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Vibroacoustical sensitivities of stiffened structures due to attached mass-spring-dampers with uncertain parameters

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ABSTRACT

The slightest manufacturing tolerances and variances of material properties can have a significant impact on structural modes. An unintentional shift of eigenfrequencies towards dominant excitation frequencies, however, may lead to increased vibration amplitudes of the structure resulting in radiated noise, which will reduce the passenger comfort inside an aircraft cabin.

The focus of this paper is on so-called non-structural masses, also known as the secondary structure, that are attached to the primary structure via clips, brackets as well as shock mounts and which constitute a major part of the overall structural mass. Using the example of a simplified fuselage panel the vibro-acoustical consequences of parameter uncertainties in linking elements is studied.

For the specification and quantification of parametric uncertainty, the Fuzzy Arithmetic provides a suitable framework. It is capable of creating deterministic parameter combinations, which then are re-transformed into fuzzy outputs and which subsequently can be used for the evaluation of the simulation results regarding target quantities and influence of each parameter on the overall system behaviour. To assess the vibrations of the fuzzy structure and by taking into account

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the excitation spectra of engine noise, modal and frequency response analyses are conducted.

Keywords: Vibro-acoustics, Parameter Uncertainties, Fuzzy Arithmetic
I-INCE Classification of Subject Number: 42

1. MOTIVATION

State-of-the-art interior noise models, both experimental and numerical, often have a "mass gap". This gap arises since the consideration of secondary structural components, which are generally small in mass but large in numbers, is challenging, not to say tedious, and therefore often neglected. Nevertheless, it is well known, that in the case of an aircraft structure the secondary masses comprise a significant part of the overall mass of the structure, wherefore the ambition to develop detailed acoustical models which include non-structural masses in an efficient way is an ongoing process.

In the numerical domain, simple approaches where the secondary masses are condensed to point masses and subsequently connected to the primary structure have already been implemented and evaluated, see Seidel [1]. To meet a more realistic representation of the acoustical model, the scope of this paper is to take into account linking elements and their inherited parametric uncertainties, which are assumed to be damping and stiffness coefficients as well as the overall mass of the connected structural components. Additionally, means to assess parametric sensitivities for a consequent identification of worst-case scenarios regarding structural response and alteration options for structural components shall be provided.

In the context of the German aviation research program LuFo V-2 Acoustic FlightLAB an acoustic demonstrator was developed, both physically at the Center of Applied Aeronautical Research (ZAL) in Hamburg and numerically via NASTRAN/PATRAN, and validated up to a frequency of 500 Hz by Wandel [2]. The present work will take the geometric and material properties of this demonstrator into account for better scientific exchange and interdisciplinary compatibility.

2. UNCERTAINTY MODELLING

In contrast to the well known and easy to implement Monte Carlo Sampling method, which randomly generates deterministic values for parameters with known probability density functions and therefore needs a lot of samples to cover the full range of uncertainty, the so-called Fuzzy Arithmetic provides a procedure to create structured parameter combinations which subsequently will be used to evaluate the results of a model subjected to uncertainty.

The concept of fuzzy sets originally was developed by Zadeh [3] as an extension of the many-valued logic, leading to fuzzy logic [4] applied to linguistic variables, and subsequently resulting in a growing interest in engineering sciences for describing parameter uncertainties in numerical calculations, for control, data analyses and management as discussed in [5]. However, standard Fuzzy Arithmetic has some limitations. Hanss in [6] shows the phenomenon of overestimation of results in engineering mechanics, depending on the actual form of the fuzzy rational expression. As a solution to overcome these drawbacks, he introduces the Advanced Fuzzy Arithmetic.

In the following subsections, a definition of FUZZY NUMBERS and, based on Hanss' ADVANCED FUZZY ARITHMETIC, the general procedure to simulate a system with uncertain input parameters will be given.

2.1. Fuzzy sets and fuzzy numbers

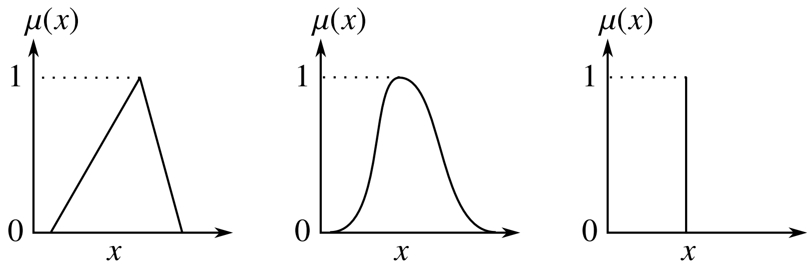
Unlike classical crisp sets, a fuzzy set does not distinguish between members and non-members. The membership to a certain set $\tilde{A}$ is described by a membership function μ which takes any value between 0 and 1

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in \mathbb{R},\ \mu_{\tilde{A}}(x) \in [0,1]\}\,. \tag{1}$$

According to HANSS a fuzzy number $\tilde{p}$ is a fuzzy set $\tilde{P} \in \tilde{P}(\mathbb{R})$ characterized by the following four conditions:

- $\tilde{P}$ is normal ($\mathrm{hgt}(\tilde{P}) = 1$).
- $\tilde{P}$ is convex.
- There is exactly one $\overline{x} \in \mathbb{R}$ with $\mu(\overline{x}) = 1$, which is called the modal value.
- The membership function $\mu(x)$, $x \in \mathbb{R}$ is at least piecewise continuous.

