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Prediction of bandgaps in membrane-type metamaterial attached to a thin plate

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ABSTRACT

This work proposes an analytical method to analyse the bandgap location and width of membrane-type metamaterial when it is attached to a thin plate structure. This method enables the bandgap prediction of such a structure by adjusting the tensile stress of the membrane directly. The accuracy of the model is verified by constructing a finite structure model for numerical simulation and comparing the results. It shows that the results given by the analytical model are primarily consistent with the simulation. The effect of membrane tensile stress and attached mass on the bandgap location and width is also investigated. It is found that the width of bandgap can be increased by increasing the membrane tensile stress and using a heavier mass attached to the membrane.

Keywords: Membrane-type, Metamaterial, Bandgap, Prediction
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1. INTRODUCTION

Metamaterials are manually engineered materials that have peculiar effective material properties that are not available in natural materials [1]. For acoustic/elastic metamaterials, their ability in attenuating the transmission of wave has attracted many research efforts. Membrane-type metamaterial is one type of acoustic metamaterial that was first proposed by Yang et al. in 2008 [2]. For acoustic metamaterials, local resonance is required for generating bandgaps. The unit cells of membrane-type metamaterials are normally formed by elastic membranes attached with lumped mass and stretched over rigid frames. Such a structure is schematically equivalent to a mass-spring resonator and thus similar to other types of metamaterial, where it possesses local resonance bandgaps in resonance frequency range. The location of bandgap is decided by the resonance frequency of the unit membrane-type resonator.

This membrane-type metamaterial can be used for sound isolation, energy harvesting and vibration absorption, with the corresponding effectiveness of their applications have

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been studied [3, 4, 5]. Mei et al. [6] used specially designed platelets to decorate the membrane and illustrated that their design with single layer membrane can absorb 86% of the acoustic wave around the resonance peak frequency, whilst the double layer design can absorb 99%. Naify et al. [7] examined both single-cell resonator's and multi-cell resonator's behaviour at low frequencies (below 200Hz) and proved that the membrane-type resonators can achieve much higher transmission loss than the prediction based on the mass-law. Otherwise, Liang Sun conducted experiments to study the membrane-type resonator's capability in structural vibration control. The results showed that the membrane resonator can effectively reduce the rectangular plate's vibration magnitudes by up to 42 dB [8].

In addition, for actual applications, the operation frequency of the metamaterial may vary according to the incident wave characteristics. Therefore, to achieve an agile bandgap location, some researchers have investigated the potential methods to tune the bandgap of membrane-type metamaterial. Langfeldt et al. [9] proposed an inflatable membrane structure, through which the stress within the membrane can be adjusted by the extent that the membrane is inflated and thus the bandgap location of the metamaterial. Chen et al. [10] proposed a membrane-type metamaterial that was magnetically controllable. The resonant frequencies of the unit cell can reach up to about 64% higher when the magnitude of input magnetic field is increased.

In the previous research on membrane-type metamaterials, the focus points were mainly about the effectiveness in different application fields or tuning of the bandgap properties. The prediction of the resonance frequencies of the designed membrane resonators before the actual fabrication, was normally conducted using finite element methods (FEM). For membrane-type metamaterial applications, it is necessary to know the bandgap properties in advance and to enable the proper design of membrane resonator parameters (tension stress, mass magnitude etc.) in accordance to the requirement. However, the utilization of FEM can be time consuming. As a result, an accurate and rapid model that can reveal the effect of resonator parameters on bandgap properties can be a useful tool for the design of membrane-type metamaterial.

In this paper, we proposed a model that can provide a prediction of bandgap properties of membrane-type metamaterial when it is applied to a thin plate structure. The model can be used to investigate the changing of bandgap properties of the membrane-type metamaterial on a thin plate, by modifying the attached mass or tuning of the membrane tensile stress. The model verification is carried out by commercial FEM software COMSOL Multiphysics.

2. MODEL AND FORMULATIONS

2.1 Estimated resonance frequency of a membrane-type resonator

The resonance frequency of membrane-type resonator can be estimated by the Rayleigh-Ritz method, which has been shown to provide a reasonably accurate estimation [11]. The structure of a membrane-type resonator can be schematically depicted as in Figure 1. The length and height of the host frame and membrane are denoted as L, H, l and h, respectively. The mass is assumed to be concentrated at a certain point at coordinates (a, b).

