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NOISE CONTROL FOR A BETTER ENVIRONMENT

Nonlinear Statistical Energy Analysis modelling of a complex structural-acoustic system

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ABSTRACT

Active noise control has been extensively studied with the aim of improving the acoustic comfort in vehicles. The current state-of-art feed-forward road noise reduction systems are limited by nonlinearities in the transmission path between the sensors and the acoustic response in the car cabin, due to nonlinear characteristics of the components in the suspension system. The aim of the present work is to explore key physical nonlinear characteristics of the vibro-acoustic response of a car by adopting a Statistical Energy Analysis (SEA) approach. A nonlinear structural-acoustic system excited with band filtered Gaussian noise has been designed to study the dependency of the dynamic response to on the degree of nonlinearity. Experimental data has shown that both structural and acoustic responses are dependent on the amplitude of the input at the driving point. Following an SEA approach, the structural subsystem has been modelled as a dissipative mechanism, allowing the derivation of a first-order equation of motion with nonlinear stiffness. Numerical solutions of the nonlinear equation are in good agreement with experimental data. The SEA model is being extended to include the acoustic response of the nonlinear system.

Keywords: Structure-borne sound, nonlinear dynamics, SEA.

1. INTRODUCTION

The vehicle-road interaction can induce vibrations on the structural components of a car that may arise the sound pressure levels in the cabin, known as road noise. Among other sources of noise, such as powertrain noise, wind noise, etc., road noise has been a major concern in automotive industry [1]. Noise-isolating materials are employed to

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reduce the sound pressure levels in a high-frequency range, above 400 Hz. However, these techniques have proven ineffective at the lower-frequency range, mainly because large barriers of damping materials would be required to effectively address the large wavelengths, affecting the weight of the car, cost, fuel consumption, etc.

Active noise control (ANC) can be viewed as a complementary technique to improve the acoustic comfort at lower frequencies, by applying a destructive interference between the noise source and a controllable secondary source [2]. Several limitations have been encountered to significantly reduce the noise levels using ANC, since the nonlinear nature of several components in the suspensions system affects the capability of a linear feed-forward control.

Vibro-acoustic modelling in the stage of design of a control system can help to characterise the effect of nonlinearities in the transmission path, and the influence, or otherwise, on the arising pressure levels in the car cabin, but the large number of degrees of freedom that a complex structure of a vehicle might have limits the capability of a numerical model, mainly due to the computational cost. The Statistical Energy Analysis (SEA) can be viewed as an alternative technique to develop a simple vibro-acoustic model by analysing a subsystem in terms of its vibrating energy, largely reducing the computational cost [3]. However the current state-of-art of SEA is limited to linear systems, and very little can be found in literature regarding to nonlinear SEA. The aim of this work is therefore, to adopt an SEA approach to model a vibrating structure as a dissipative mechanism only, allowing to describe it as a single degree of freedom system, where a nonlinear analysis can be performed in time domain.

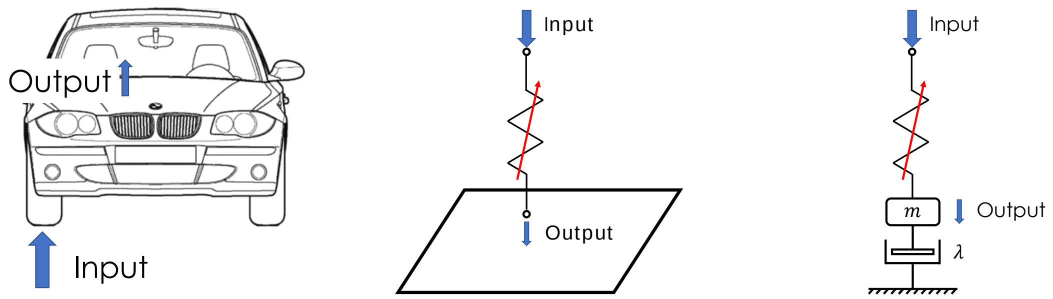
2. NONLINEAR MODELLING IN TIME DOMAIN

Analytical solutions to nonlinear differential equations are seldom available, and are limited to particular nonlinear relationships between the force and displacement, e.g. a cubic nonlinearity of a Duffing oscillator [4]. Therefore, it is common practice to perform a numerical analysis to find a solution of a nonlinear equation, either in time or frequency domain. Even though there are versatile solvers available in packages for scientific computing, the analysis becomes computationally infeasible when the system has multiple degrees of freedom, as it is the case of a structural system.

