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Theoretical validation and improvement of the traditional in-situ decoupling method for the multi-coordinate coupled system

Wang, Zengwei
jokage@sjtu.edu.cn

Shi, Xi
xishi@ sjtu.edu.cn

Peng, Zhike
z.peng@ sjtu.edu.cn

Shanghai Jiao Tong University
School of Mechanical Engineering, Shanghai Jiao Tong University, Shanghai, 200240, P.R. China

ABSTRACT
Accurate dynamic characteristics of substructures and parameters of the coupling links are important for transfer path analysis and fault diagnosis, since complex mechanical structures often consist of many substructures and moving parts. In this paper, a procedure is derived for revealing relationships between frequency response functions of substructures and that of the multi-coordinate coupled system to theoretically validate the traditional in-situ decoupling method. In order to improve the traditional in-situ decoupling method, simpler expressions for substructure FRFs between internal DOFs and internal /interface DOFs are derived. It is found that the traditional in-situ decoupling method is theoretically equivalent to the virtual decoupling method, which helps clarify the difference between the two decoupling methods.

Keywords: In-situ decoupling, Inverse sub-structuring, FRF
I-INCE Classification of Subject Number: 40

1. INTRODUCTION

Accurate information of component's dynamics is necessary for dynamic modeling, model updating, vibration transfer path analysis and fault diagnosis of complex mechanical structures which consist of several substructures. In some practical cases, substructures cannot be measured separately but only when coupled to neighboring substructures [1]. As such problems are encountered, it has to identify the dynamic behavior of substructures from the known dynamic behavior of the complete system and/or that of the remaining part. That is where structural decoupling comes in.

So far, many different structural decoupling methods have been proposed in literature, and mainly divided into two classes: the standard decoupling method [2-4] and the in-situ decoupling method [5-8]. Both methods are based on relationships between the dynamic behavior of substructures and that of the complete system. The former one requires that the remaining part be characterized a priori. The latter one assumes that a compliant interface separates the substructures, which may be represented as a series of links. Therefore the in-situ decoupling method can identify substructure's dynamics solely through in-situ measurements of the whole assembly, which avoids the need of physical disassembly. This alleviates some measurement difficulties related to determination of the dynamic behavior of substructures.

The in-situ decoupling method was first proposed by Zhen et al. [9,10] for the analysis of the dynamic behavior of a single-coordinate coupled packaging system. Subsequently, this traditional method was further studied and directly extended for the multi-coordinate coupled packaging system [11-13]. Although the traditional method has been proposed for many years, it seems that it is more suitable for identification of component's dynamics of a system with less coupled coordinates. Because for the multi-coordinate coupled system, the traditional method involves too many matrix inversion operations and the process of calculating the dynamic characteristics of substructures is very complicated. This implies that a lower numerical accuracy is obtained. Beside, although authors of reference [11] have given explicit equations for calculating frequency response functions (FRFs) of subsystems from that of the system and have validated them numerically and experimentally, there is no apparent theoretical evidence that this method can be directly used for the multi-coordinate coupled system. Therefore, the goal of this paper is to further reveal relationships between FRFs of subsystems and that of the multi-coordinate coupled system and to theoretically validate and improve of the traditional in-situ decoupling method based on the findings.

It should be mentioned that some different approaches were also proposed. It mainly includes the decoupling method in [14-16], transmissibility based method [7,17] and virtual decoupling method [18-20]. These methods may be considered mathematically equivalent to each other since they are based on the same assumption (conservation of force across the link) [16].

The remainder of the paper is organized as follows. Section 2 gives a brief introduction of the traditional in-situ decoupling method. The theoretical core of the research is presented in Section 3 and Section 4. Section 3 gives the theoretical validation of the traditional method for the multi-coordinate coupled system. The theoretical improvement of the traditional in-situ decoupling method is presented in Section 4. The paper ends with a conclusion in Section 5.

2. TRADITIONAL IN-SITU DECOUPLING METHOD FOR ONE-COORDINATE COUPLED SYSTEM

Consider a system with two substructures A and B coupled at $c(a)$ and $c(b)$ by one link. It is shown in Fig. 1, where $o(x)$, $i(x)$, $c(x)$ are different degrees-of-freedom (DOFs) of substructure X (X=A or B). The dynamic stiffness of the link is denoted by K_{AB}. According to the traditional in-situ decoupling method, there is the following relationships between FRFs of the complete system and FRFs of substructure A and B:

$$\begin{bmatrix} H_{S,o(a)i(a)} & H_{S,o(a)c(x)} & H_{S,o(a)i(b)} \\ H_{S,c(x)i(a)} & H_{S,c(x)c(x)} & H_{S,c(x)i(b)} \\ H_{S,o(b)i(a)} & H_{S,o(b)c(x)} & H_{S,o(b)i(b)} \end{bmatrix} = \begin{bmatrix} H_{A,o(a)i(a)} & H_{A,o(a)c(x)} & 0 \\ H_{X,c(x)i(a)} & H_{X,c(x)c(x)} & H_{X,c(x)i(b)} \\ 0 & H_{B,o(b)c(x)} & H_{B,o(b)i(b)} \end{bmatrix} - \begin{bmatrix} \alpha H_{A,o(a)c(a)} \\ H_{X,c(x)c(x)} \\ \beta H_{B,o(b)c(b)} \end{bmatrix} D \begin{bmatrix} \alpha H_{A,c(a)i(a)} & H_{X,c(x)c(x)} & \beta H_{B,c(b)i(b)} \end{bmatrix} \tag{1}$$

