of the fluid is $\rho_f = 1.225$ kg/m^3 and the speed of sound is $c_f = 340$ m/s.

The comparison of the vibration responses of the coupled system computed by different circumferential wave modes of the shell is illustrated in Fig.3. The structural responses of the shell are measured at Q_π ($x = L/2, \theta = \pi$) in the normal direction. The stiffness and damping coefficients of each isolator are: $k_1 = 2.5\times10^4$ N/m, $k_2 = 6k_1$ and $c = 50$ N·s/m, and the gap of two suspension springs in each isolator is $\Delta = 4\times10^{-3}$ m. The force acting on the rigid boy is: $F = F_0 \sin(2\pi f_0 t)$ with $F_0 = 800$ N and $f_0 = 18$ Hz. The horizontal axis of the figure represent response frequencies normalized by the forcing frequency of the external force. The predominant response of the coupled system is that the vibrating rigid body periodically interacts with the secondary suspension springs per certain forcing cycle, and the system parameters are repeated periodically. This results in complex nonlinear vibration behaviors of the system. In general, the vibration responses of the shell are dominated by $n = 0:8$, and the contribution of higher-order modes corresponding to $n > 8$ can be neglected. The frequency responses of the shell exhibit a series of super-harmonics υf_0 ($\upsilon = 2,3,\cdots$). It is observed from Fig.3(b) that the displacement amplitudes of the shell corresponding to υf_0 ($\upsilon = 2,3,\cdots$) decrease as the order of the harmonic is increased.

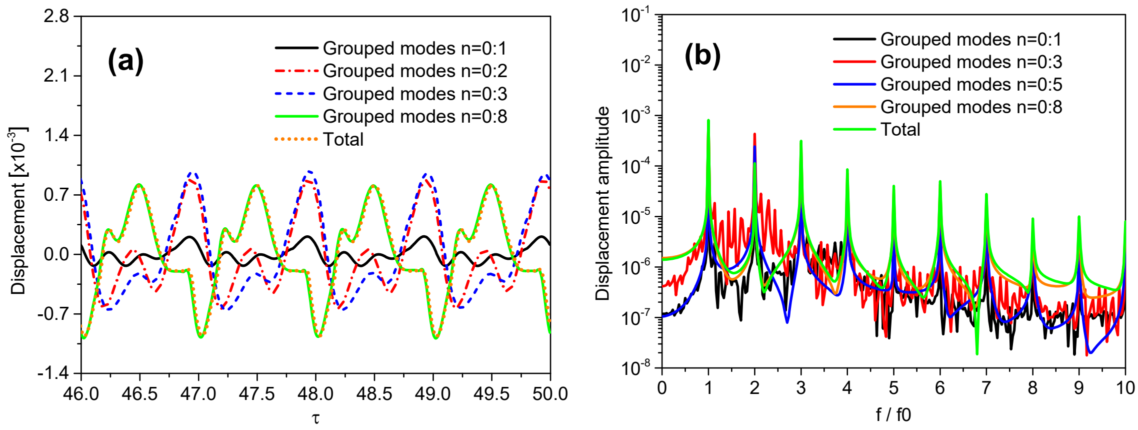

Fig.3 Vibration responses of the shell: (a) time-histories of displacement; (b) frequency spectra responses

The radiated sound responses of the coupled stiffened composite cylindrical shell and piecewise isolation system are examined. The gap of two suspension springs in each isolator is 4×10^{-3} m. The system is under a harmonic excitation $F_{rgd} = F_0 \sin(2\pi f_0 t)$ applied on the rigid body with $F_0 = 300$ N and $f_0 = 18$ Hz. The dependence of the sound pressure responses of the coupled system on the contribution of the circumferential wave modes of the shell is shown in Fig.4. The time-histories of the sound pressure are measured at $\hat{Q}_0$ ($X = 0$ m, $Y = 0$ m, $Z = 20$ m). The results show that the contribution the $n = 0:1$ modes of the shell to the radiated sound of the coupled system is significant. The sound pressure responses determined by $n = 0:2$ and $n = 0:3$ modes are almost overlapped. This implies that the contribution of the $n = 3$ modes of the shell to the radiated sound of the coupled system can be neglected. It is observed from Figs.3(a) that the time-histories of the sound pressure radiated from the coupled system are dominated by $n = 0:8$ modes. The frequency spectra shows that a peak right above the forcing frequency f_0 is presented. However, the energy of the radiated sound is distributed over a broad frequency interval, as a series of super-harmonics υf_0 ($\upsilon = 2, 3, \cdots$) exist in the frequency spectra of the sound pressure. The largest sound pressure amplitude does not appear at the excitation frequency, but at the third-order super-harmonic. In addition, several strong peaks with amplitudes comparable to that at the excitation frequency can be observed in the frequency spectra of the sound pressure. Consequently, the radiated acoustic pressure of the coupled structural system is dominated by super-harmonics.

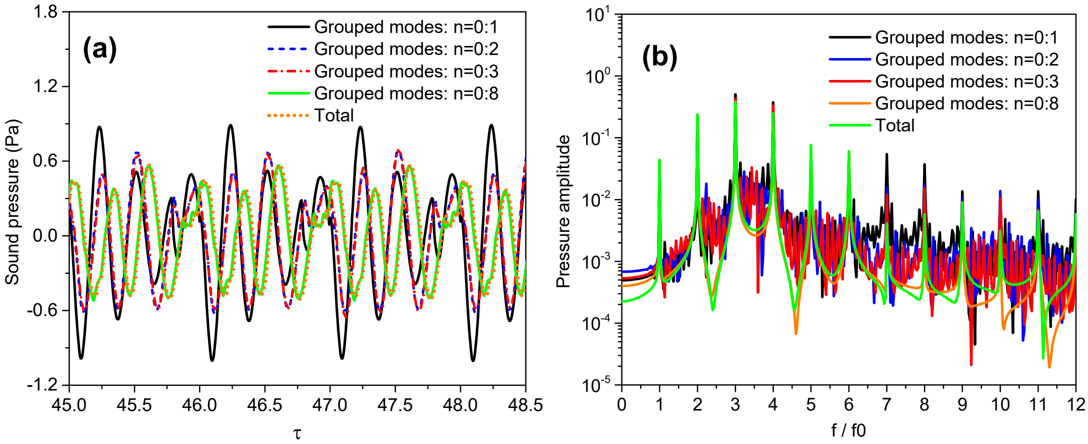

Fig.4 Sound pressure: (a) time-history response; (b) frequency spectra response

4. CONCLUSIONS

The structural and acoustic responses of a coupled stiffened composite cylindrical shell and piecewise isolation system immersed in an infinite acoustic medium are analyzed. A modified variational method is employed to formulate the numerical model of the structural system, and the circumferential rings and longitudinal stringers attached to the shell are treated as discrete elements. Each isolator is represented by a piecewise linear spring and a viscous damper. A time domain Kirchhoff-boundary element method is adopted to model the exterior acoustic fluid. Compatibility conditions on the structure-fluid interface are taken into account in the analysis in order to achieve reasonable structural and acoustic responses of the system. For the coupled system under a harmonic force (excitation frequency f_0) acting on the rigid body, significant peaks at υf_0 ($\upsilon = 1, 2, 3, \cdots$) are found in the frequency spectra of the structural responses of the cylindrical shell as well as the acoustic responses of the radiated sound pressure.

5. ACKNOWLEDGEMENTS

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