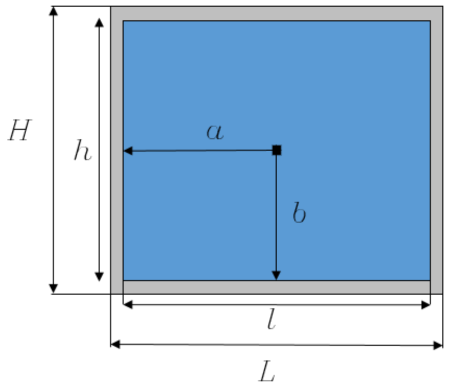

Figure 1. Configuration of a membrane-type resonator

The expressions for the strain energy and kinetic energy of the resonator can be given as [11]:

$$S_{max} = \frac{1}{2}\iint D\{(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2})^2 - 2(1-v)[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - (\frac{\partial^2 w}{\partial x \partial y})^2]\}dxdy + \frac{1}{2}\iint T[(\frac{\partial^2 w}{\partial x^2})^2 + (\frac{\partial^2 w}{\partial y^2})^2]dxdy \quad (1)$$

$$K_{max} = \frac{\omega^2}{2}\{\iint m_s w^2(x,y)dxdy + m_R(a,b)\, w^2(a,b)\} \quad (2)$$

where D is the bending stiffness of the membrane:

$$D = \frac{Et^3}{12(1-v^2)} \quad (3)$$

E, t and v are the Young's modulus, thickness and Poisson's ratio of the membrane respectively; T is the tension stress per unit length on membrane; $m_R(a,b)$ is the mass located at coordinates (a,b); m_s is the membrane mass per unit area; $w(x,y)$ and $w(a,b)$ are the transverse displacement of the membrane and mass at the indicated coordinates; ω is the natural frequency of the resonator.

Based on equations (1) and (2), the natural frequency can be estimated as:

$$\omega^2 = \frac{2U_{b,max}}{\iint m_s w^2(x,y)dxdy + m_R(a,b)\, w^2(a,b)} \quad (4)$$

The following mode shape function is used for estimating the natural frequency [11]:

$$w(x,y) = A_{mn}\sin\left(\frac{\pi x}{l}\right)\sin\left(\frac{m\pi x}{l}\right)\sin\left(\frac{\pi y}{h}\right)\sin\left(\frac{n\pi y}{h}\right) \quad (5)$$

For a membrane-type resonator, we mainly focus on the first order resonance frequency. Substituting the corresponding mode shape function into equation (4), the membrane-type resonator's lowest natural frequency is given as:

$$\omega_{11} = \frac{1}{2\pi}\sqrt{\frac{\frac{\pi^4 D}{4l^3h}(3h^4+3l^4+2l^2h^2) + \frac{3(l^2+h^2)T\pi^2}{16lh}}{\frac{9lhm_s}{64} + m_R sin^4(\frac{\pi a}{l})sin^4(\frac{\pi b}{h})}} \quad (6)$$

2.2 Dispersion relation

To obtain the dispersion relation curve of the system, the Plane Wave Expansion (PWE) method is used. This method is proved to be able to provide an accurate prediction for dispersion relation of a thin plate structure with the periodically attached spring-mass resonators [12]. As the membrane-type metamaterial can be simplified as spring-mass resonators, the PWE method is utilized in this work to predict the bandgap property.

As illustrated in the previous discussion, the first order resonance frequency of the membrane resonator can be derived, and the mass magnitude is already known. As the result, according to the equation $\omega_n = \sqrt{\frac{k_R}{m_R}}$, the equivalent stiffness of the unit cell can be obtained.

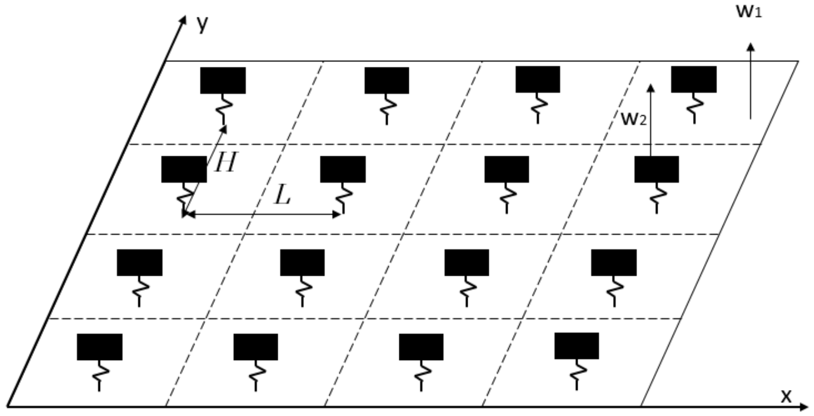

Figure 2. Configuration of a thin plate with periodically attached membrane-type metamaterial

The system configuration can be simplified as Figure 2. According to the dimension of membrane resonator frame, the outer dimension of a single unit cell is $L \times H$.