Obeying these rules, easily fuzzy number representations of parameter uncertainties can be developed. Examples of plausible fuzzy shapes are shown in figure 1.



(a) Triangular (b) Quasi-Gaussian (c) Crisp

Figure 1: Fuzzy Number representations

2.2. The General Transformation Method

A simulation of a fuzzy-parameterized system with n uncertain parameters $\tilde{p}_i$, $i = 1, 2, \ldots, n$ and a fuzzy output $\tilde{q} = F(\tilde{p}_1, \tilde{p}_2, \ldots, \tilde{p}_n)$ can be conducted by

1. *Decomposition of input fuzzy parameters*

   The μ-axis is subdivided into m intervals of length $\Delta\mu = 1/m$ (see figure 2), resulting in $(m+1)$ membership levels and their corresponding discrete values

   $$\mu_j = \frac{j}{m}, \quad j = 0, 1, \ldots, m. \tag{2}$$

   Secondly, each fuzzy number $\tilde{p}_i$ is decomposed into α-cuts, leading to

   $$P_i = \left\{X_i^{(0)}, X_i^{(1)}, \ldots, X_i^{(m)}\right\}, \quad i = 1, 2, \ldots, n, \tag{3}$$

   where each set P_i consists of the intervals

   $$X_i^{(j)} = [a_i^{(j)}, b_i^{(j)}], \quad a_i^{(j)} \le b_i^{(j)}, \quad j = 1, 2, \ldots, m. \tag{4}$$

2. *Transformation of input intervals*

   The intervals $X_i^{(j)}$ of each level of membership μ_j are transformed into arrays $\hat{X}_i^{(j)}$ of the form

$$\hat{X}_i^{(j)} = \underbrace{\left(\left(\gamma_{1,i}^{(j)}, \gamma_{2,i}^{(j)} \ldots, \gamma_{(m+1-j),i}^{(j)}\right), \ldots, \left(\gamma_{1,i}^{(j)}, \gamma_{2,i}^{(j)}, \ldots, \gamma_{(m+1-j),i}^{(j)}\right)\right)}_{(m-j+1)^{i-1}\cdot(m-j+1)-\text{tuples}}, \tag{5}$$

   with

$$\gamma_{l,i}^{(j)} = \underbrace{(c_{l,i}^{(j)}, \ldots, (c_{l,i}^{(j)})}_{(m-j+1)^{n-1}\text{ elements}} \tag{6}$$

   and

$$c_{l,i}^{(j)} = \begin{cases} a_i^{(j)} & \text{for} \quad l = 1 & \text{and} \quad j = 0, 1, \ldots, m, \\ \frac{1}{2}\left(c_{l-1,i}^{(j+1)} + c_{l,i}^{(j+1)}\right) & \text{for} \quad l = 2, 3, \ldots, m-j & \text{and} \quad j = 0, 1, \ldots, m-2, \\ b_i^{(j)} & \text{for} \quad l = m-j+1 & \text{and} \quad j = 0, 1, \ldots, m. \end{cases} \tag{7}$$