2.2.1. SEA model of a structural system

As the aim of this work is to explore the characteristics of the influence of stiffness nonlinearities in the response of a structural system, a model of a flat thin plate excited through a nonlinear spring is proposed as a case study. An SEA-based model of the plate considers that this structural system, in absence of boundaries or other coupled elements, is a dissipative mechanism, i.e. the input power is dissipated within the plate only. This assumption allows to describe the plate as an equivalent damper. A schematic representation of the proposed system is shown in figure 1.

The inertial element shown in the equivalent system in figure 1c takes into account the mass m of the accelerometer placed on that point to collect measurements in a physical system, plus any other element of the physical nonlinear device placed on that point.



(a) Physical complex system. *(b) Simple model: nonlinear device and infinite plate.* *(c) Equivalent mass-damper-spring system.*

Figure 1: Dynamic modelling of the vibrations in a structural component of a car (output) induced by the road-vehicle interaction (input). The interface between the input and output is represented by the nonlinear spring.

In a general case, the interface between the nonlinear spring and the plate can be modelled as a rigid flat disc embedded on the plate with six degrees of freedom, i.e three displacements and three rotations, and the dissipative characteristic λ can be viewed as the point impedance of the connection that has the form $\mathbf{Z} = \mathbf{D}/j\omega$, where $\mathbf{D}$ is the 6×6 dynamic stiffness matrix of the system [5]. However, considering out-of-plane motion only, the damping coefficient λ is equivalent to the out-of-plane point impedance Z_p that is given by [6]

$$Z_p = \frac{8\rho h^2 c_L}{\sqrt{12}}, \tag{1}$$

where ρ and h are the density and thickness of the plate, respectively, and c_L is the longitudinal wave speed given by

$$c_L = \sqrt{\frac{\mathrm{E}}{\rho(1-\nu^2)}}, \tag{2}$$

where E and ν are the Young's modulus and Poisson's ratio of the material of the plate, respectively. A flat thin brass plate is used in this model, whose the properties given in table 1.

Table 1: Material properties of brass.

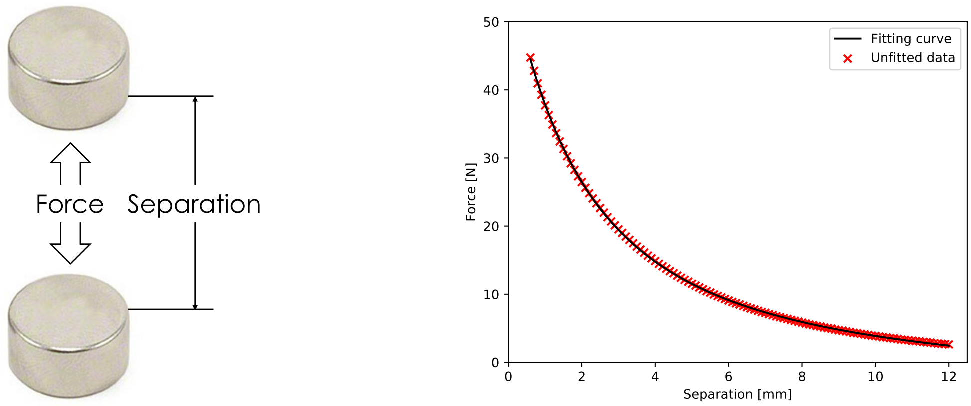
E [GPa]	ν	ρ [kg/m^3]	h [mm]
105	0.346	8470	0.5

2.2.2. Nonlinear stiffness

The repulsive force between two magnets has a nonlinear relationship with the separation, and this stiffness-type nonlinearity has been adopted in this case study. If the magnets are sufficiently small to be represented as single points, the force is inversely proportional to the square of the separation, however, this relationship is more complicated in the near-field depending on the geometry of the magnets. An approximate relationship between two cylindric magnets has the form

$$F = -Ar + \frac{B}{(r+C)^2}, \tag{3}$$

where r is the separation and A, B and C are constants that can be found by fitting the force-separation data provided by the manufacturer (www.magnetexpert.com). A pair of two N42 disc neodymium magnets shown in figure 2a was used as a nonlinear device. Data of force-separation and the fitting curve given by equation 3 are shown in figure 2b.