The equation of motion for the system can be given by these equations:

$$\begin{cases} D\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 w_1(x,y) - \omega^2 m_s w_1(x,y) = \sum_R f_1(X,Y)\delta[(x-X, y-Y)] & (7) \\ -\omega^2 m_R w_2(X,Y) = f_2(X,Y) & (8) \end{cases}$$

where (x,y) and (X,Y) are the coordinates of points on the plate and the location of resonators, $w_1(x,y)$ and $w_2(X,Y)$, are the transverse displacements of plate and resonator at different points. Here, f_1 and f_2 are forces that are applied on the thin plate and resonator masses, while δ is Dirac function. Applying the Bloch theorem and changing the coordinates of points into a lattice vector, the equation of motion can be transformed into the matrix form [12]. The equation can then be solved by applying a truncation in the plane wave number series. The mass magnitude and the equivalent stiffness are integrated into the equation and can therefore reveal the effect to the bandgap structure.

3. RESULTS AND DISCUSSION

3.1 Bandgap of an infinite structure

In PWE model, the structure is assumed to have periodical boundary conditions. As the result, the calculated bandgap is based on an infinite structure configuration. In this section, examples of membrane resonators with various mass and tension stress are considered. The corresponding change of the bandgap location and width are investigated.

The parameters of the membrane resonator are indicated in Table 1. The mass is 2.7g and it is attached in the middle of the resonator. The materials for membrane and frame are chosen as silicon rubber and epoxy, respectively. The parameters of the plate where the resonators are attached to, are defined as: Young's modulus $E = 200GPa$; Poisson's ratio v =0.3 and density ρ =7850 kg/m^3. The thickness of the plate is 2mm, which is smaller than the flexural structural wavelength of interest.

Table 1. Parameters of the membrane resonator

Membrane		Frame		Mass	
Young's modulus (MPa)	1.9	Young's modulus (GPa)	2.65	Magnitude (g)	2.7
Poisson's ratio	0.48	Poisson's ratio	0.41	Radius (mm)	5
Density (kg/m^3)	980	Density (kg/m^3)	1100	Height (mm)	4
Thickness (mm)	0.2				
l (mm)	50				
h (mm)	50				

In order to examine the effect of tension stress to the membrane resonator, the applied stresses to the membrane are set as 2MPa, 3MPa, 6MPa and 10MPa, respectively. The bandgap structures of the resonator with different stress are shown in Figure 3.