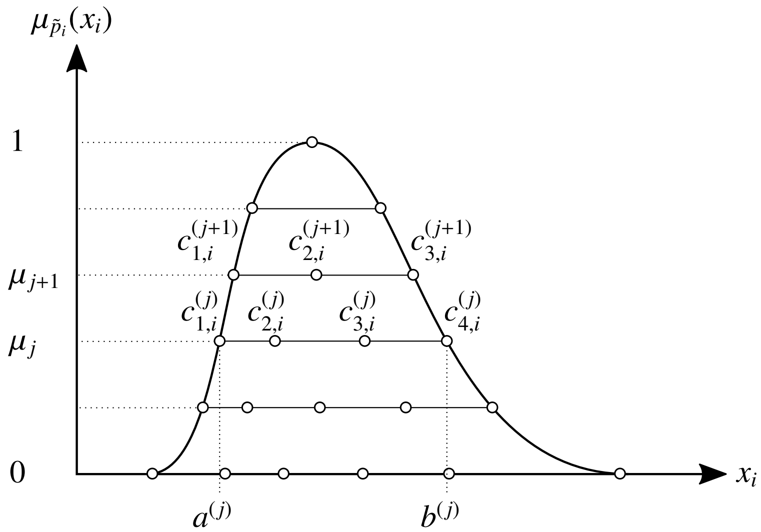


Figure 2: Decomposition pattern according to the General Transformation Method

3. *Model evaluation*

   The evaluation of the given fuzzy-parameterized system expressed by the functional F then can be carried out by evaluating each column of the arrays separately using the means of classical arithmetic embedded in deterministic solvers. The output $\tilde{q}$ then can be combined into the arrays $\hat{Z}^{(j)}$, where the kth element ${}^k\hat{z}^{(j)}$ is determined by

$${}^k\hat{z}^{(j)} = F\left({}^k\hat{x}_1^{(j)}, {}^k\hat{x}_2^{(j)}, \ldots, {}^k\hat{x}_n^{(j)}\right), \tag{8}$$

   with ${}^k\hat{x}_i^{(j)}$ as the kth element of $\hat{X}_i^{(j)}$.

4. *Re-transformation of the output array*

   Re-transforming the arrays $\hat{Z}^{(j)}$ according to the recursive formulas

$$\left[a^{(j)}, b^{(j)}\right] = \begin{cases} \left[\min(a^{(j+1)}, {}^{k}\hat{z}^{(j)}),\ \max(b^{(j+1)}, {}^{k}\hat{z}^{(j)})\right], & j = 0, 1, \ldots, m-1 \\ {}^{k}\hat{z}^{(j)}, & j = m \end{cases} \tag{9}$$

   gives the decomposed representation of the fuzzy-valued output $\tilde{q}$, expressed by the set

$$Q = \left\{Z^{(0)}, Z^{(1)}, \ldots, Z^{(m)}\right\}. \tag{10}$$

5. *Re-composition of output intervals*

   By recomposing the intervals $Z^{(j)}$ of the set Q according to their levels of membership μ_j, the fuzzy representation of $\tilde{q}$ can be obtained.

2.3. Analysis of the fuzzy-valued output

For the quantification of each individual influence mentioned above a general methodology is proposed in [6]. Additionally to the in the last section explained step how to re-transform the output array $\hat{Z}^{(j)}$, its values can be used to determine the GAIN FACTORS $\eta_i^{(j)}$

$$\eta_i^{(j)} = \frac{1}{(m-j+1)^{n-1}\left(b_i^{(j)} - a_i^{(j)}\right)} \sum_{k=1}^{(m-j+1)^{n-i}} \sum_{l=1}^{(m-j+1)^{i-1}} \left({}^{s_2}\hat{z}^{(j)} - {}^{s_1}\hat{z}^{(j)}\right) \tag{11}$$

with

$$s_1(k,l) = k + (m-j+1)(l-1)(m-j+1)^{n-i} \tag{12}$$

$$= k + (l-1)(m-j+1)^{n-i+1} \tag{13}$$

$$s_2(k,l) = k + [(m-j+1)\,l - 1]\,(m-j+1)^{n-i} \tag{14}$$

The values $a_i^{(j)}$ and $b_i^{(j)}$ are the lower and upper bounds of the Interval $X_i^{(j)}$, and ${}^{s_1}\hat{z}^{(j)}$ and ${}^{s_2}\hat{z}^{(j)}$ denote the s_1-th and s_2-th element of $\hat{Z}^{(j)}$, respectively.

To obtain a non-dimensional form of the influence measures with respect to the different physical dimensions of $\tilde{p}_i$, the STANDARDIZED MEAN GAIN FACTORS κ_i can be determined as an overall measure of influence according to

$$\kappa_i = \frac{\sum_{j=1}^{m-1} \mu_j \left|\eta_i^{(j)}\left(a_i^{(j)} + b_i^{(j)}\right)\right|}{2\sum_{j=1}^{m-1} \mu_j} = \frac{1}{m-1} \sum_{j=1}^{m-1} \mu_j \left|\eta_i^{(j)}\left(a_i^{(j)} + b_i^{(j)}\right)\right|. \tag{15}$$

