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Design of periodic pipes for structural vibration filtering

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ABSTRACT

This paper concerns the design of periodic pipes seen as structural waveguides in which flexural, torsional and longitudinal waves can propagate. To control the propagation of these vibrations and reduce their possible sound borne annoyance, Bragg band gaps effects can be reached by architecting the variations of the cross section. As a result, the tailored structural waveguide acts as a vibration filter. In industrial contexts, these filters can be used in a complex assembly for decoupling vibroacoustic subsystems. In many practical cases, all kinds of waves usually co-exist because of inevitable structural couplings. As a consequence, optimization of the geometrical and mechanical features of the unit cell has to be performed to mitigate at the same time several kinds of waves. For this purpose, dispersion relations are derived and analyzed based on the Floquet method and numerical Finite Element simulations. Demonstrators of finite size are also studied in order to illustrate the potential for practical applications.

Keywords: Floquet method, Periodic, Automotive
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1. INTRODUCTION

Vibration damping is becoming an important step in the automotive industry. Indeed, the trend is for the weight of structures to be reduced, which increases wave propagation phenomena. To do this, the classic solutions are the use of an elastic coating. Which leads to heavy treatments and an increase in the mass of the structure. These solutions are not compatible with the current lightweight philosophy. This is why the design of lightweight materials is a major technological and scientific challenge to meet the needs of industry.

One of the solutions combining structural lightening and vibration damping is the use of periodic waveguides. These structures are made of an assembly of several identical unit cells. Many studies have been conducted to evaluate the potential of periodic structures in mitigating vibrations as [1–5]. All have highlighted the existence of a frequency range where the propagation of vibration is forbidden, known as Bragg stop bands that can be reached for any type of waves: flexural, longitudinal and torsional.

Usually the effects of periodic waveguides are studied on structures that do not reflect the reality of the industry. Indeed, the geometry of an industrial structure leads to waves coupling phenomena. It is possible to design stop bands with the properties of unit cells who built the waveguide. The goal of the work presented here is to build a unidirectional waveguide to mitigate at the same time several kinds of waves.

This paper is presented in two sections. The method used to predict stop bands, from a unit cell is described in section 1, and in section 2, the unit cell is optimized in order to obtain a multiple stop band that is able to mitigate all types of waves over the same frequency range.

2. FLOQUET MODEL OF A PERIODIC PIECEWISE CONSTANT BEAM

The model used in this work for calculating the dispersion relations is based on the Floquet method described in [6] and [1]. It is here applied to a structural waveguide of periodic piecewise constant mechanical properties, as represented in Figure 1. The unit cell consists of the assembly of two straight segments of pipe of circular cross-sectionwith same internal diameter butdifferent external diameter and material. All geometrical and mechanical parameters are defined in Table 1.

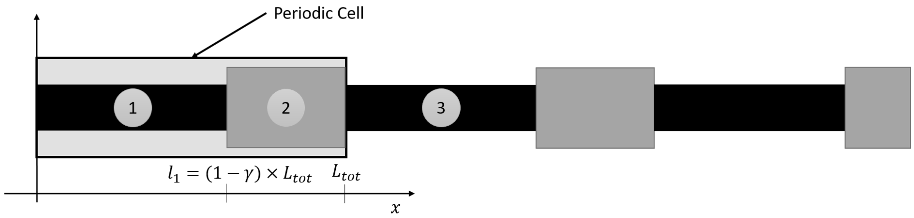

Figure 1: Example of a periodic structure as they are used in this work

To predict where the band gap is tuned, we calculated the Bloch waves numbers for each frequency, $k_b(\omega)$ this method is convenient for the prediction of the structural

Parameters	definition
L_{tot}	length of the unit cell [m]
γ	length's ratio
$l_1 = (1-\gamma) \times L_{tot}$	length of the first section of the unit cell [m]
$l_2 = \gamma \times L_{tot}$	length of the second section of the unit cell [m]
R_1	radius of the first section of the unit cell [m]
β	radius' ratio
$R_2 = \beta \times R_1$	radius of the second section of the unit cell [m]
E	Young's modulus [Pa]
ρ	density [kg/m^3]
ν	Poisson's coefficient

Table 1: Parameters used to describe the unit cell

damping, because it gives acces to the complex wavevectors.