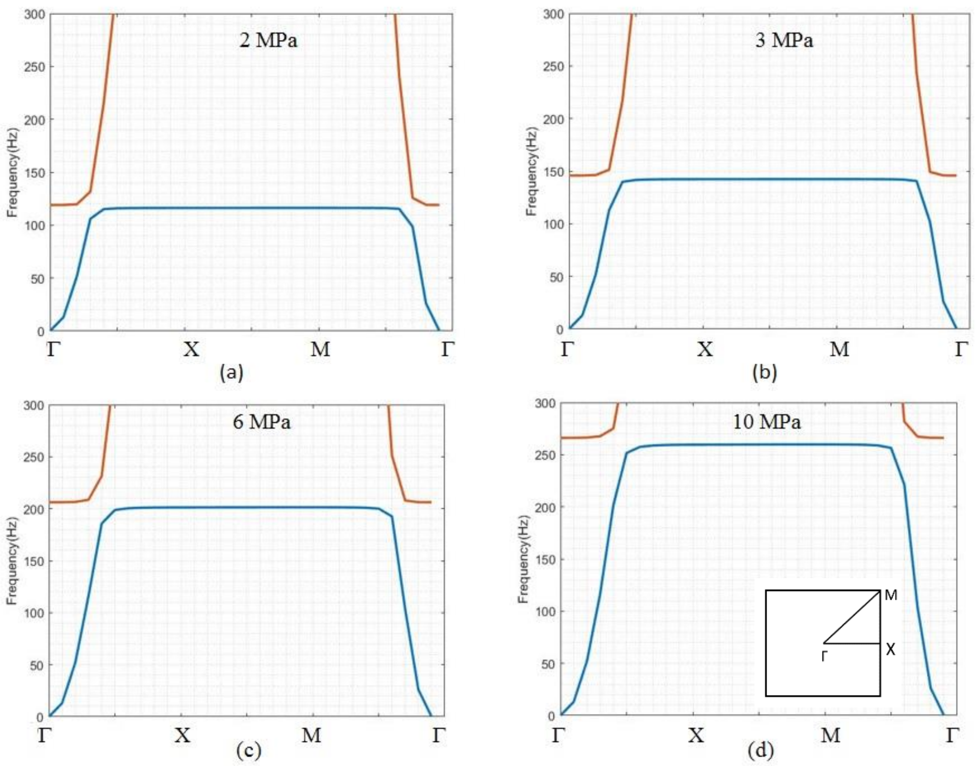

Figure 3. Bandgap structure of membrane-type metamaterial with (a) 2MPa, (b) 3MPa, (c) 6MPa and (d) 10MPa applied stress. The inset presents the corresponding Brillouin zone.

According to Figure 3, the bandgap shifts to a higher frequency range as the stress level is increased. When 2 MPa stress is applied to the membrane, a full narrow bandgap exists between 116.3 – 119.1 Hz. Once the stress is increased to 10 MPa, the bandgap shifts to 259.8 – 266.1 Hz. The changing trend of bandgap fits the pattern of the bandgap change for membrane-type metamaterial. The detailed location of the bandgap and width are provided and compared with the simulation results in following section.

Moreover, the adjustment of mass attached to the membrane is investigated. Three different masses are used in this study: 2.7g, 5.4g and 10.8g, respectively while the membrane tensile stress is set at 2 MPa. The corresponding bandgap structure is given in Figure 4. The 2.7g example is the same as Figure 3(a). According to the figure, the bandgap location increases when a lighter mass is used, which is consistent to what has been found in previous research [13].

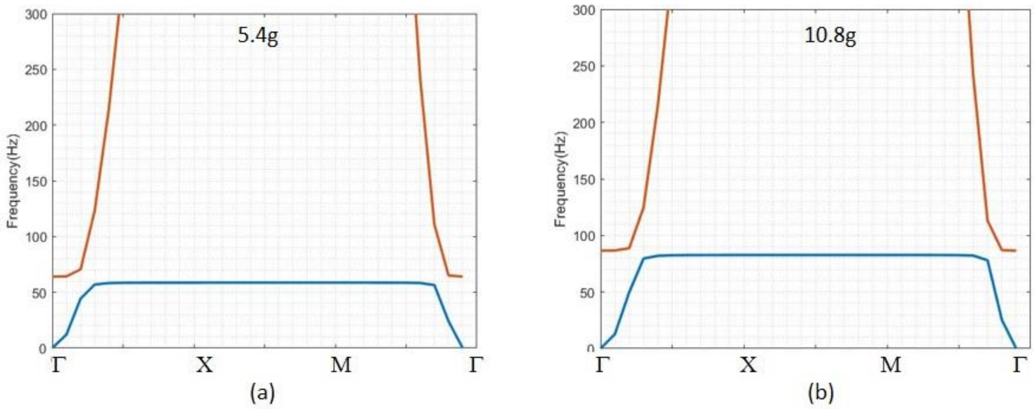

Figure 4. Bandgap structure of membrane-type metamaterial with attached mass of: (a) 5.4g and (b) 10.8g. The bandgap range: (a) 82.7 – 86.6 Hz and (b) 58.7 – 64.1 Hz.

3.2 Finite structure and numerical simulation

A finite structure model is constructed to verify the accuracy the proposed theoretical method. As shown in Figure 5, a thin steel plate is attached with 5×5 unit cells of membrane-type metamaterial. The dimension of the thin plate is $500 \times 300 \times 2$mm, and the outer dimension of one unit cell is 60×60mm.