(a) N42 Neodymium magnets (15 mm diameter).

(b) Repulsive force vs separation.

Figure 2: Nonlinear repulsive force between two magnets.

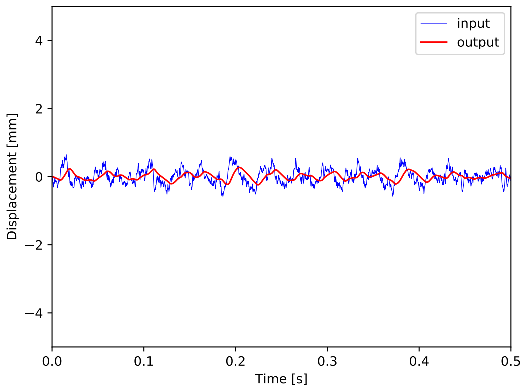
2.2.3. Numerical simulation

The equation of motion of the single degree-of-freedom system, where the response $x(t)$ is the displacement of the plate at the point where one magnet is attached and the input is the prescribed displacement $y(t)$ of the moving magnet above, is given by

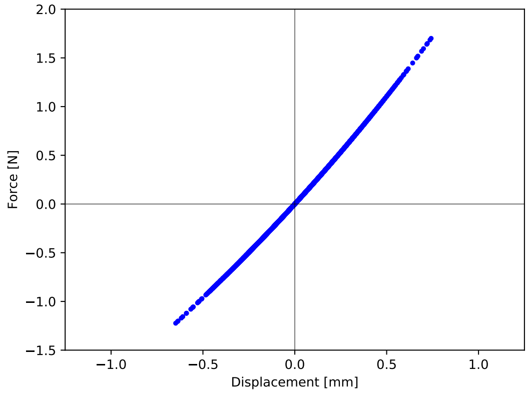
$$m\ddot{x}(t) + \lambda\dot{x}(t) = A\left[y(t) - x(t)\right] + B\left[\frac{1}{(C + r_o - [y(t) - x(t)])^2} - \frac{1}{(C - r_o)^2}\right], \tag{4}$$

where r_o is the separation between the magnets when the system is static. The right-hand side of equation 4 is therefore the dynamic force exerted on the plate, oscillating about the equilibrium.

The input $y(t)$ can have any spectrum, either narrow or broadband. In this study, a random process is considered as the input. In the frequency domain, the ratio between the acceleration and displacement is the square of the radial frequency ω^2. If the acceleration has a flat constant broadband spectrum S_o, then a realistic prescribed input, in terms of displacement, can be viewed as a Brownian process [7] that has a spectrum of the form S_o/ω^2. Numerical solutions of equation 4 were obtained using the ode45 solver in MATLAB for low and high amplitudes of the input. Results for a short recording time of 0.5 s are shown in figures 3 and 4 for an input $y(t)$ with a standard deviation $\sigma = 0.22$ mm and $\sigma = 1.35$ mm, respectively, and a frequency band of 20 – 2000 Hz.

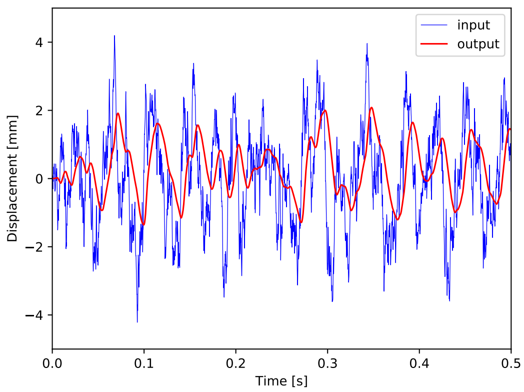


(a) Time data.