The method is implemented for each of the 3 types of wave of interest. Only the derivations corresponding to the bending waves are recalled below for brevity. The structural waveguide is assumed to behaves a 1D beam so that in each segment i, the flexural displacement w_i which harmonic flexural motion satisfies to the Euler-Bernoulli harmonic motion equation :

$$\frac{\partial^4 w_i(x)}{\partial x^4} - k_i^4 w_i(x) = 0, \tag{1}$$

Where $k_i^4 = \frac{\rho S_i \omega^2}{E_i I_i}$ is the flexural wavenumber, S_i the cross section, E_i the flexural rigidity and where the time convention $e^{i\omega t}$ is omitted. Solution of Eq. 1 can be written as the combinaison of four waves :

$$w_i(x) = \sum_{j=1}^{4} B_{ij} e^{ik_{ij}x}. \tag{2}$$

To find the unknown amplitudes B_{ij}, 4 continuity conditions for the displacement, the slope, the bending moment and the shear force are written at the interface between segments 1 and 2 and between segments 2 and 3. The periodicity of the structure is discribed with 4 additional Floquet conditions, given by $w_i(0) = e^{ik_b} w_3(L_{tot})$ for the displacement for example, with k_b the Floquet number. All details can be found in [6].

All conditions are reported in Eq. 2, leading to a following set of linear equations,

$$\begin{bmatrix} M \end{bmatrix} . \begin{bmatrix} B_{11} \\ ... \\ B_{33} \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}, \tag{3}$$

where M is a frequency dependent $[12 \times 12]$ matrix. For each considered ω i a given frequency range, the solutions for k_b are obtained by solving numerically $\det(M(\omega)) = 0$.

The same procedure is also applied to solve the wave equation corresponding to both torsion and compression 1D motions.Finally, 3 linear systems as Eq. 3 are independently solved fair each ω, giving all the dispersion relations of such a structure seen as a 1D multiwave structural waveguide.

3. DESIGN AND OPTIMIZATION OF A MULTIWAVES FILTER

First, the results fromthe Floquet method are validated by comparison with FEM simulations (3D elasticity module of the structural mechanics COMSOL package). Figure 2 shows that torsion and compression movements arevery well represented by the Floquet model. An increasing drift with frequency and observed for bending movement, which reflects the validity limit of the Euler-Bernoulli low frequency model.

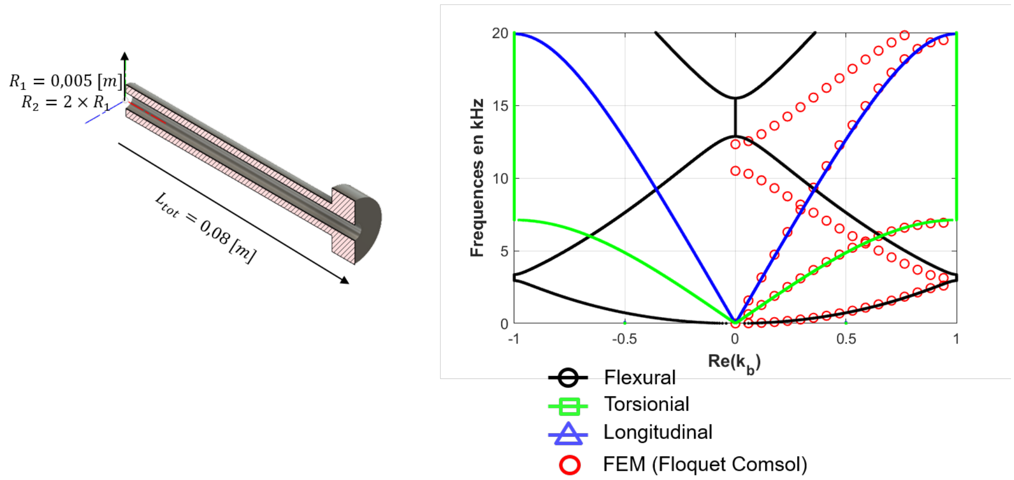

Figure 2: Dispersion curve for the unit cell modelised at the left.