Finally, the DEGREES OF INFLUENCE ρ_i as a relative measure of influence can be determined via

$$\rho_i = \frac{\kappa_i}{\sum_{q=1}^{n} \kappa_q} = \frac{\sum_{j=1}^{m-1} \mu_j \left|\eta_i^{(j)}\left(a_i^{(j)} + b_i^{(j)}\right)\right|}{\sum_{q=1}^{n} \sum_{j=1}^{m-1} \mu_j \left|\eta_q^{(j)}\left(a_q^{(j)} + b_q^{(j)}\right)\right|}, \tag{16}$$

where

$$\sum_{i=1}^{n} \rho_i = 1. \tag{17}$$

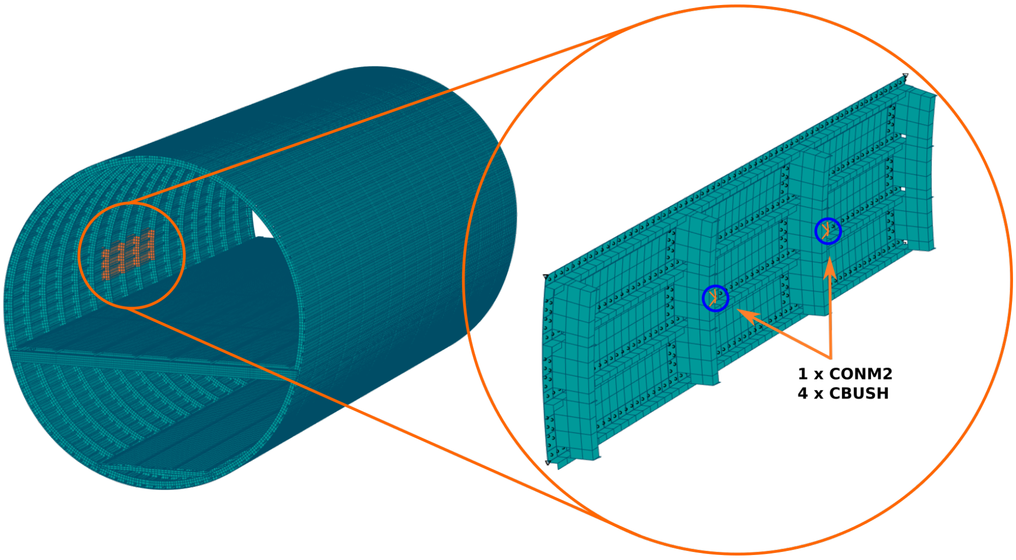


Figure 3: Acoustic Flight-LAB Demonstrator and isolated test setup

3. APPLICATION

In order to assess the influence of parametric uncertainties from linking elements in airframes during vibro-acoustical analyses, a small fuselage panel was isolated from the geometry of the Acoustic Flight-LAB Demonstrator (see figure 3) to help to identify the impact of selected masses in a limited domain.

3.1. Test setup

The test setup consists of a panel, which comprises a skin field of approximately $50\,cm$ x $160\,cm$ stiffened by four frames as well as four stringers. It is simply supported on its corners and two generic masses of $\mathrm{m} = 1\,kg$ are attached to the middle of the two innermost frames via four spring-dashpot elements, respectively. The Finite Element (FE) representation of the given structure is composed of elements formulated by the numerical solver MSC.NASTRAN. The primary structure is defined by CQUAD4-elements referring to PSHELL-cards, and the attached secondary masses are discretized by CONM2- and CBUSH-elements and their corresponding PBUSH-entries. For further details on the element definitions used in MSC.NASTRAN, please refer to the MSC.NASTRAN Quick Reference Guide [7].

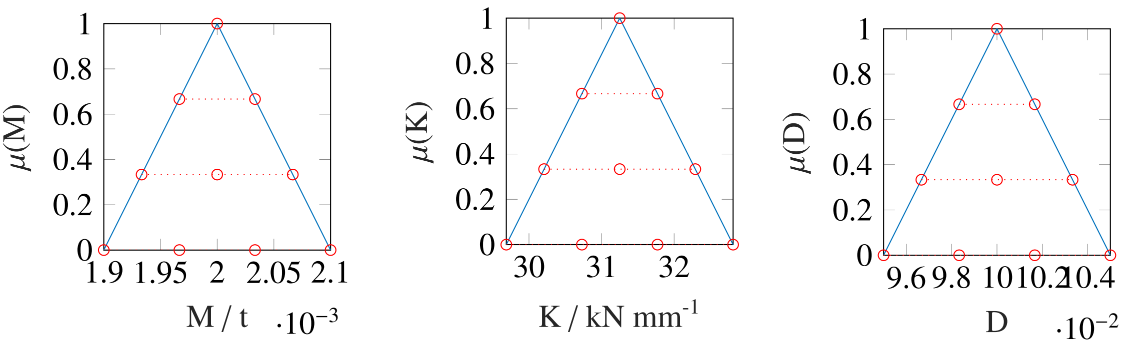
The uncertain properties for the mass of each CONM2-element and the stiffness as well as damping coefficients for the PBUSH-cards can be found in table 1.