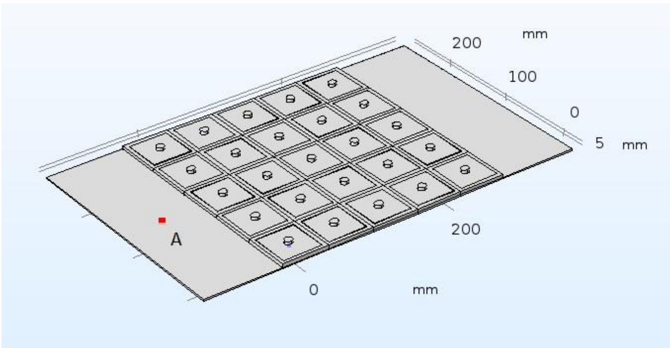

Figure 5. Configuration of finite structure

In order to examine the bandgap performance of the structure, the frequency domain analysis is conducted on the model. In the theoretical model, the structure is assumed to be an infinite structure. However, this cannot be realized in the actual application so a certain number of periodicity is required to let the finite structure generates the bandgap behaviour. In this work, we adopted 5 by 5 unit arrays which is considered to be enough for the bandgap forming. The left edge of the plate is fixed and transverse excitation signal is applied from the right edge, while the response signal is picked up from point A. Other boundary conditions of the structure are set as free boundary conditions and the biaxial pre-stress condition is applied to the membrane. The properties of the membrane resonator are as described in Table 1.

The frequency scanning range is set from 50 to 300 Hz and the membrane tensile stresses are set as 2 MPa, 3 MPa, 6 MPa and 10 MPa, respectively. The results are described in Figure 6.

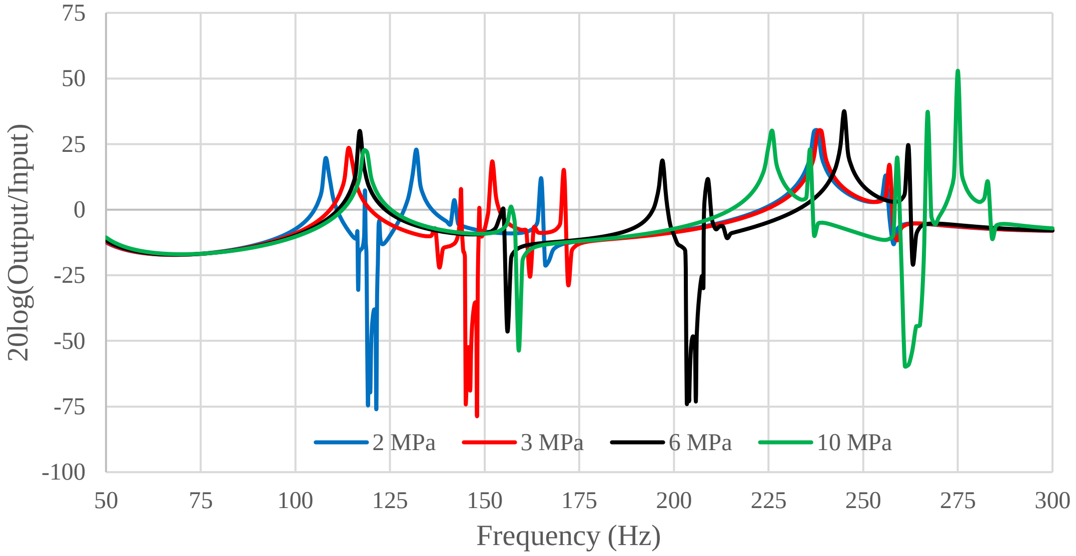

Figure 6. Frequency response of a plate with attached membrane-type metamaterial. The response curves of the membrane resonator applied with different membrane tensile stress is presented by: Blue line (2MPa); Red line (3MPa); Black line (6MPa) and Green line (10MPa).

As shown in the figure, the bandgap ranges are: 118.4 – 121.8 Hz (2 MPa), 144.8 – 148.6 Hz (3 MPa), 203.4 – 208 Hz (6MPa) and 260 – 267 Hz (10 MPa). It can be observed that the vibration transmission in the bandgap range is efficiently attenuated within this bandgap range. As expected, the resonance frequency increases as the membrane tensile stress level is increased. In addition, the bandgap width also increases as a higher level of stress is applied to the membrane.

The effect of decorated mass attached on the membrane is further investigated. In the finite structure, the membrane tensile stress level is kept at 2 MPa while the mass is changed to either 2.7g and 10.8g. Figure 7 shows the results where the bandgap ranges are: 118.4 – 121.8 Hz (2.7g) and 59 – 66 Hz (10.8g). The results are similar to the theoretical results given in the former section, with the bandgap width increases as heavier mass is used, while the bandgap location decreases as expected.

