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Concert hall geometry optimization with a simplified acoustic diffusion model

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ABSTRACT

The design of rooms for musical interpretation is the most complex from an acoustic point of view, as each type of music requires a room with specific and differentiated acoustic characteristics. Over the last few decades, significant progress has been made in relating subjective assessments of the acoustic quality of a room to a series of objective (physically measurable) parameters. In particular, in concert hall design, the reverberation time (RT) and the strength factor (G), related to loudness, have been considered of crucial importance. In this research, an optimal preliminary design methodology of concert hall geometry is proposed with the objective of keeping the mentioned acoustical parameters within desirable ranges. The methodology is based on a combination of a simplified acoustic diffusion model (SADM) and the simulated annealing (SA) optimization technique. The SADM allows one to accurately and quickly evaluate the effects of different geometries on acoustic parameters and the SA method is used to direct the search of the optimum set of design geometric variables.

Keywords: Optimal design, Concert halls, Acoustic diffusion model
I-INCE Classification of Subject Number: 25, 51, 76

1. INTRODUCTION

The design of rooms for musical interpretation presents a considerable complexity from an acoustic point of view. This is mainly due to the fact that human perception of sound depends on different magnitudes such as the sound level, the direction of

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propagation and attenuation over time. In addition, each type of music requires a room with specific and differentiated acoustic characteristics.

Historically, reverberation time (*RT*) has been considered of great importance in concert hall design because the sound quality is directly affected by this parameter. It is defined as the time it takes for sound, at a certain frequency, to decay by 60 dB in a closed space. This parameter strongly depends on the density and intensity of the reflections produced in the room, once sound emission stopped. The reflections will be basically defined according to the geometric characteristics of the room and the absorption of the materials involved. Another significant parameter for room acoustics is the strength factor (*G*). It is defined as the ratio of the sound energy that comes from a non-directive source measured at a seat, relative to the same sound energy from the same source measured in a free field at 10 m. This parameter is expressed in decibels and is closely related to the subjective parameter "loudness". The loudness corresponds to the degree of amplification produced by the room and makes it possible to evaluate the distribution of the sound as well as determining where the transmitted energy is deficient for some frequency. The strength factor depends fundamentally on the distance a seat is from the stage and on the absorption and shape of the enclosure. Both parameters (*RT* and *G*), in combination, are of great importance in the study of concert hall acoustics [1]. In particular, in large halls for symphonic music, the ideal ranges of these parameters at mid-frequencies (average value of 500 Hz and 1000 Hz bands), when compared with the acoustic quality ratings of concert halls, are between 1.9 and 2.1 s for RT_{mid} (occupied halls) and 4 and 7.5 dB for G_{mid} (empty halls) [1]. These target values represent common practice for the design of concert halls.

This paper presents an optimal preliminary design methodology of concert hall geometry in order to achieve a desirable acoustic quality for symphonic music while keeping RT_{mid} and G_{mid} values, at different positions in the audience area, within the ideal ranges. The design variables considered here correspond to the principal dimensions of the hall. The proposed methodology combines the use of a simplified acoustic diffusion model (SADM) and the simulated annealing (SA) optimization technique. The former is an approximate two-dimensional simplification of the acoustic diffusion model (ADM), introduced some years ago [2, 3]. The main advantage of using the SADM in this optimal design situation (iterative process) is the important reduction in computational time in comparisons with the ADM and geometric models although maintaining a similar accuracy [4]. The SADM is used to predict the acoustic parameters during the optimal search while the SA technique directs the exploration in order to reduce the number of required SADM simulations. This methodology has been successfully applied by the authors in previous studies related to industrial buildings [5, 6].

2. PROBLEM FORMULATION

The study is concerned with a large concert hall for symphonic music. The adopted hall presents a fixed stage area and an audience area that can vary according to its principal dimensions allowing the investigation of shoe-box and fan-shaped halls. Then, the design geometric variables are: large (L), width of the rear wall (W) and height (H) of the hall (see Fig. 1).

This research aims to investigate the early optimal design stage of concert halls, for a predefined number of seats, where only the basic dimensions and shape are studied while material properties are not evaluated. Therefore, the absorption coefficients of the interior surfaces are considered constants. This assumption is based on the fact that, except for audience and stage area, only a small amount of absorption is needed and absorption coefficients about 0.1-0.2 are generally used. Moreover, the absorption coefficients of audience and orchestra area are similar for most cases, so they are considered as constants [7, 8].

