Estimation of the sound pressure in a bent pipe from non intrusive acceleration measurements by an inverse problem procedure

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ABSTRACT

This work involves the theoretical and experimental study of the vibroacoustic coupling occurring in a bent pipe line in the plane wave domain. In this type of piping line, resulting forces located in the elbows are generated by the internal sound pressure. The objective is then to estimate the internal pressure field as a function of the accelerations measured at each pipe bent. For this purpose, a modal extraction method of the structure is used in order to obtain the mode shapes and damping at each bend. Then, knowing these parameters, an inverse problem method is applied to estimate the pressure field, knowing the vibration field. An experimental validation of this prediction method demonstrated its robustness and simplicity of implementation, the industrial application of which concerns in particular the transport of gases inducing significant pressure gradients related to reciprocating compressor sources.

Keywords: Vibroacoustic coupling, Unbalanced forces, Modal identification, Regularization method, Inverse problem, Non-intrusive measurement.
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1. INTRODUCTION

In the context of industrial oil processing sites, gases are conveyed along pipelines using compressors that can act as acoustic sources. The induced acoustic pulsations may lead to vibrations due to vibroacoustic coupling between the gas column and the pipe. Sometimes, strong vibrations can threaten the safety of an idustrial installation because of either a possible undersizing of the pipe supports or high pressure levels due to acoustic resonances along the pipe (Tison & al [1], Nakamura [2]).

To treat these problems, the diagnotic studies require the access to pressure and acceleration measurements. But direct sound pressure measurements is rarely possible for safety reasons, related to the dangerousness of the internal fluid and the extreme experimental conditions (measurement points not accessible, large thermal and pressure variations). The resolution of the inverse problem is then relevant because it connects the internal pressure of the fluid to the vibration of the structure that can be achieved by non-intrusive acceleration measurements.

In a U shaped pipe, composed of straight segments bent at 90^o angles, the forces generated by the gas propagation are located in the non-axis-symmetric sections, *i.e.* the elbows. The pressure phase shifts or amplitude discrepancies between the elbows lead to a non-balancing of the forces (Blodgett [3]). This work therefore involves the development of an inverse procedure, to measure the internal pressure of a pipe based on acceleration measurements at the elbows.

Many physical models have been developed to take into account the structural fluid interaction in pipes (DeJong [4], Wiggert & Tijsseling [5]). These models allow light or strong couplings to be considered beyond the acoustical cut-off frequencies. The resolutions require the use of numerical models (Wiggert & al [6], Lavooij & Tijsseling [7], Zhang & Tijsseling [8]) involving parameters data such boundary conditions which are sometimes difficult to estimate precisely. In practical case, the procedure is applied to weak couplings associated with light fluids. It is valid as long as the studied frequencies are included in the flexion mode domain and are lower than the first acoustic cut-off frequency. But the interest of this procedure lies in the fact that it is carried out experimentally and requires only a few measurement points.

This article is organized in two parts. A first theoretical part deals with the modelling of vibroacoustic coupling. For this purpose, the forces resulting from the internal pressure distribution are computed assuming plane wave hypothesis. Integrating this force into the equation of motion leads to a relationship between pressure and acceleration, which introduces the direct problem and the inverse problem. Then second part deals with the experimental application of this vibroacoustic model. First, an experimental modal analysis of a U shape pipe is proposed to estimate the accelerance. Using this configuration. The method is then tested on experimental signals obtained with a swept sine acoustic source provided by a loudspeaker in order to test the direct and inverse procedure.

2. MODELING OF THE VIBROACOUSTIC COUPLING

Modeling the vibroacoustic coupling between the fluid and the structure requires to understand how forces generated by the sound pressure field are projected onto the inner walls of the pipe.

