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Reflection coefficients of Enhanced Acoustic Black Holes at a beam termination

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ABSTRACT

The design of lightweight and stiff structures with high vibration damping properties is an important issue in mechanical engineering. The insertion of Acoustic Black Holes (ABH) in a flat panel or in a beam termination is an innovative technique for passive structural vibration damping without added mass. ABH consist in local heterogeneities of the stiffness and the damping and are known as efficient absorbers for flexural waves above their cut-on frequency. The goal of the paper is to identify and to push the current limitations of this existing strategy by developing "enhanced acoustic black holes" (eABH) absorbers based on the integration of thermal adaptive systems. The capability of such techniques to enhance the control of the flexural rigidity and the local damping is investigated numerically. The reflection coefficients, R, of several architectures of ABH beam termination are computed using the Impedance Matrix method. Such a method is adapted to the accurate computation of R of a structural wave guide with varying properties. Comparing R of several architectures allows us to discuss the potentialities of such eABH.

Keywords: Reflection, Impedance, Loss factor
I-INCE Classification of Subject Number: 40

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1. INTRODUCTION

The control of vibrations under mass constraints is a challenging objective in the transport industry. The design of lightweight and stiff structures with high vibration damping properties is therefore an important issue in mechanical engineering.

An answer to this issue is the insertion of Acoustic Black Holes (ABH) in a flat panel or in a beam termination. This is an innovative technique for passive structural vibration damping without added mass. ABH consist in local heterogeneities of the stiffness and the damping and are known as efficient absorbers for flexural waves above their cut-on frequency [11, 8, 9].

On the one hand, the stiffness heterogeneity is the result of the power-low profile of the beam. On the other hand, the damping is generally induced by a uniform adhesive tape. Although, the local heterogeneity of the damping depends on the tapered medium because of the mixing law [5] and not on the damping layer whose inherent loss factor is constant. That is to say that the ABH efficiency can be improved by monitoring the damping as well as the stiffness. The stiffness is easy to handle by the geometry of the medium, while this is not the case for the damping because it does not only depend on the damping layer itself.

Now, consider the damping layer with space dependent loss factor. This could be a start to handle the damping heterogeneity. Developing "enhanced acoustic black holes" (eABH) absorbers based on the integration of thermal adaptive systems constitutes a solution to this issue [3]. These adaptive systems can be based on the use of shape memory polymer (SMP) layers like the tBA-PEGDMA whose elastic modulus and loss factor strongly depend on temperature [1]. To this end, the SMP provides varying and high values of loss factor, up to 250%.

The objective of this paper is to discuss the potentialities of such eABH, focusing on beams. To help the discussion, the reflection coefficients, R, of several architectures of ABH beam terminations are computed using the Impedance Matrix method [6, 2, 4]. Such a method is adapted to the accurate computation of R of a structural wave guide with varying properties. This method is introduced in the first part. In a second part, in order to show the benefit of a loss factor gradient, a parametric study is done. The numerical results are compared.

2. STRUCTURAL WAVE GUIDE MODEL THROUGH THE IMPEDANCE MATRIX METHOD

The Impedance Matrix method is used to estimate the reflection matrix, $\mathbf{R}$, of a beam. The method consists in 2 steps [6, 2, 4]: first, derivation of the expression of the impedance matrix, $\mathbf{Z}$, of the model through the state formalism; second, determination of the expression of $\mathbf{R}$ depending on $\mathbf{Z}$ by switching from the state to the wave formalism.

2.1. Governing Equations

In the framework of Euler–Bernoulli model in harmonic regime, the state vector is composed of the displacement w, the slope ψ, the shear force T and the bending moment M. The governing equations can be written in the state-space as

$$\frac{\partial \mathbf{X}}{\partial x} = \mathbf{H}\mathbf{X}, \tag{1}$$

with

$$\mathbf{X} = \left\{ \begin{array}{c} w \\ \psi \\ \hline T \\ M \end{array} \right\} = \left\{ \begin{array}{c} \mathbf{x}_c \\ \mathbf{x}_f \end{array} \right\}, \text{ and } \mathbf{H} = \left[\begin{array}{cc|cc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1/EI \\ \hline -\rho S\omega^2 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{array} \right] = \begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_2 \\ \mathbf{H}_3 & \mathbf{H}_4 \end{bmatrix}. \tag{2}$$

$\mathbf{H}$ is the transition matrix where ω is the pulsation, ρ is the material density of the beam, $S = bh$ is the beam cross-section which is rectangular, $I = bh^3/12$ is the moment of inertia of the cross-section and E is the Young's modulus [10].