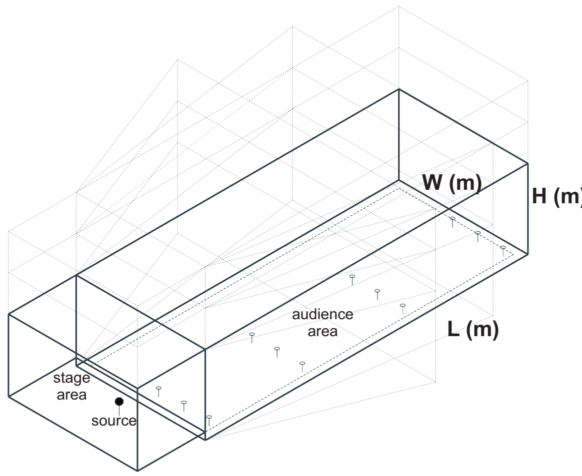

Figure 1 - Geometric variables definition and locations of the source and receiver points.

The acoustic parameters G_{mid} and RT_{mid} are estimated in the audience area. In particular, when using an omnidirectional sound source with a known sound power level $\mathrm{SWL_f}$, the parameter G_{f} , for a frequency band f of interest, can be expressed as follows [9]:

$$G_{\mathrm{f}} = \mathrm{SPL_f} - \mathrm{SWL_f} + 31\ \mathrm{dB}, \tag{1}$$

where $\mathrm{SPL_f}$ is the sound pressure level calculated at every receiver point. On the other hand, the reverberation time RT, is evaluated from the sound decay function as will be explained in section 3.1.

The optimization approach consists in determinating the mentioned geometric variables from the knowledge of G_{mid} and RT_{mid} values (average values of 500 Hz and 1000 Hz bands) at a number of receiver points. Then, the design problem is formulated as follows:

$$(L,W,H)_{\mathrm{opt}} = \arg\ \min\ OF, \tag{2}$$

where $(\ldots)_{\mathrm{opt}}$ designates the set of the optimal variables and OF is the objective function defined as the following root mean square error function:

$$OF(L,W,H) = \sqrt{\sum_{i=1}^{M} \frac{1}{M} (\overline{G_{\mathrm{mid}}} - G_{\mathrm{mid,i}})^2} + \sqrt{\sum_{i=1}^{M} \frac{1}{M} (\overline{RT_{\mathrm{mid}}} - RT_{\mathrm{mid,i}})^2}, \tag{3}$$

where M is the total number of receiver points i, $\overline{G_{\mathrm{mid}}}$ = 5.75 dB and $\overline{RT_{\mathrm{mid}}}$ = 2 s. The last ones represent the mean values of the ideal ranges of the strength factor and the reverberation rime, respectively [1]. The geometric variable ranges are based on common recommended parameters of existing concert halls. In addition, some relations between the number of seats and the volume and audience area are considered based on common practice [8]. Thus, the objective function is subject to the following constraints:

$$
\begin{gathered}
18 \text{ m} \le L \le 45 \text{ m}, \\
18 \text{ m} \le W \le 35 \text{ m}, \\
10 \text{ m} \le H \le 25 \text{ m}, \\
0.40 \text{ m}^2/\text{seat} \le Ss / N \le 0.68 \text{ m}^2/\text{seat}, \\
8 \text{ m}^3/\text{seat} \le V / N \le 10 \text{ m}^3/\text{seat},
\end{gathered} \tag{4}
$$

where Ss/N is the ratio of the total audience area (Ss) to total seat number N and V/N is the volume per seat.