2.2.1. Description of the forces applied to the piping line

When a plane acoustic wave propagates into a bent piping line, resulting forces are applied at the elbows. This can be demonstrated with the Ostrograsky's theorem (Kaltz [9]). Indeed, a uniform pressure distribution acting on any closed surface $S_{\text{tot}} = S_p + S_{\text{in}} + S_{\text{out}}$ leads to a zero resulting force, so:

$$\vec{F} = \int_{S_p} P d\vec{S} = -PS_{\text{in}}\vec{n}_{\text{in}} - PS_{\text{out}}\vec{n}_{\text{out}}, \tag{1}$$

with $\vec{n}_{\text{in}}$ and $\vec{n}_{\text{out}}$ being respectively the outgoing unit vectors located on the S_{in} input and S_{out} output sections (fig.1). In the studied case, the experimental setup (fig.2) only involves perpendicular elbows ($\theta = \pi/2$), this corresponds to the most cases encountered in industrial application. Then the resulting force applied to an i^{th} elbow are:

$$\vec{F}_i = PS_i\sqrt{2}\vec{n}_i. \tag{2}$$

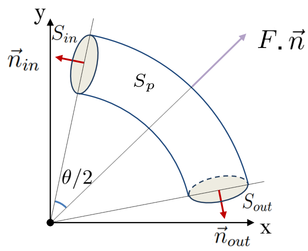


Figure 1: Geometric variables of a bent pipe segment.

where $i = [1, 2, 3, 4]$ corresponds to the elbows of the experimental setup of fig.2.

Depending on the geometry of the pipe and the distribution of the acoustic pressure field, these forces may not cancel each other out and lead to vibratory excitation, called "unbalanced forces" or "shaking forces" in the literature (Blodgett [3], Tison & al [1]) :

$$\Delta F_j = (P_{j+1} - P_j)S\sqrt{2}, \tag{3}$$

with $j = 1$ to 3 and $P_{j+1} - P_j$ the maximum pressure deviation over a period between the ith and ith + 1 elbows.

2.2.2. Direct problem: response of the pipe to the internal pressure excitation

As a preliminary step to the implementation of an inverse method, a modal model of the pipe dynamics is built. From the discretization described above, the motion equations of the pipe are written in matrix form:

$$[\mathbf{M}]\ddot{\mathbf{X}} + [\mathbf{C}]\dot{\mathbf{X}} + [\mathbf{K}]\mathbf{X} = \mathbf{F}, \tag{4}$$

with $\mathbf{F} = [F_1, \ldots, F_N]$ the N×1 vector of the normal forces to the pipe wall, N the number of degrees of freedom, $\mathbf{X} = [x_1, \ldots, x_N]$ the N×1 vector of the displacements and $[\mathbf{M}]$,$[\mathbf{C}]$ and $[\mathbf{K}]$ respectively the mass, damping and stiffness N×N matrices of the pipe.

Following the modal expansion principle, $\mathbf{X}$ can be developped such as

$$\mathbf{X} = [\mathbf{\Psi}]\mathbf{R} \tag{5}$$

with $\mathbf{R}$ the K×1 vector of modal coordinates, where K is the number of used modes and $[\mathbf{\Psi}] = [\psi_1 | \ldots | \psi_K]$ the N×K matrix of the mode shapes.

Orthonormalizing the mode shapes $\phi_k = \psi_{n,k}/\sqrt{m_k}$, projecting on the modes ϕ_k and assuming a harmonic excitation ($\mathbf{R} = \tilde{\mathbf{R}}e^{j\omega t}$) leads to the solution of the motion equation defined in terms of acceleration:

$$\mathbf{a} = -\omega^2 [\mathbf{\Phi}][\mathbf{H}][\mathbf{\Phi}]^t \mathbf{F} = [\mathcal{H}]\mathbf{F}, \tag{6}$$

with

$$[\mathbf{H}] = \operatorname{diag}\left(\frac{1}{\omega_k^2 + 2j\xi_k\omega_k\omega - \omega^2}\right), \tag{7}$$

where $\omega_k = 2\pi f_k$ is the modal pulsation and ξ_k is the modal damping of the pipe.