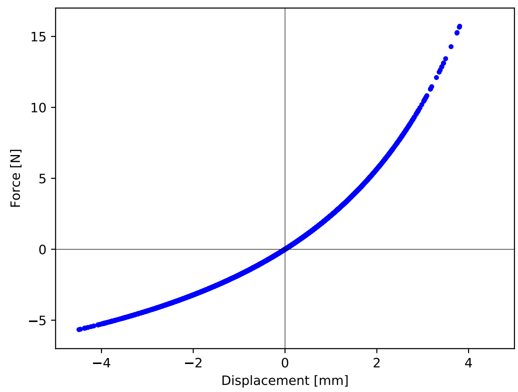


(b) Force vs displacement.

Figure 3: Brownian input and plate response. $\sigma = 0.22$ *mm.*



(a) Time data.



(b) Force vs displacement.

Figure 4: Brownian input and plate response. $\sigma = 1.35$ *mm.*

The degree of linearity of a single-input-single-output system can be estimated by the coherence in the form

$$\gamma^2 = \frac{|S_{xy}|^2}{S_{xx}S_{yy}}, \tag{5}$$

where S_{yy} and S_{xx} are the averaged auto-spectrum of the input and output, respectively, and $|S_{xy}|^2$ is the modulus square of the averaged cross-spectrum. Coherence is equivalent to the ratio between the response computed with the transfer function of the system, or the first Wiener Kernel for nonlinear systems [8], and the actual response of the system. For a linear case, in absence of noise, $\gamma^2 = 1$. On the other hand, for nonlinear systems, a reduction in coherence is expected depending on the amplitude of the input. Simulations were performed with four different amplitudes of the input signal with a frequency band of 20 – 2000 Hz. Coherence for each of the four cases is shown in figure 5.

It can be seen that coherence is near to one in the frequency band where the system is nearly linear, i.e. low-amplitude input, and decreases through the frequency band as the amplitude of the input signal is increased.

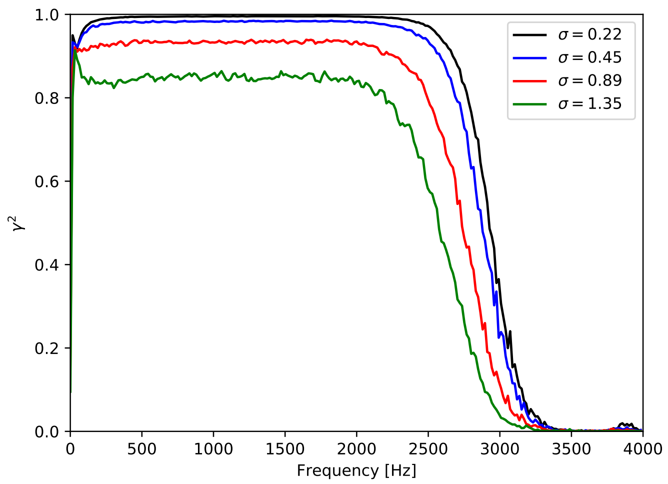


Figure 5: Coherence for different amplitudes of the input.

3. EXPERIMENTAL VALIDATION

The equivalence between a plate and a damper is valid for an infinite plate model with no borders. However, physical systems have boundary condition and energy is not only dissipated within the system, but can be also transferred to a coupled system or radiated to an acoustic medium. In this section, the accuracy of the present model is validated against experimental data collected from a physical structural-acoustic system that is excited with a random input through the nonlinear interface, i.e. the pair of magnets described in the previous section.

3.3.1. Experimental setup

The test rig consists of a brass thin plate with dimensions 0.83×0.6 m, whose properties are given in table 1, placed on an acoustic cavity of 0.83×0.6×0.54 m. The plate is camped on the edges of a box with rigid walls enclosing the acoustic medium. One of the magnets is attached on the top of the plate and the other one on the tip of a rocking beam. The plate is excited then by the force exerted through the magnets due to the oscillation of the beam. A gaussian white noise signal in a frequency band of 20 – 2000 Hz is is given as an input to a test shaker that drives the beam. A schematic representation of the rig can be seen in figure 6. Figure 7 shows a detail of the nonlinear interface between the magnets and the accelerometers placed to collect data of the prescribed input (the motion of the top magnet) and the response of the plate at the driving point (bottom magnet).