3. ACOUSTIC MODEL

3.1 3D Model

The ADM is an energetic model, based on the theory of diffusion, which accurately predicts the sound field and the temporal sound decay in enclosures of arbitrary shape. Then, the reverberant acoustic energy density $w_{\mathrm{f}}(\mathbf{r},t)$ at position $\mathbf{r}$ and time t in a hall of volume V, is obtained as the solution of the following equations [2, 3]:

$$\frac{\partial w_{\mathrm{f}}(\mathbf{r},t)}{\partial t} - D\,\nabla^2\, w_{\mathrm{f}}(\mathbf{r},t) + \sigma_{\mathrm{f}} w_{\mathrm{f}}(\mathbf{r},t) = q_{\mathrm{f}}(\mathbf{r},t) \quad \text{in V}, \tag{5}$$

$$-D\frac{\partial w_{\mathrm{f}}(\mathbf{r},t)}{\partial \mathbf{n}} = A_{\mathrm{f}} c w_{\mathrm{f}}(\mathbf{r},t) \quad \text{on } \partial\,\text{V}, \tag{6}$$

where ∇^2 is the Laplace operator, D is a diffusion coefficient, σ_{f} is a coefficient of volumetric absorption, $q_{\mathrm{f}}(\mathbf{r},t)$ is the source power per unit volume, $\mathbf{n}$ is the exterior normal to the boundaries, A_{f} is an absorption factor and c is the speed of sound. The diffusion coefficient $D = c\lambda/3$ takes into account the room morphology of interior surfaces area S through the mean free path $\lambda_{\mathrm{r}} = 4V / S$. The absorption term $\sigma_{\mathrm{f}} = m_{\mathrm{f}} \times c$ considers the atmospheric attenuation in the room, where m_{f} is the absorption coefficient of air [10]. Equation (6) corresponds to the boundary conditions on the room limits, where the absorption factor $A_{\mathrm{f}} = c\alpha_{\mathrm{f}} / 2\left(2-\alpha_{\mathrm{f}}\right)$ is adopted [11].

In order to obtain the strength factor G_{f}, the system of equations (5) and (6) should be solved for the stationary case. Then, the local reverberant sound field is obtained and the total sound pressure level is calculated taking into account the direct sound field contribution [3]:

$$\mathrm{SPL}_{\mathrm{f}}(\mathbf{r}) = 10\log_{10}\left\{\rho c\left[\int_{\mathrm{V_s}} \frac{q_{\mathrm{f}}(\mathbf{r})}{4\pi r^2}\mathrm{dV_S} + c w_{\mathrm{f}}(\mathbf{r})\right]\frac{1}{P_{\mathrm{ref}}^2}\right\}, \tag{7}$$

where the source term $q_{\mathrm{f}}(\mathbf{r}_{\mathrm{j}}) = Ws_{\mathrm{f}}\delta\left(\mathbf{r}-\mathbf{r}_{\mathrm{j}}\right)$ is modeled as an omnidirectional point source with a sound power Ws_{f}, being $r = \|\mathbf{r} - \mathbf{r}_{\mathrm{j}}\|$ the distance between the receiver and the source and $\mathbf{r}_{\mathrm{j}}$ an arbitrary source point. Then, the parameter G_{mid} is calculated from equation (1) by averaging the SPL_{500Hz} and SPL_{1000Hz} values obtained from expression (7).

On the other hand, in order to estimate the reverberation time RT_f, it is necessary to obtain the temporal function of sound energy in the hall by solving the system of equations (5) and (6) for the time-dependent case. In this instance, the source term $q_f(\mathbf{r}_j,t) = E_0\delta\left(\mathbf{r}-\mathbf{r}_j\right)\delta\left(t-t_0\right)$ is modeled considering an impulse sound source, being E_0 the energy emitted by the source at time t_0 [3]. From the solution of $w_f(\mathbf{r},t)$, the parameter RT_{mid} is calculated from the average value of 500 and 1000 Hz bands by applying the Schroeder integration to the sound pressure level SPL decay curve:

$$\mathrm{SPL}_f(\mathbf{r},t) = 10\log_{10}\left(\frac{w_f(\mathbf{r},t)\rho c^2}{P_{\mathrm{ref}}^2}\right), \tag{8}$$

being ρ the air density and $P_{\mathrm{ref}} = 2 \times 10^{-5}$ Pa. In particular, the RT_{30} value is computed here, which is calculated using the sound decay from -5 dB to -35 dB [3].