Table 1: Parameter uncertainties

parameter	**modal value**	**deviation**	**remark**
$\tilde{p}_1$ = mass	$\overline{x}_1 = 1\,kg$	triangular shape, ±2%	
$\tilde{p}_2$ = stiffness	$\overline{x}_2 = 32.5\,kN/mm$	triangular shape, ±5%	all six *dof*
$\tilde{p}_3$ = structural damping	$\overline{x}_3 = 0.1$	triangular shape, ±5%	all six *dof*

3.2. Simulation results

The decomposition of the uncertain parameters MASS, STIFFNESS and DAMPING according to section 2.2 using four α-cuts leads to the fuzzy presentation depicted in figure 4. Subsequently an overall of 100 deterministic combinations is generated which are used for MODAL ANALYSES and FREQUENCY RESPONSE ANALYSES. While in the case of MODAL ANALYSIS the eigenfrequencies up to 500 Hz can be calculated directly, in the latter analysis the mean tangential displacement of each node located on the skin will be taken into account to obtain a physical figure representing an acoustical sound source.



(a) Mass (b) Stiffness (c) Damping

Figure 4: Uncertain parameters

3.2.1 Modal Analysis

The MODAL ANALYSIS yields 64 modes in the frequency range 10 Hz to 500 Hz, which are shown in a fuzzy representation in figure 5.

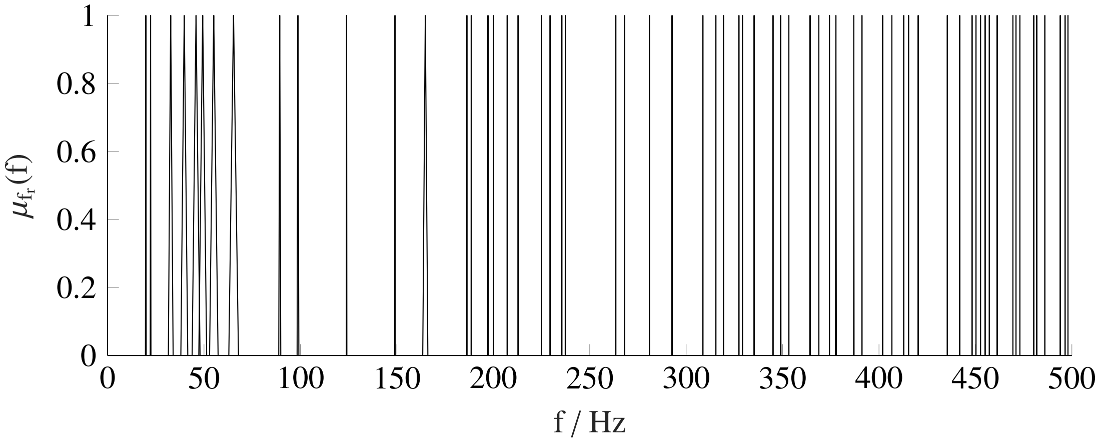
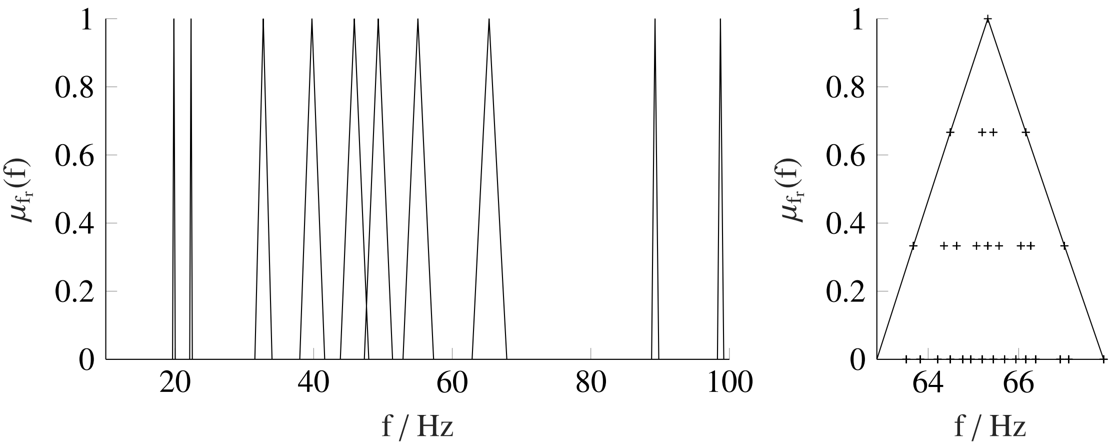


Figure 5: Fuzzy modes #1 - #64

For the sake of clarity, only the fuzzy results of the modes #1 to #10 are depicted in figure 6a. Obviously, the fuzziness of the input parameters has the biggest impact on modes #3 to #8, resulting in an area of frequencies from about 30 Hz up to 70 Hz in which an exciting frequency under some unfavourable parameter combinations most certainly will match a resonant frequency of the structure. For example, the 8th mode spans from