The impedance $\mathbf{Z}$ is defined by the relationship between the kinematic and force subvectors $\mathbf{x}_c$ and $\mathbf{x}_f$:

$$\mathbf{x}_f = j\omega\mathbf{Z}\mathbf{x}_c. \tag{3}$$

Replacing Eq.(3) into Eq.(1) leads to the Riccati equation

$$\frac{\partial \mathbf{Z}}{\partial x} = -\mathbf{Z}\mathbf{H}_1 - j\omega\mathbf{Z}\mathbf{H}_2\mathbf{Z} + \frac{\mathbf{H}_3}{j\omega} + \mathbf{H}_4\mathbf{Z}. \tag{4}$$

This equation can be numerically solved (cf. Sec.2.2). The second step consists in switching from the state to the wave formalism. To do so, the matrix $\mathbf{E}$, whose columns are the eigenvectors of $-j\mathbf{H}$, can be numerically computed. At last, $\mathbf{R}$ can be computed as

$$\mathbf{R} = [j\omega\mathbf{Z}\mathbf{E}_2 - \mathbf{E}_4]^{-1}[\mathbf{E}_3 - j\omega\mathbf{Z}\mathbf{E}_1]. \tag{5}$$

2.2. Magnus scheme

In order to solve the Riccati equation, Eq.(4), a fourth-order Magnus scheme is used [7]. The beam model is discretised and Eq.(1) becomes

$$\mathbf{X}(\tilde{x}_{n+1}) = e^{\Omega_\mathbf{n}}\mathbf{X}(\tilde{x}_n), \text{ with } e^{\Omega_\mathbf{n}} = \begin{bmatrix} \mathbf{O}_1 & \mathbf{O}_2 \\ \mathbf{O}_3 & \mathbf{O}_4 \end{bmatrix}. \tag{6}$$

Here, $\tilde{x}_n$ is the axial discrete coordinate and $\Omega_\mathbf{n}$ is the fourth-order Magnus matrix defined by

$$\Omega_\mathbf{n} = \frac{\Delta}{2}(\mathbf{H}_a + \mathbf{H}_b) + \frac{\sqrt{3}}{12}\Delta^2[\mathbf{H}_b, \mathbf{H}_a]. \tag{7}$$

The parameter $\Delta = \tilde{x}_{n+1} - \tilde{x}_n$, is the spatial step of the numerical scheme, $\mathbf{H}_a$ and $\mathbf{H}_b$ defined thereafter are part of the transition matrix $\mathbf{H}$ (cf. Eq.(2)), $[\mathbf{H}_b, \mathbf{H}_a] = \mathbf{H}_b\mathbf{H}_a - \mathbf{H}_a\mathbf{H}_b$, is the commutator between $\mathbf{H}_b$ and $\mathbf{H}_a$,

$$\mathbf{H}_a = \mathbf{H}(\tilde{x}_n + (\frac{1}{2} - \frac{\sqrt{3}}{6})\Delta) \text{ and } \mathbf{H}_b = \mathbf{H}(\tilde{x}_n + (\frac{1}{2} + \frac{\sqrt{3}}{6})\Delta). \tag{8}$$

Replacing Eq.(3) into Eq.(6) leads to an iterative scheme which can be used to compute the value of $\mathbf{Z}$ for each axial position on the beam. The free boundary condition imposes $Z_{(x=0)} = 0$ and thus can initiate the iterative expression of the impedance:

$$\mathbf{Z}(\tilde{x}_{n+1}) = \frac{1}{j\omega}[\mathbf{O}_3 + j\omega\mathbf{O}_4\mathbf{Z}(\tilde{x}_n)][\mathbf{O}_1 + j\omega\mathbf{O}_2\mathbf{Z}(\tilde{x}_n)]^{-1}. \tag{9}$$

3. RESULTS

The variations of the reflection coefficient, R, of propagating waves is shown in Figure 1. This result involves a $100\mu m$ thick damping layer with a loss factor of 100%.