3.2 2D Model

The simplified acoustic diffusion model (SADM) is obtained from a 3D-2D reduction of the ADM applying the Kantorovich methodology [12]. Hence, the density of reverberant energy $w_f(\mathbf{r},t)$ can be represented, as the product of two functions: one, corresponding to the variation of the energy in the plane and, the other, considering the variation in the height [4]:

$$w_f(\mathbf{r},t) \approx \tilde{w}_f(\mathbf{r},t) = P_f(x,y,t)\times Z(z), \tag{9}$$

where $P_f(x,y,t)$ is an unknown function and $Z(z)$ is a function selected a priori in order to approximate the vertical variation of the reverberant energy density. This methodology presents the advantage that only part of the solution is chosen in advance, while the remainder thereof is determined according to the nature of the problem.

The simplest way to approximate the vertical variation of the reverberant energy density is by means of the second order polynomial $Z(z) = 1+a_1z+a_2z^2$. Polynomial coefficients are determined from the boundary conditions defined in the two extreme planes of the room (floor and ceiling):

$$D\frac{\partial Z(z)}{\partial z} = \pm A_f\, Z(z). \tag{10}$$

Substituting equation (9) into equations (5) and (6), multiplying by $Z(z)$, and integrating along the vertical direction, the following system of equations corresponding to the SADM is obtained [4]:

$$\frac{\partial P_f(x,y,t)}{\partial t} - D_{Z1}\nabla_p^2 P_f(x,y,t) + (D_{Z2}-\sigma_{Z,f})P_f(x,y,t) = q_{Z,f}(x,y,t) \ \text{ en } \Omega_r, \tag{11}$$

$$D_{Z1}\frac{\partial P_f(x,y,t)}{\partial \mathbf{n}} + P_f(x,y,t)cA_{Z,f} = 0 \ \text{ sobre } \partial\Omega_r. \tag{12}$$

where ∇_P^2 is the Laplace operator in the plane and $\partial\Omega_r$ represents the perimeter of the hall. From above equations, the following definitions are made,

$$D_{Z1} = \int_0^H D\, Z(z)^2 dz, \tag{13}$$

$$D_{Z2} = \int_0^H D\left(\frac{\mathrm{d}^2 Z(z)}{\mathrm{d}z^2} Z(z)\right) dz, \tag{14}$$

$$\sigma_{Z,f} = \int_0^H \sigma_f Z(z)^2 dz, \tag{15}$$

$$q_{Z,f} = \int_0^H q_f\; Z(z)\; dz, \tag{16}$$

$$A_{Z,f} = \int_0^H A_f Z(z)^2 dz. \tag{17}$$

Once the approximated reverberant energy density is calculated as the solution of the previous equations, the acoustic parameters G_{mid} and RT_{mid} can be obtained as was indicated in previous section. In this paper, the system of equations (11) and (12) is solved numerically to perform the acoustic calculation by means of the finite element method.

4. NUMERICAL OPTIMIZATION MODEL: SIMULATED ANNEALING

The simulated annealing method (SA) is a heuristic technique that has the advantage of being flexible with respect to the evolutions of the problem and easy to implement. The main characteristic of the algorithm is to avoid local convergence in problems of great scale [13]. The algorithm starts by defining an initial trial for an array having the desired variables (principal dimension of the hall) $\mathbf{X}_0$ within the feasible domain of the problem. Then, it successively generates, in a reduced domain of the neighborhood of the actual array $\mathbf{X}$, new trials $\mathbf{X}'$ which are accepted as current according to the change in the objective function $OF = OF(\mathbf{X}') - OF(\mathbf{X})$. If this change is negative, the new trial is accepted as the new current array. If the change is positive, the acceptability is decided according with the probability of Boltzmann distribution. The latter depends on a specific parameter which directs the convergence of the algorithm, as it approaches to zero. So, initially, when this parameter is high, there is a large probability of accepting configurations with a greater OF value, but as the procedure advances and the parameter decreases, the probability of acceptance becomes considerably smaller, until it finally converges to the optimal solution.

The optimization procedure is implemented in Matlab by linking finite element solutions of the SADM, using the software FlexPDE, with SA technique in an iterative manner [5, 6].

