

Discrete vibrational black holes

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ABSTRACT

Discrete structure that implements approximately the vibrational black hole effect for flexural waves is proposed. It is a rod/plate with grooves whose depth increases gradually. The bending stiffness of such a structure gradually decreases, while the linear mass remains constant. The dependences on parameter change, at which the structure behaves similarly to the vibrational black hole and the exact analytical solution of corresponding equation are found.

Keywords: Discrete, flexural waves, Black Hole

I-INCE Classification of Subject Number: 42,43,47

INTRODUCTION

Various structures, which are focused on the absorption of flexural waves are investigated (see [1 - 6], for example) in the extensive literature on vibrational black holes (VBH). The basis of such structures is a sharpened according to the power law the rod/plate. At the exponent of power law $n \geq 2$, the velocity of flexural waves propagation for sufficiently high frequencies when approaching the edge of the sharpness tends to zero. The wave that entered the initial thick section of rod does not reach the end of the sharpness in any finite time. This feature provides, from the theoretical point of view, effective absorption of the wave. In practice, the ideal sharpening up to a zero thickness is unrealizable. Therefore, for effective absorption, a small amount of absorbing material must be added to the tip of the sharpening (see for example [2]). Other opportunities for effective work imperfect black hole - scattering on the irregularities sharpened edges [3,4], nonlinear effects [5,6] have been discussed. In this paper, we propose a VBH design for flexural waves in the rod. This design does not require sharpening the end of the rod to very small thicknesses for practical implementation.

DESIGN

A sketch of the considered BH is shown in fig.1. The grooves are deposited on the initially homogeneous rod with thickness H and length L . The depth of the grooves $h(x)$ gradually increases

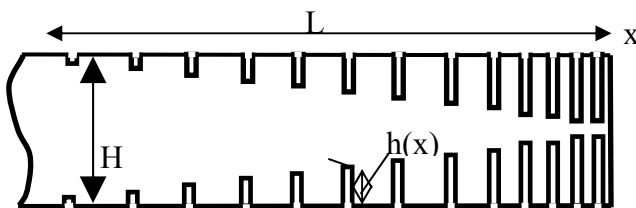


Fig.1. Sketch of BH for bending oscillations.

by approaching the end of the rod. The groove, obviously, "softens" the rod, reducing the local bending stiffness. As a consequence, flexural wave velocity propagation decreases and black hole effect appears. In the general, the displacement field in such a

construction can only be found numerically. The following approach is proposed as a reasonable analytical approximation.

ANALYTICAL SOLUTION

It allows homogenizing the discrete structure of the design with grooves. The starting point of the consideration is the standard equation of flexural vibrations of the inhomogeneous rod for transverse displacements $\eta(x)$:

$$-\rho S(x)\omega^2\eta + (E \cdot I(x)\eta'')'' = 0, \quad (1)$$

ρ, E - the density and Young's modulus of the rod material, $S(x)$ - the cross-sectional area, $I(x)$ - the moment of inertia of the cross-section. The first term in (1) describes the force of inertia, the second term – the bending moment associated with the stretching/compression of the layers of the rod. For a rod with a rectangular cross-section of width a and thickness s , $S = a \cdot s$, $I = \frac{1}{12} a \cdot s^3$. In the proposed

calculation method, it is assumed that the presence of grooves in the rod ensures the absence of tensile/compressive stresses in the layers between the grooves. This means that the gap between the opposite grooves $s(x) = H - 2h(x)$ should be substituted to $I(x)$. If the depth of the grooves is increased, the effective cylindrical stiffness of such a grooved rod is decreased. On the contrary, the linear mass of the rod will be constant. Equation (1) is rewritten as

$$-\rho H\omega^2\eta + [E \cdot \frac{1}{12} s(x)^3 \eta'']'' = 0. \quad (2)$$

Following papers [1, 7], we consider further the power dependence of $s(x)$:

$$s(x) = H \cdot (x/L)^\alpha \quad (3)$$

and the power dependence of solution $\eta(x)$:

$$\eta(x) = x^\gamma. \quad (4)$$

Substituting (3, 4) into (2), we obtain:

$$-x^\gamma b + \frac{d^2}{dx^2} [x^{3\alpha} \frac{d^2}{dx^2} x^\gamma] = 0 \quad (5)$$

$$b = 12 \frac{\rho\omega^2}{EH^2} L^{3\alpha} \quad (6)$$

After differentiating in (5), we get:

$$-x^\gamma b + \gamma(\gamma-1)(3\alpha+\gamma-2)(3\alpha+\gamma-3)x^{3\alpha+\gamma-4} = 0 \quad (7)$$