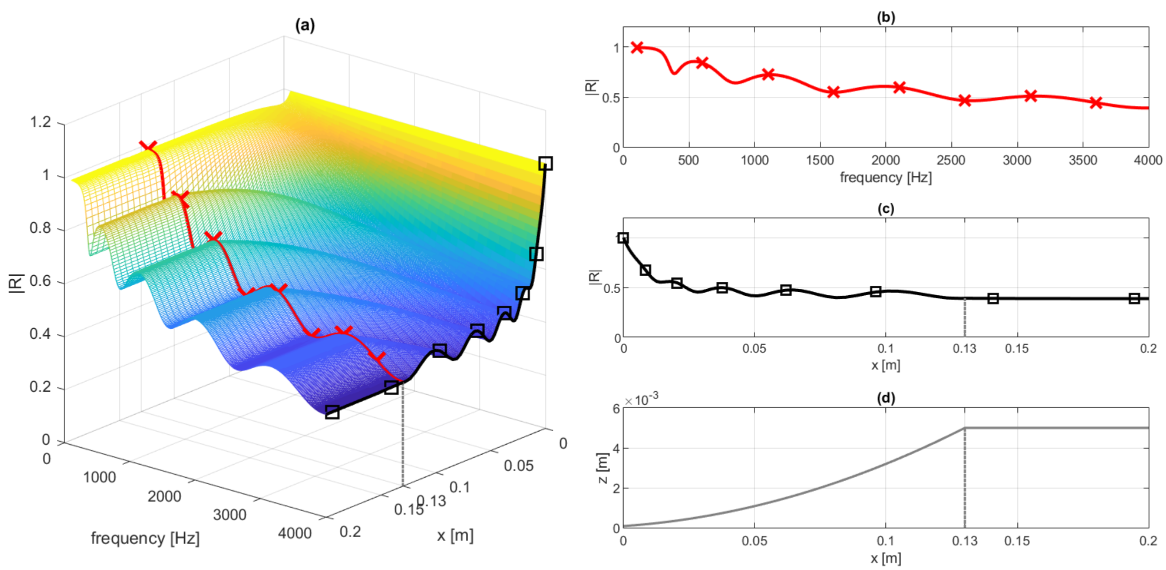

Figure 1: *(a) Modulus of the reflection coefficient, R, along x and the frequency. (b) The red line is R at the end of the ABH profile. (c) The black line is R at the higher frequency of the studied frequency range. (d) The grey line is the thickness profile of the ABH beam.*

Figure 1 gathers several characteristics of the ABH reflection coefficient. For a given value of x, it shows the variations with the frequency. For a given frequency, it shows the reflection profile according to x.

In order to reduce R, an investigation on the effect of variations of the damping layer loss factor, η, is performed. A parametric study is done (cf. Fig. 2), where η is supposed to be of the form $\eta(x) = \eta_1 \frac{x}{L} + \eta_0$, where L is the length of the ABH termination and η_1 and η_0 are varying constants. We consider three cases described in the Table 1.

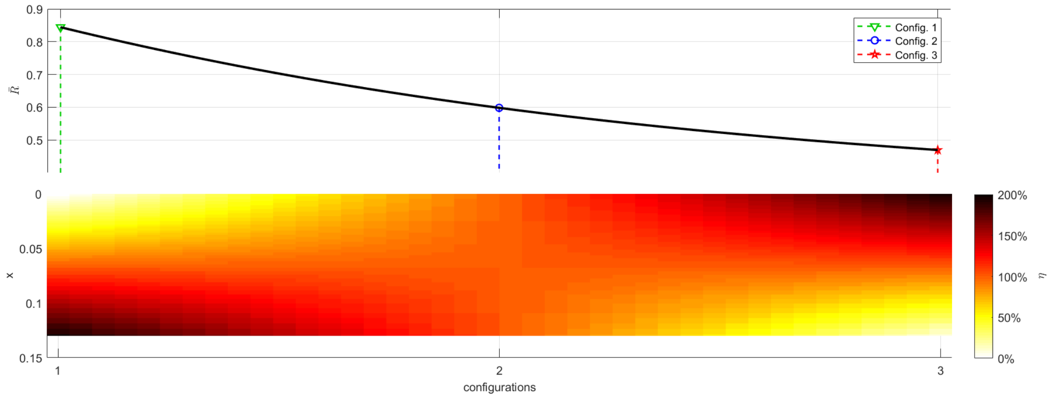

Figure 2: *Parametric study of linear variations of the damping layer loss factor, η. Three configurations are highlighted (cf. Tab.1). The curve represents the averaged reflection coefficient in the frequency range for each configuration.*

Table 1: *Characteristics of the viscoelastic damping layer.*

	η_1	η_0
config. 1	2	0
config. 2	0	1
config. 3	-2	2

The averaged reflection coefficient, $\bar{R} = \frac{1}{f_{max}-f_{min}} \int_{f_{min}}^{f_{max}} |R| df$, is a way to estimate the wave attenuation in a given frequency range. Figure 2 shows that Config. 3 seems to be advantageous compared to the others. This is confirmed by Figure 3, where the reflection coefficients of configurations 1, 2 and 3 are shown.