5. NUMERICAL EXAMPLE

The selected concert hall presents a fixed stage area (170 m^2) located 0.5 m above the audience area. An omnidirectional point source located centered on the stage at 1.2 m height and 3 m away from the front wall was considered. The acoustical parameters

RT_{mid} and G_{mid} were evaluated in 12 receiver points located in the audience area at 1.1 height (see Figure 1). Due to the hall symmetry, only half of the audience area was analyzed. During the study, the relative position of the receivers as a function of the distance to the interior surfaces is modified in accordance with the shape of the hall. A total seat number N = 1900 was selected (this value is in accordance with empirical knowledge of the best concert halls for symphonic music of the world [8]). The adopted atmospheric absorption coefficients are $m = 5 \times 10^{-4}$ and 1×10^{-3} m^{-1} for the 500 and 1000 Hz bands, respectively. The absorption coefficients of the different surfaces, shown in Table 1, were selected based on common materials used in concert halls [8, 14].

Surface	α **500 Hz band**	α **1000 Hz band**
Ceiling	0.10	0.08
Floor	0.06	0.06
Walls	0.11	0.08
Stage area, unoccupied	0.06	0.06
Stage area, occupied - symphonic orchestra	0.25	0.37
Audience area, seats unoccupied - medium upholstered	0.68	0.70
Audience area, seats fully occupied - medium upholstered	0.80	0.83

Table 1 - Adopted absorption coefficients.

Figures 2(a) and (c) show RT_{mid} and G_{mid} values at the receiver points for the optimized hall. The latter presents a fan shape with the following geometric parameters: L = 30.4 m, W = 35 m and H = 15.1 m. The volume and area per seat are 8.15 m^3 per seat and 0.41 m^2 per seat, respectively. The optimal solution presents a OF = 2.41 and it is reached around of 500 iterations. The total calculation time (considering 1000 iterations) was about 1.84 h. Figures 2(b) and (d) present the results for a shoe-box hall whose dimensions (L = 44.75 m, W = 18 m and H = 15.1 m) allow to consider the same area and volume audience that the optimized hall. In this case, the objective function is OF = 3.54. From Figure 2, it is shown that the optimized hall (fan-shaped hall) generates an RT_{mid} and G_{mid} distribution closer to the ideal values compared to the shoe-box hall (despite having both the same area and volume audience). Results confirm that the G_{mid} values vary significantly, depending the receiver location and the hall shape while RT_{mid} values have a little sensitive to changes in locations and hall geometries. These assumptions have been verified by other authors [7, 15].

In addition, SADM calculations are compared with the ADM and the ray tracing technique implemented in CATT-Acoustic. A finite element mesh of about 6.6×10^2 triangular elements and 11×10^3 tetrahedral elements was used to solve the SADM and the ADM, respectively. The ray tracing simulation was performed with 150×10^4 sound rays. Results demonstrate that the SADM practically coincides with the ADM and presents a good agreement with the ray tracing technique. The employed computation times (for a single iteration) are 7 s for the ADM and less than 1 s for the SADM (stationary solution), and 400 s for the ADM and 4 s for the SADM (time-dependent solution). Figure 3 shows the evolution of the objective function as a function of the number of iterations until the optimal solution (OF = 2.41) is reached.

In this example, the reflections on the interior surfaces were considered completely diffuse. For this kind of surfaces the MDA shows accurate predictions. On the other hand, the efficiency of the present methodology for analyzing sound decay for specular disproportionate rooms with non-homogeneous absorption distributions is under study [16, 17]. However, in general, high levels of diffusivity are required in concert halls based on the positive correlation between preference and high values of the scattering coefficients [15]. In fact, for instance, common used values of scattering coefficients are around 0.65 for audience and stage area [7, 15]. So, the assumption of highly reflective surfaces is almost always right is concert halls.