2.2.3. Inverse problem: identification of the pressure from pipe vibration measurements

From the above model, an inverse procedure can also be applied to estimate the pressure field from the measure of the resulting acceleration. In the practical case of noisy acceleration input data, the problem requires to be regularized to avoid instabilities when inverting the matrix (Idier [10]). To do that, the Tikhonov method is applied (Tikhonov [11]) leading to:

$$\tilde{\mathbf{F}} = ([\mathcal{H}]^T[\mathcal{H}] + \beta[\mathbf{I}])^{-1}[\mathcal{H}]^T\mathbf{a}. \tag{8}$$

The higher the β value, the more stability is ensured but the more the solution is altered. This means that there exists an optimal value of β which can be determined by using the L-curve method (Hansen [12]).

3. RESULTS

This section describes the application of the modal method described above to the specific case of a laboratory demonstrator of an industrial pipeline.

3.3.1. The experimental setup

The experimental setup (fig.2) consists of a 44 mm inner diameter PVC pipe of 3.85 m full length (from the loudspeaker to the termination). This pipe contains 4 elbows sepated by segments of lengths 0.365 m, 0.310 m, 0.375 m respectively (fig.2) with an open termination or a closed termination.

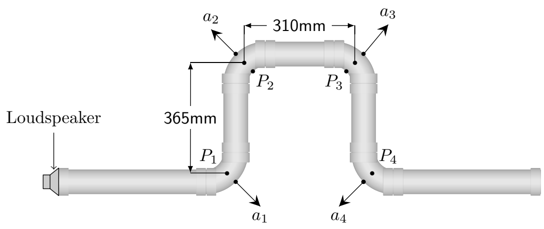


Figure 2: (a) Experimental setup showing the U-shaped pipe : the loudspeaker is connected at the inlet ant the 4 microphones and accelerometers are located at the 4 elbows.

This pipe is acoustically excited by a loudspeaker whose cut-off frequency is 40 Hz. Acoustic pressure measurements inside the pipe are performed using 1/4" G.R.A.S 40BP microphones located at each elbow and associated to G.R.A.S Power Module Type 12AQ conditioner. Accelerations are measured using an ICP PCB 356A01 accelerometer of negligeable masse that can be moved at various points along the U-shaped pipe (fig.2.(a)), at which the vibration response is useful to measure. All is driven in the Labview environment with a National Instrument USB-4431 DAQ device.

3.3.2. Modal analysis of the pipe

The direct model requires the prior identification of the modal parameters f_k, ξ_k and ϕ_k of the pipe. This is carried out according to an impact testing method. An impact hammer (PCB 086D05) excites the pipe at the $N = 60$ points of a mesh along the pipe. Two steps are involved in the modal identification procedure. The first step leads to the extraction the modal frequency and damping ratios, using the high resolution technique ESPRIT, Ege & al [13]. The second step leads to the estimation of the order of the model and to the extraction of the mode shapes (Paiva & al [14]).

Twelve modes are then identified in the range $[0 - 250]$Hz and three of them are shown in fig. 3. Some modes display a global motion such as rotation around x (mode 1) or around z (mode 3) and translation along y (mode 5) while others display more complex motion such as twisting (mode 4) or breathing (mode 11). The validity of the estimated modal parameters is checked by comparing synthesized and measured transfer functions. The equation 2 implies that in the plane wave domain, the force vector $\mathbf{F}$ is restricted to a $[4, 1]$ vector with only 4 non-zero located at the 4 elbows. So, only the 4 corresponding components of the 12 extracted eigen vectors are considered such that $[\boldsymbol{\mathcal{H}}]$ is a $[4, 4]$ matrix.

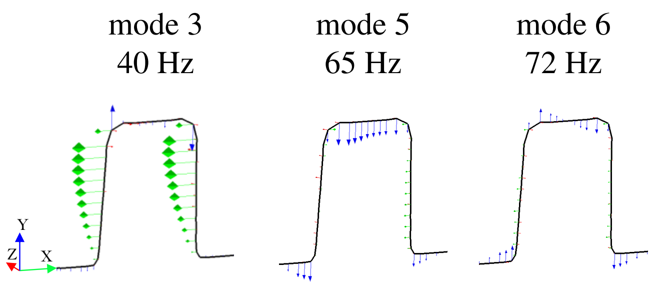


Figure 3: 3^{rd}, 5^{th} and 6^{th} mode shapes. the green and blue arrows indicate the x and y axis.