The objective is now to tune the stop strips in such a way as to obtain a total vibration filterable to mitigate the 3 types of waves in the same frequency range. Achieving this in a monolithic guide is probably impossible. However, by choosing a bi-material structure, it is possible to slow down the torsion and compression waves in order to bring their correponding stop bands down to low frequencies where bending bands appear. Aluminium ($E = 71E9$ [Pa]; $\rho = 2700$ [kg/m^{-3}]; $\nu = 0.3$) for segment 1 and nylon ($E = 2E9$ [Pa]; $\rho = 1150$ [kg/m^{-3}]; $\nu = 0.4$) for segment 2 are here choosen for their large mechanical properties contrast.

The *fminsearch* function of MatLab based on a Nelder-Mead algorithm is run to reach an optimal design. Objectives of the optimization's function are to obtain the largest stop band in low frequency for simultaneously each type of waves, Ref [7]. The tuning parameters are L_{tot}, γ, R_1 and β.

Figure 3 shows the result of the process for both the geometrical layout (left) and dispersion relation (right). Stop bands are aligned for the 3 types of waves over two wide frequency ranges so that between 5 and 15 kHz, the structural guide behaves mainly as a total filter. Performances are correctly verified with COMSOL simulation, more representative of the phenomena.

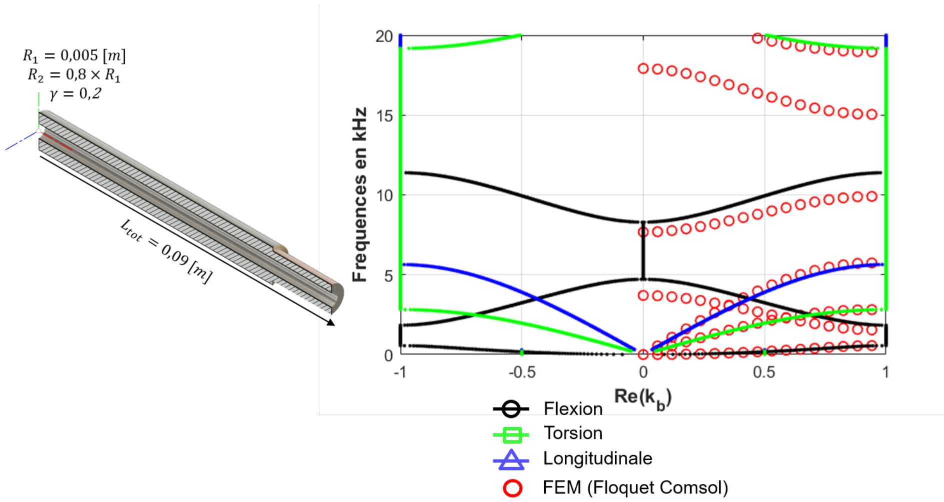

Figure 3: Dispersion curve for an optimized unit cell composed by two materials with different sound speed property.

4. CONCLUSIONS

This paper presents a methodology for the design of "total" structural filter able to mitigate all waves likely to propagate in an elongated bar, seen as a structural guide. To do that, 2 materials are combined according to a simple periodic piecewise constant geometry. As a result, stop bands for all waves are aligned in a wide frequency range corresponding typically to "middle frequency" NVH problems. The Floquet method, with its very low computation cost, is effective when implemented in a numerical optimization procedure to obtain a pre-design. FEM simulations arethen useful to obtain more precisely the dispersion relations and thus the frequency template of the resulting filter.

Further works may concern the optimization of this type of filters at lower frequencies. Numerical analysis and then experimental demonstration of the performance of filters of finite lengths based on such designs would also be interesting to evaluate the application potential.

5. ACKNOWLEDGEMENTS

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