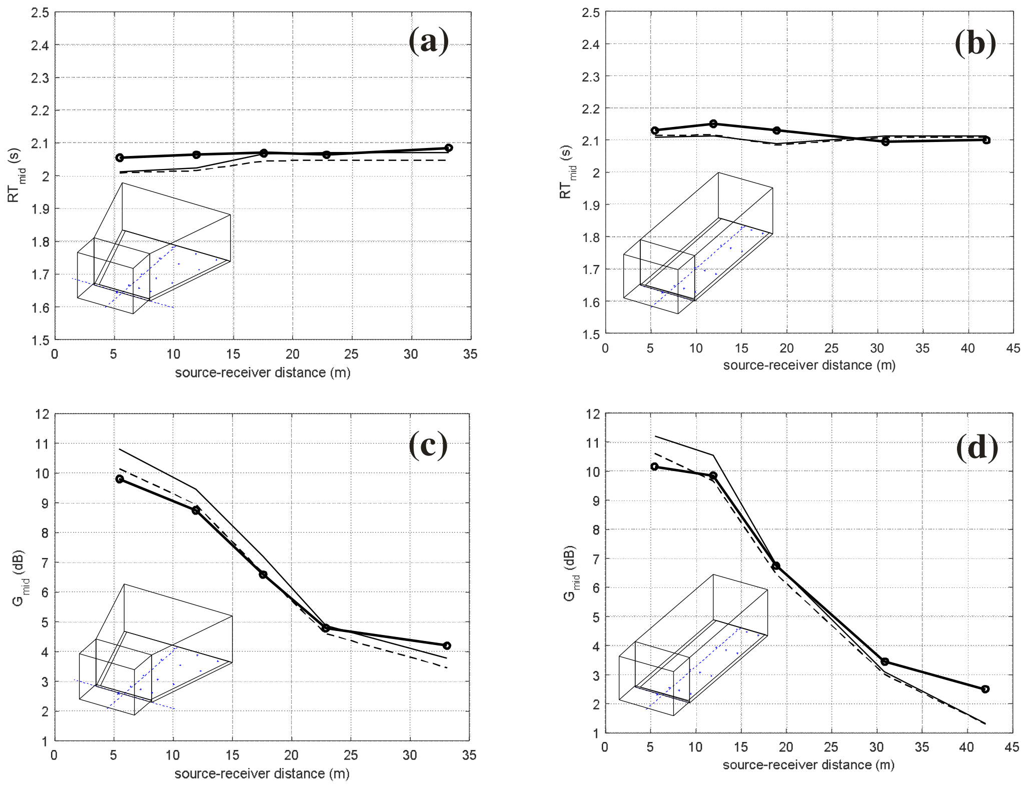

Figure 2 - G_{mid} and RT_{mid} results for the optimized fan-shaped hall (left) and a shoe-box hall with the same area and volume audience (right): SADM (--); ADM (–) and CATT (-•-).

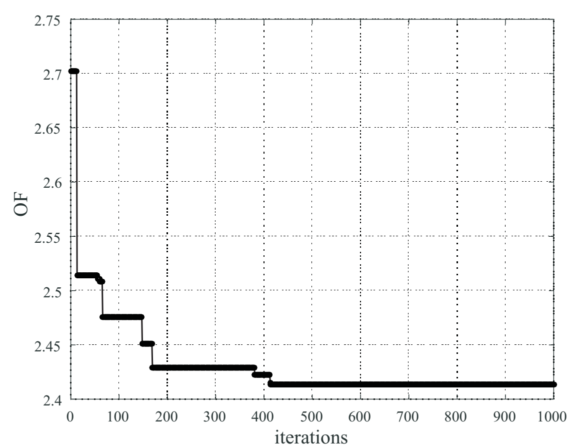

Figure 3 - Evolution of the objective function as a function of the number of iterations.

6. CONCLUSIONS

A methodology for the early optimal design of concert hall geometry was formulated, based on considering recommended ideal values of the reverberation time

(RT_{mid}) and the strength factor (G_{mid}) at different receiver points in the audience area. The simplified diffusion model (SADM) has been proposed in order to calculate the RT_{mid} and G_{mid} distribution. Thus, the optimal design formulation has been satisfactorily solved by the combined employment of the Simulated Annealing technique and the finite element solution of the SADM. Results have shown that the use of the SADM significantly improves the convergence time while maintaining almost the same accuracy than the ADM and the ray tracing technique. This represents the major advance in the proposed methodology due to the great number of iterations required.

However, it should be noted that there are other acoustical parameters to consider in order to describe the overall acoustic performance of concert halls. In particular, those parameters related to early reflections, such as Early Decay Time (EDT), Clarity (C_{80}), Definition (D_{50}), Center time (CT), among others. These parameters cannot be estimated by the diffusion model because it is accurate mainly in predicting the late part of the decay process [18]. This limitation could be solved by using a combined model where late reflections are predicted by the diffusion model and the first order reflections are calculated by an image-source model.

The methodology presented here is used for the early stage design of concert hall geometry, being necessary a more robust model (e.g. ray tracing technique) in order to obtain a more detailed acoustic design.

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