Fig. 4 show very good agreement with the measurement for the co-localized transfer function $\mathcal{H}_{2,2}$ (the index $i = 2$ defining the ith elbow), as well as for other transfer functions $\mathcal{H}_{2,1}$, $\mathcal{H}_{2,3}$ and $\mathcal{H}_{2,4}$ (not shown here for the sake of conciseness). It shows that in the frequency range $[0 - 250]$Hz, the experimental set-up is very well represented by considering only 12 modes.

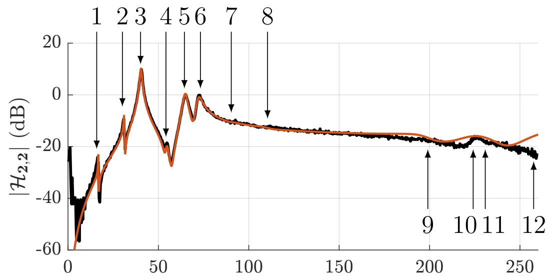


Figure 4: Measured (black) and synthesized (red) accelerances along the outgoing normal $\vec{n}_2$ of elbow 2, $\mathcal{H}_{2,2} = a_2/F_2$. The module with the indicated frequencies location of the 12 extracted modes, the modes from 1 to 8 are the elbow flexion modes and the modes from 9 to 12 are the pipe flexion modes (fig.3).

3.3.3. Experimental application : internal pressure from swept sine excitation

For the application of identification method, the case of the forced vibrational response to an acoustic frequency swept sine in the range $[5, 200]$Hz is considered. At the pipe inlet, the acoustic excitation is generated by the loudspeaker, while at the outlet the open termination is applied. at the pipe inlet (fig.2). Sound pressure and acceleration (along the outgoing normal $\vec{n}$) are measured at each of the 4 elbows.

Fig. 5 plots the acceleration at elbow 2 identified by the direct method (grey lines) for which the measured pressures are introduced into the eq. (6) in the case of the open termination. The identified accelerations are in very good agreement with the measurements (black lines), which experimentally validates the principle of describing the U shaped pipe structure by only degrees of freedom at its 4 elbows. Here, acceleration can appear out the mode shapes due to high amplitude force excitation in presence of acoustic standing waves along the pipe. Note that in the case of coincidence between acoustical and mechanical mode, strong vibro-acoustic coupling can result in peaks of very high amplitude.

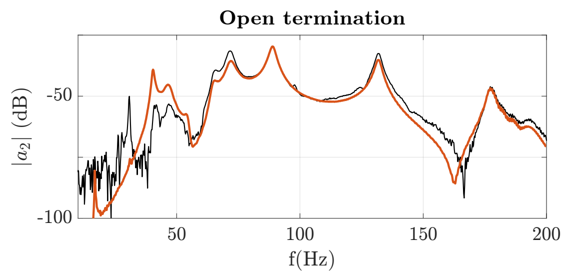
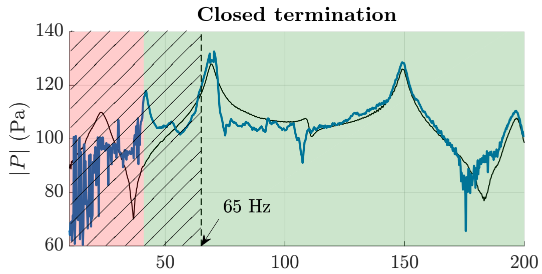
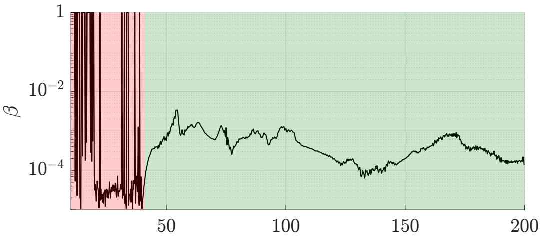


Figure 5: Measurement (black) and identification from the direct method (red) of the acceleration at elbow 2 in the case of an open termination.