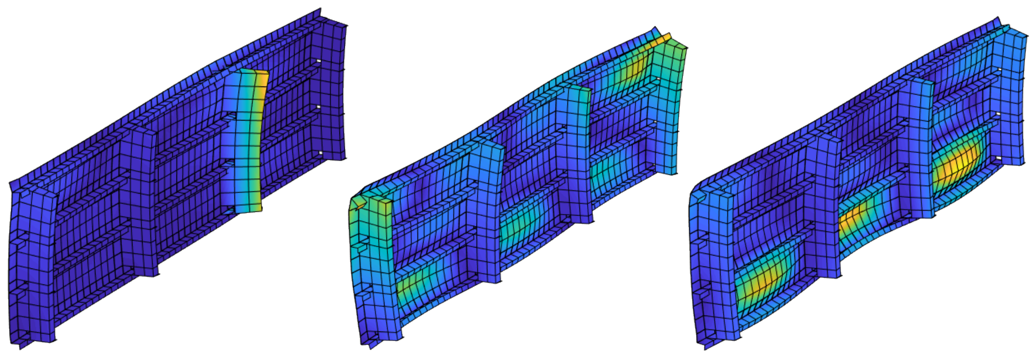
63 Hz to 68 Hz (see figure 6b, each crosses indicates the result of a deterministic parameter combination).



(a) Fuzzy modes #1 - #10 (b) Fuzzy mode #8

Figure 6: A detailed look on modes #1 - #10

Additionally from figure 7 it can be seen, that the transition from global to local skin modes occurs from mode #13 to mode #14, hence the assumption can be made, that parametric uncertainties in non-structural masses most likely will have a bigger effect on the structural response below the first skin mode than above.



(a) $\overline{f}_{13} = 164.75\,Hz$ (b) $\overline{f}_{14} = 186.3\,Hz$ (c) $\overline{f}_{15} = 188.56\,Hz$

Figure 7: Modeshapes #13 to #15

3.2.2 Frequency Response Analysis

The results from the Frequency Response Analyses are shown in figures 8 and 9. The red line represents the crisp result, where the membership is $\mu(A(f)) = 1$. The grey lines edge the upper and lower bounds of each α-level, where the colour gradient represents the membership value. Throughout the full frequency domain the impact of the uncertainties is observable but with the supplementing information obtained during the modal analysis, it is obvious that the fuzziness of eigenfrequencies and the fuzzy frequency response are correlated.

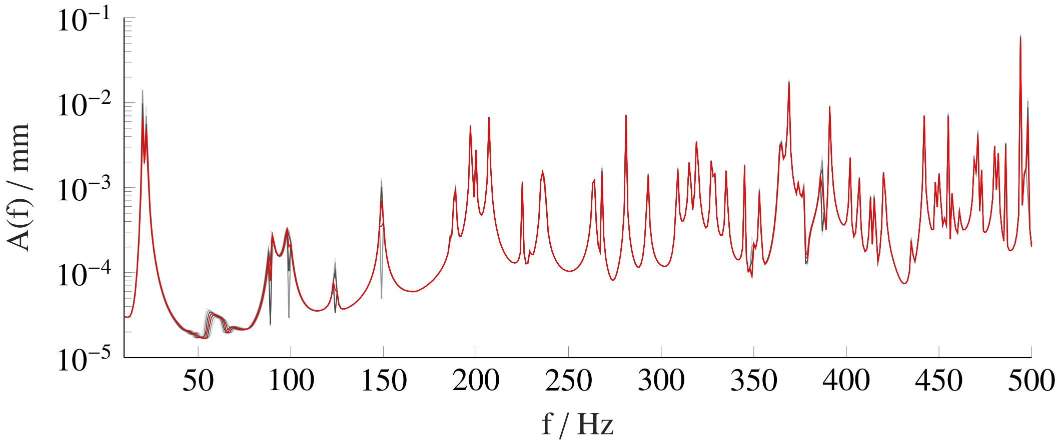
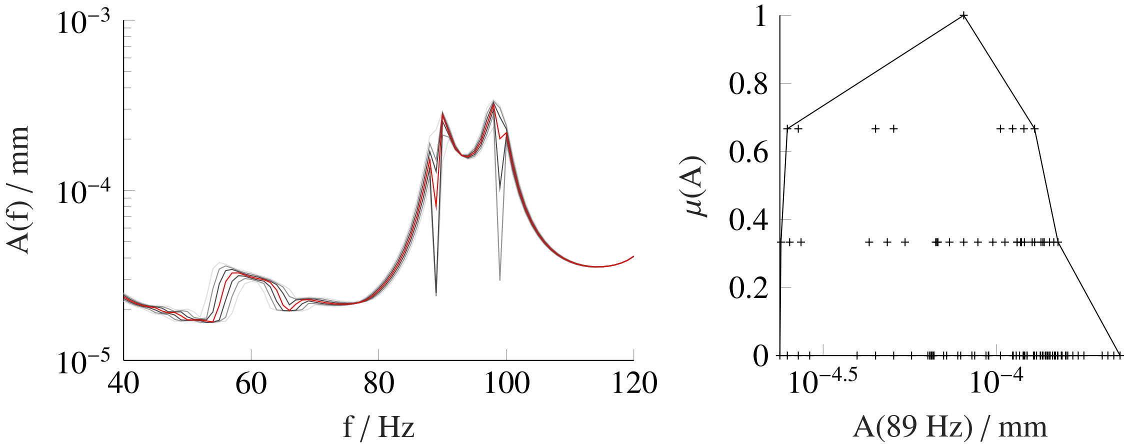