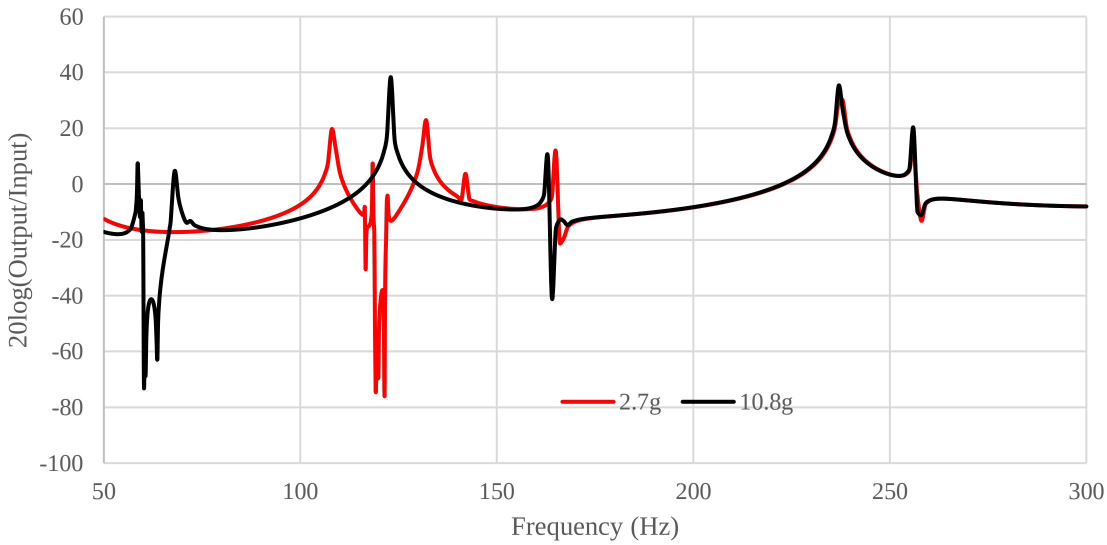

Figure 7. Frequency response of plate when the decorated mass is 2.7g (Red line) and 10.8g (black line).

The bandgap structure worked out by the proposed analytical method and numerical simulation are presented in Table 2.

Table 2. Bandgap property data comparison: Analytical model versus Finite Element Method simulation

Stress (MPa)	2		3		6		10	
	Analytical	FEM	Analytical	FEM	Analytical	FEM	Analytical	FEM
Upper edge (Hz)	119.1	121.8	145.8	148.6	206.1	208	266.1	267
Lower edge (Hz)	116.3	118.4	142.4	144.8	201.3	203.4	259.8	260
Band width (Hz)	2.8	3.4	3.4	3.8	4.8	4.6	6.3	7
Mass (g)	2.7		10.8					
	Analytical	FEM	Analytical	FEM				
Upper edge (Hz)	119.1	121.8	65.4	66				
Lower edge (Hz)	116.3	118.4	58.7	59				
Band width (Hz)	2.8	3.4	6.7	7				

As shown in Table 2, the bandgap location and width estimated by the proposed analytical method is basically consistent with the numerical results. Small amount of error exist, yet within an acceptance range. Therefore the proposed analytical model is accurate enough to predict the bandgap properties of membrane-type metamaterial attached to a thin plate.

4. CONCLUSIONS

In this work, an analytical method has been proposed to predict the bandgap properties of membrane-type metamaterial that is periodically attached to thin plate. Using the developed method, the effect of changing the membrane tensile stress level and the attached mass on the bandgap characteristics, is investigated. It is found that the increase of membrane tensile stress can lead to the increase of bandgap width and the shifting of bandgap location to higher frequencies. It is also observed that the use of heavier decorated mass increases the bandgap width, while the bandgap location is shifted to lower frequencies. Results from the analytical model are compared with numerical simulation results. It is found that the results are consistent, demonstrating the accuracy of the developed analytical model. The model allows the parameters of membrane-type metamaterial to be adjusted directly, and is able to predict the change in the corresponding bandgap characteristics rapidly. This is in contrast to numerical models (e.g. FEM-based models) that can take much longer to calculate the results. Therefore, the developed model can be used as an effective tool for the design of membrane-type metamaterial.

5. ACKNOWLEDGEMENTS

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