Fig. 6 shows the results of the inverse method when identified accelerations are introduced into Eq. (8). In industrial pipe applications that motivate this work, vibration issues are mainly related to the establishment of acoustic resonances.



(a)



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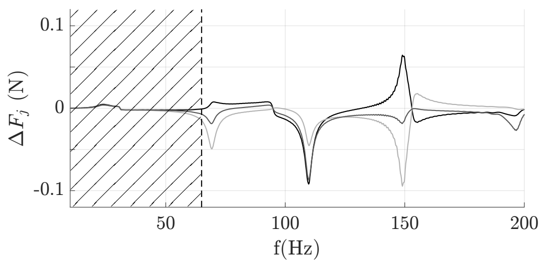

(c)

Figure 6: Experimental results of the inverse indentification of presure inside the pipe for the open closed termination. (a) measured (black) and identified (blue) pressures at elbow 2 ; (b) regularization parameter β ; (c) unbalanced forces ΔF_j (eq.3) between each pair of elbow, from black ($j = 1$) to light grey ($j = 3$).

At low frequency ($f < 40$Hz), a red zone is defined using the term β (Fig. 6(b)) which very high (β>1) and unstable. Indeed, in this frequency band, the acceleration measurements are very noisy. This significant noise is due to the loudspeaker whose cut-off frequency is around 40Hz. So below this frequency the radiated sound field is to low to induce vibrations. However, in a hatched area larger than the red area, significant identification errors are still observed. This corresponds to frequency thresholds beyond which unbalanced forces become large enough to generate vibrations.

From Fig. 6(c) which represent the unbalanced forces (from black $j = 1$ to light grey $j = 3$) defined in Eq. (2), the estimated threshold frequencies are 65 Hz for the open termination and 85 Hz for the closed termination. Below these frequencies, when the wavelength is large regarding to the inter-elbow distance, the pressure phase shift is too small and the resulting unbalanced forces are weak. Then, the measured vibrations are due to other excitations that are not modeled. Here, the vibration pollution corresponds to the solid transmission of the loudspeaker vibrations through its connexion with the pipe structure.

Above these frequency thresholds, Fig.6(a) show a very accurate identification of the pressure in the pipe, especially at the acoustic resonances. This result is the main insight of this work, since it is obtained from only 4 acceleration measurements, which gives this inverse method a great potential for application to fast and simple in situ characterizations.

4. CONCLUSION

This work deals with the development of a procedure which estimates the pressure field from acceleration measurements. First, a theoretical model has been proposed to connect the internal pressure field to the vibration field of the pipe. This development has led to a direct problem solving model (Eq.6), to obtain a vibration field from a pressure field, as well as an inverse problem solving model (Eq.8) to obtain a pressure field from a vibration field, based on the Tikhonov regularisation method from which the β stabilization parameter is obtained from the L-curve method.

Then, an experimental modal analysis has been carried out to obtain the modal components of the pipe used for the experimental setup. The modal identification of the pipe required the application of a two step method. The modal fit leads to a good representation of the accelerance transfer functions which permits to apply the direct and inverse problems.

To do that, a swept sine signal was generated in the pipe from a loudspeaker connected to the inlet. Open or closed acoustic terminations has been used to generate different pressure fields. The confrontation of the direct procedure with the measurement permits to validate the simplification of the continuous system to a discrete system with only 4 degrees of freedom. Concerning the inverse procedure, a first frequency threshold of validity has been defined by the term β which becomes high ($>10^{-1}$) when the acceleration signals are too noisy, and a second threshold is justified by the shaking forces (Eq.3) which have to be significant enough regarding to external solicitations, to apply the inverse procedure. These external solicitations can be induce by solid transmission from the loudspeaker to the pipe. When the shaking forces high enough (about 65 Hz in that case, for the closed termination), the solution of the inverse problem agrees with the measurement. Thus, in an industrial context, the inverse procedure can be used as a diagnostic tool to correctly treat critical vibrations. The advantage of this procedure is that it is simple, non-intrusive and requires few measurement points.

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