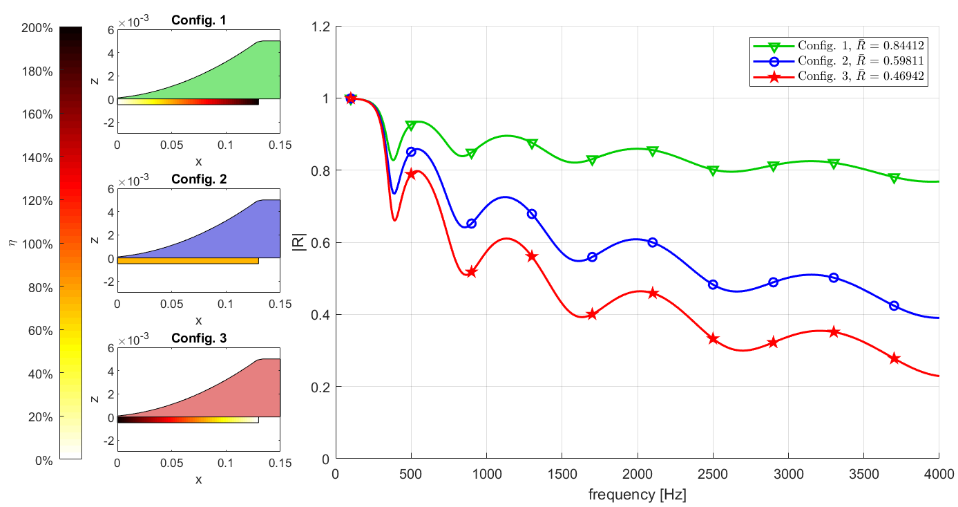

Figure 3: *Modulus of the reflection coefficient for the damping layer configurations 1 to 3.*

Figure 3 clearly illustrates the benefit of the gradient property of the damping layer.

4. CONCLUSION

To conclude, the benefit of a loss factor gradient in the damping layer has been investigate. Positive results justify the potentialities of the so-called enhanced ABH. A damping layer with varying properties seems to improve the efficiency of ABH waves absorbers.

In practice, spatial variations of the loss factor can be achieved using imposed temperature gradient at the extremity. In the case of shape memory polymer (SMP), such temperature variations induce also variations of the elastic storage modulus which should be taken into account. This varying properties are degrees of freedom to create strong drops of the reflection coefficient.

References

[1] Pauline Butaud, Emmanuel Foltête, and Morvan Ouisse. Sandwich structures with tunable damping properties: On the use of shape memory polymer as viscoelastic core. *Composite Structures*, 153:401 – 408, 2016.

[2] Jacques Cuenca. *WAVE MODELS FOR THE FLEXURAL VIBRATIONS OF THIN PLATES - Model of the vibrations of polygonal plates by the image source method - Vibration damping using the acoustic black hole effect*. Theses, Université du Maine, October 2009.

[3] Jacques Cuenca, Adrien Pelat, François Gautier, and Neil Ferguson. Improving the acoustic black hole effect for vibration damping in one-dimensional structures. In Société Française d'Acoustique, editor, *Acoustics 2012*, Nantes, France, April 2012.

[4] Vivien Denis. *Vibration damping in beams using the acoustic black hole effect*. Theses, Université du Maine, October 2014.

[5] E.M.Kerwin Jr. D.Ross, E.E.Ungar. Damping of plate flexural vibrations by means of viscoelastic laminae. *J.E.Ruzicka(Ed.), Structural Damping*, page 49–87, 1960.

[6] V.B. Georgiev, J. Cuenca, F. Gautier, L. Simon, and V.V. Krylov. Damping of structural vibrations in beams and elliptical plates using the acoustic black hole effect. *Journal of Sound and Vibration*, 330(11):2497 – 2508, 2011.

[7] A. Iserles, A. Marthinsen, and S. P. Nørsett. On the implementation of the method of magnus series for linear differential equations. *BIT Numerical Mathematics*, 39(2):281–304, Jun 1999.

[8] V.V. Krylov and F.J.B.S. Tilman. Acoustic 'black holes' for flexural waves as effective vibration dampers. *Journal of Sound and Vibration*, 274(3):605 – 619, 2004.

[9] V.V. Krylov and R.E.T.B. Winward. Experimental investigation of the acoustic black hole effect for flexural waves in tapered plates. *Journal of Sound and Vibration*, 300(1):43 – 49, 2007.

[10] D.J. Rixen M. Geradin. *Mechanical Vibrations: Theory and Application to Structural Dynamics*. John Wiley & Sons, third edition edition, 2014.

[11] M.A.Mironov. Propagation of a flexural wave in a plate whose thickness decreases smoothly to zero in a finite interval. *Soviet Physics—Acoustics*, 34:318–319, 1988.