3.3.2. Results

The system was tested with two different amplitudes of the time domain gaussian white input signal to the shaker, 10 V for the lower amplitude and 40 V for the higher. The testing time was 50 s, and the acceleration data collected by the accelerometers were further processed to recover the power spectrum in units of squared displacement. The processed data of the accelerometer on the tip of the beam was used as a prescribed displacement $y(t)$ in time domain model of equation 4 to simulate the response of the plate at the driving point. The averaged power spectrum of the experimental data and the

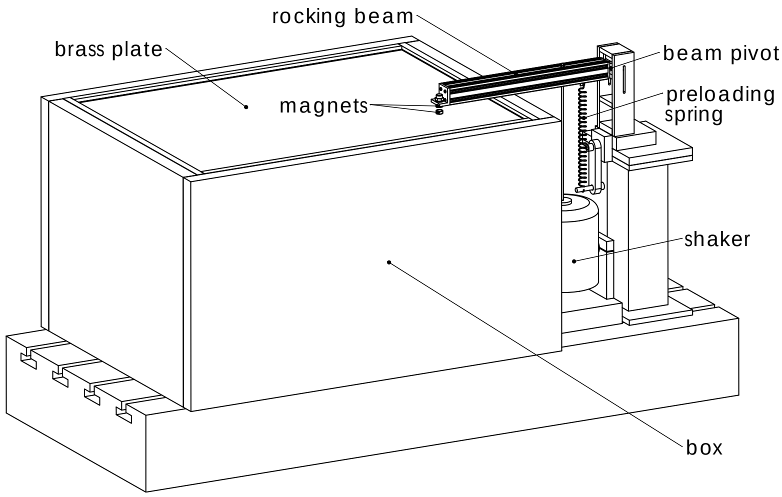


Figure 6: Experimental setup: The brass plate placed on the top of the acoustic box is excited by the repulsive force generated due to the oscillations of the rocking beam.

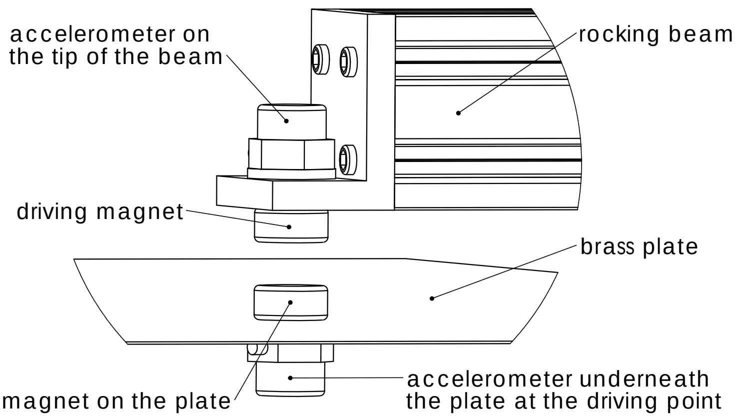


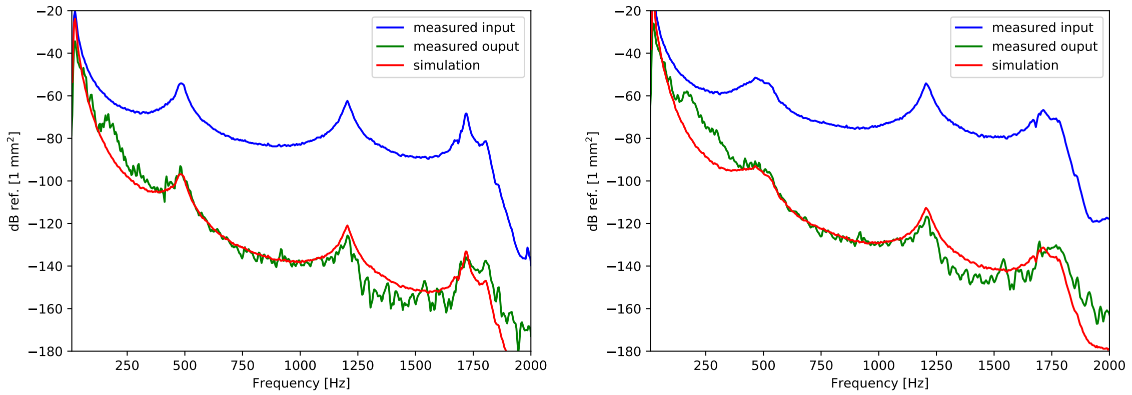
Figure 7: Detail of the tip of the beam and the plate at the diving point: the position of the pair of magnets is shown with the accelerometers that record data of their motion.

results of the simulation are presented in figure 8.