Figure 8: Fuzzy frequency response 10 Hz to 500 Hz



(a) Fuzzy frequency response 40 Hz to 120 Hz

(b) Fuzzy frequency response at 89 Hz

Figure 9: A detailed look on fuzzy frequency response 40 Hz to 120 Hz

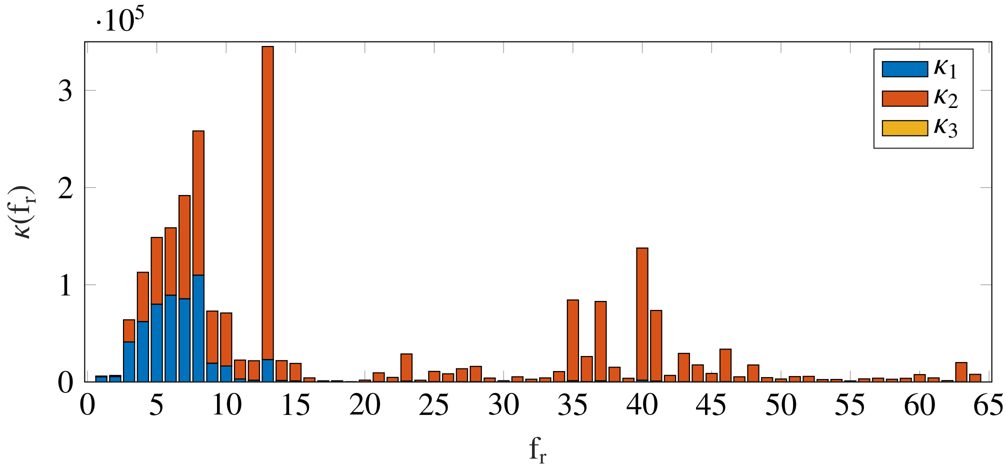
3.3. Sensitivities

As seen in the preceding section the fuzzy results $\tilde{q}$ of eigenfrequencies and frequency response only represent the overall influence of the uncertain parameters $\tilde{p}_1, \tilde{p}_2$ and $\tilde{p}_3$ combined. However, there is evidence that each model parameter contributes differently to the overall degree of fuzziness, depending on the analysed frequency as well as the parameter definition.

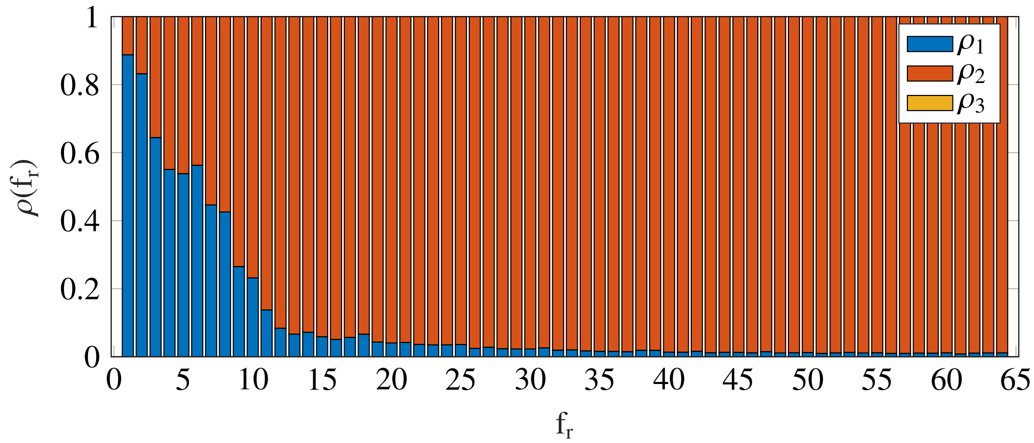
Applying the procedure of section 2.3 the *degrees of influence* for the three parameters are obtained.