Equating in (7) exponents, we obtain the expression for α :

$$\alpha = 4/3 \quad (8)$$

Substituting (8) into (6, 7), we obtain the equation for the exponent γ :

$$\gamma(\gamma-1)(2+\gamma)(1+\gamma) - b = 0. \quad (9)$$

$$b = 12 \frac{\rho \omega^2}{E} \left(\frac{L^2}{H} \right)^2. \quad (10)$$

Change of variable is used to solve the algebraic equations of the fourth order (9):

$$\gamma = \mu - \frac{1}{2}, \quad (11)$$

which, when substituted in (9), gives the biquadrate equation for μ :

$$[\mu^2 - (1/2)^2] \cdot [\mu^2 - (3/2)^2] - b = 0$$

Solving this equation, and substituting the solutions in (11), we obtain expressions for exponents in solutions (4):

$$\gamma_{1,2,3,4} = -\frac{1}{2} \pm \sqrt{\frac{5}{4} \pm \sqrt{1+b}}. \quad (12)$$

The obtained formulas (3, 8, 10, 12) are similar to the formulas in [7], which describe the standard VBH – parabolic sharpened rod. It can be easily verified that for low frequencies ($b \rightarrow 0$) all the roots of eq. (12) are real. Vibrations are in phase at all points of the rod, wavelike motions are absent. In the other limiting case high frequencies ($b \rightarrow \infty$) eq. (12) yields two real roots and two complex conjugate roots. The complex roots correspond to propagating waves, and the real roots, to evanescent waves, similar to the case of homogeneous rod. The critical value of b , which corresponds to the appearance of an imaginary component, is

$$b^* = \frac{9}{16}. \quad (13)$$

The corresponding critical frequency is

$$\omega^* = \sqrt{\frac{E}{12\rho} b^*} \cdot \frac{H}{L^2} = \frac{1}{\sqrt{12}} c_Y \frac{3H}{4L^2}, \quad (14)$$

$c_Y = \sqrt{E/\rho}$ is propagation velocity of the longitudinal Young waves in a rod.

It should be mentioned, that for standard VBH in the form of parabolically sharpened rod, the critical frequency ω^*_{par} (eq. (17) in [7]) is five times higher:

$$\omega^*_{par} = \sqrt{\frac{E}{12\rho} b_{par}^*} \cdot \frac{H}{L^2} = \frac{1}{\sqrt{12}} c_Y \frac{15H}{4L^2}.$$

It follows from the formula (3), that the thinning of the uncut part of the rod

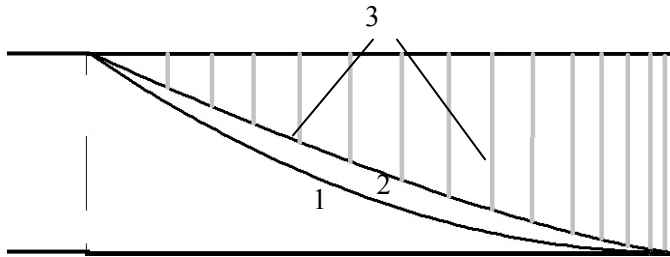


Fig.2. Comparison of standard VBH and discrete VBH.

- 1 parabolic sharpening of the standard VBH;
- 2 the envelope of discrete grooves of suggested VBH;
- 3 the grooves in discrete BH.

$s(x) \sim x^{4/3}$ is slower as compared to the thinning of the traditional VBH [7] $h(x) \sim x^2$. The profiles are compared in fig. 2. It is evidently, that the profile of the envelope of the groove in the terminal part of the VBH (curve 2) does not require such precision in

manufacturing as parabolic profile of standard VBH (curve 1).

CONCLUSIONS

In conclusion, we note two advantages of the proposed design. The first one is strong reducing the threshold frequency of the BH-effect. The second one is weakening the accuracy of manufacturing requirements. Taking into account the approximate nature of the analytical model used here, it is necessary to carry out additional experimental and numerical verifications.

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