The peaks of the averaged power spectrum of the experimental input (blue curve) shown in figure 8, correspond to the modes of the beam within the frequency band. And the peak of the averaged power spectrum of the experimental output (green curve), corresponds to the first acoustic mode of the cavity.

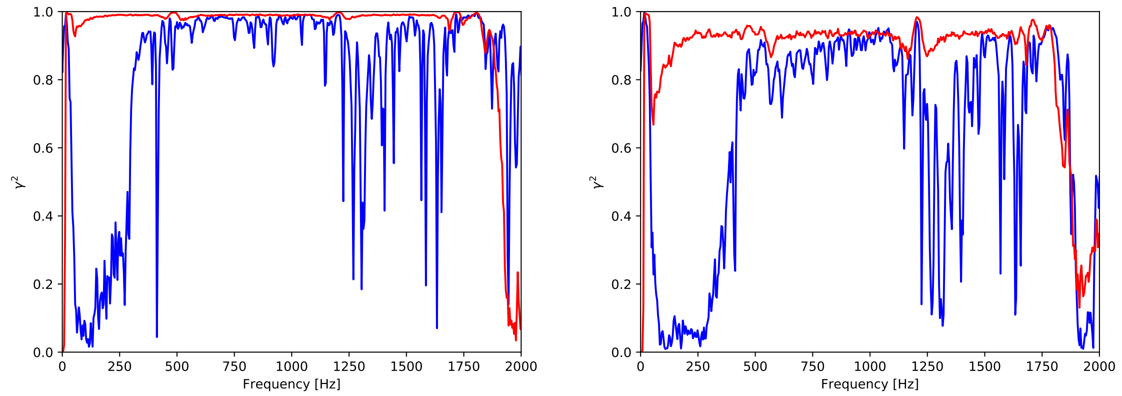
In general, there is a good agreement between the experimental data and results from simulations, particularly in the band of $480 - 1200$ Hz. However, to identify the effect of the nonlinearity in the response of the system, the coherence between the measured data, as well as the results from simulations, and the measured input have been computed and plotted in figure 9.

As it was expected, the experimental reduction in coherence (blue) shown in figure 9, indicates that the response of the system is dependent on the amplitude of the prescribed motion of the driving magnet, and is well estimated by the model. The reduction of



(a) 10 V signal to shaker. *(b) 40 V signal to shaker.*

Figure 8: Averaged power spectrum of the measured input (blue) and output (green), and comparison with the results from simulations (red).



(a) 10 V signal to shaker. *(b) 40 V signal to shaker.*

Figure 9: Experimental coherence (blue) and the coherence computed with the results from simulations (red).

coherence below 250 Hz was found to be due to low frequency noise originated by the shaker. Additionally, above 1200 Hz, the displacement is small and close to the noise floor, therefore, the further reduction in coherence above this frequency is due to uncorrelated noise in the measured signals.

4. CONCLUSIONS AND FURTHER WORK

The representation of a structural system as a damper allows to reduce the vast number of degrees of freedom of a physical structural system to a single degree of freedom of a nonlinear mass-spring-damper system, whose solution can be computed numerically. Even though a physical system cannot be always represented as an infinite system with no boundaries, experimental evidence has shown that the approximation is accurate, as it estimates the averaged power spectrum of the response, as well as the reduction in coherence between the input and the response due to the nonlinear interface in the force transmission path.

This work has been focused in the study of the force transmission path and the effect of the nonlinearity in the response of a structural system, and it is limited to simple systems, e.g. a flat thin plate. More complex systems, as is the case of a coupled structural-acoustic system, require a further expansion and the analysis becomes infeasible in time domain, therefore, the current stage of this research is focussed in deriving a generalised hybrid SEA model that includes a nonlinear interface of the power input to the system.

5. ACKNOWLEDGEMENTS

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