3.3.1 Modal Analysis

In figure 10 the overall influence of both the parameters $\tilde{p}_1$ = MASS and $\tilde{p}_2$ = STIFFNESS in the modes below the first skin mode #15 stands out. Additionally a shift from a mass-dominated *relative gain factor* towards a stiffness-dominated *relative gain factor* can be observed: A steep drop from about 90% mass influence at mode #1 to about 90% stiffness influence at mode #15, from where the relative influence of $\tilde{p}_1$ still decreases, but with a much smaller rate. The fact, that $\tilde{p}_3$ = DAMPING doesn't contribute to the overall influence is reasonable, since in eigenvalue analyses only the mass and stiffness matrices are taken into account.



(a) Absolute modal gain factors κ_i

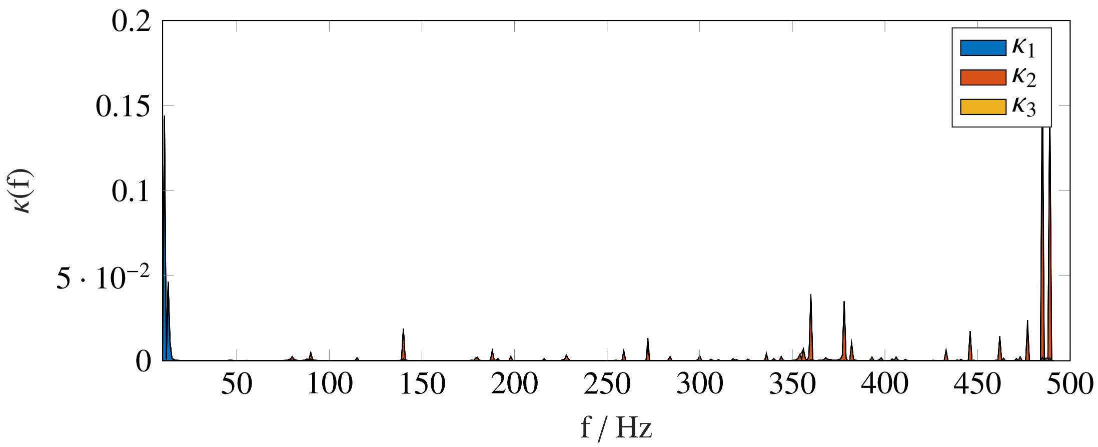


(b) Relative modal gain factors ρ_i

Figure 10: Modal gain factors

3.3.2 Frequency Response Analysis

The relative influence of $\tilde{p}_1$ as well as $\tilde{p}_2$ is comparable to the results obtained from the preceding modal analysis, except that now the damping as well contributes to the global uncertainty. A clear trend for $\tilde{p}_3$ as in mass-to-stiffness throughout the frequency domain is not observable, nevertheless, it should be noted that the overall relative contribution of structural damping in the linking elements is rather low when compared to the other two parameters.



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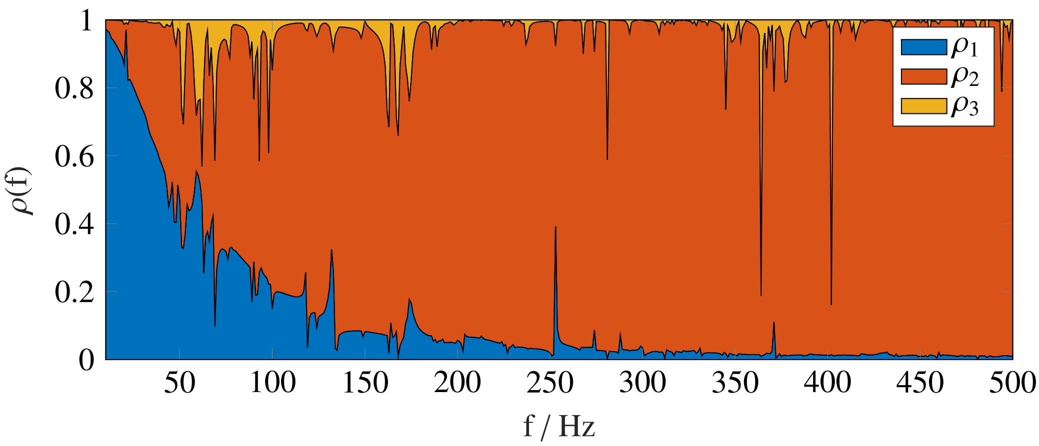

(b) Relative gain factors ρ_i

Figure 11: Frequency response gain factors

4. CONCLUSION

Fuzzy Arithmetic has been implemented successfully for the case of an aircraft panel with uncertain parameters within its secondary structure. For the current application, it has been shown, that this method provides a means to identify structural sensitivities regarding mode eccentricities as well as response behaviour when subjected to external forces. Dominant parameters can be identified for each frequency domain which allows in a subsequent step the alteration of selected structural components and properties towards a more favourable vibro-acoustical configuration.

5. ACKNOWLEDGEMENTS

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