where

$$\alpha = \begin{cases} 1 & \text{for } x = a, X = A \\ -1 & \text{for } x = b, X = B \end{cases}, \quad \beta = \begin{cases} 1 & \text{for } x = b, X = B \\ -1 & \text{for } x = a, X = A \end{cases},$$

$$D = \left(H_{A,c(a)c(a)} + H_{B,c(b)c(b)} + K_{AB}^{-1} \right)^{-1}$$

$H_{S,o(a)i(a)}$ is the FRF of the system between $o(a)$ and $i(a)$, and $H_{A,o(a)i(a)}$ is the FRF of the subsystem A. More details can be found in [5].

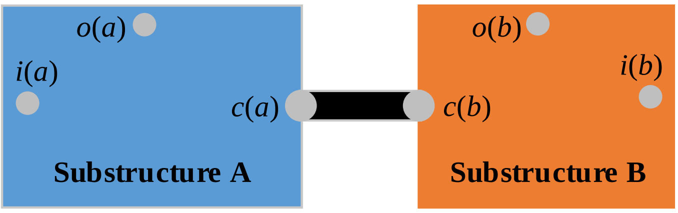

Fig. 1 - A system consisting of two substructures connected by one link.

After some algebraic operations, the dynamic stiffness of the link and FRFs of subsystems A and B can be expressed as

$$K_{AB} = \frac{H_{S,c(a)c(b)}}{H_{S,c(a)c(a)} H_{S,c(b)c(b)} - H_{S,c(a)c(b)}^2} \tag{2}$$

$$H_{A,c(a)c(a)} = \frac{H_{S,c(a)c(a)} H_{S,c(b)c(b)} - H_{S,c(a)c(b)}^2}{H_{S,c(b)c(b)} - H_{S,c(a)c(b)}} \tag{3}$$

$$H_{B,c(b)c(b)} = \frac{H_{S,c(b)c(b)} H_{S,c(a)c(a)} - H_{S,c(b)c(a)}^2}{H_{S,c(a)c(a)} - H_{S,c(b)c(a)}} \tag{4}$$

For the multi-substructures one-coordinate coupled system, the above equations can directly be used for decoupling the subsystem one by one. However, Eq. (2), Eq. (3) and Eq. (4) cannot be directly expanded in the form of matrices for identification of the dynamic stiffness of the link and FRFs of subsystems for the multi-coordinate coupled system, since it requires performing complicated matrix operations.

3. THEORETICAL VALIDATION OF TRADITIONAL IN-SITU DECOUPLING METHOD FOR THE MULTI-COORDINATE COUPLED SYSTEM

Although explicit equations for calculating FRFs of subsystems from that of the multi-coordinate coupled system have been given in [11-13] and have already been validated numerically and experimentally, no apparent theoretical validation was ever conducted. These equations are obtained from some obvious relationships between FRFs of substructures and that of the system, and as a result, the traditional method involves too many matrix inversion operations, especially for the process of calculating the FRFs of substructures between internal DOFs and interface DOFs. Besides, no explicit equations was derived for FRFs of substructures between internal DOFs.

In this section relationships between FRFs of substructures and that of the multi-coordinate coupled system will be further revealed and the traditional in-situ decoupling method will be theoretically validated. Based on the findings, the traditional in-situ decoupling method will be theoretically improved in the next section.

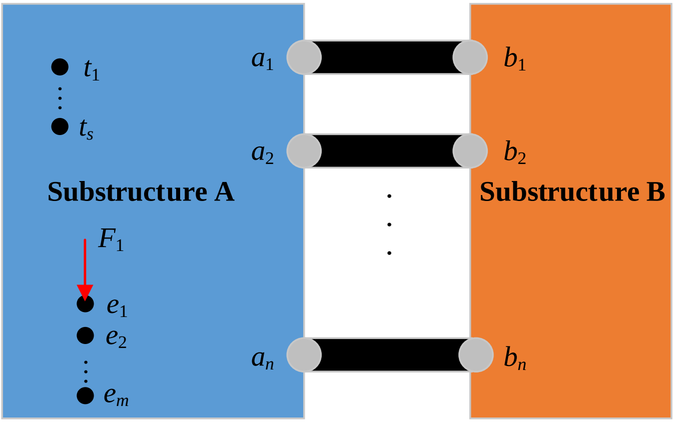

Fig. 2 - A system consisting of two substructures connected by several links.

Consider a multi-coordinate coupled system with substructures *A* and *B* as shown in Fig. 2. In order to facilitate the problem description and formula derivation, a set of terms different from the one of Fig. 1 are used to represent four sets of DOFs shown in Fig. 2. The term *a* denotes interface DOFs on the side of substructure *A* and b denotes interface DOFs on the side of substructure *B*. The terms *t* and *e* denote the internal DOFs of substructure *A*. The subscript number indicates the number of DOFs. If the coupling links are considered to behave as massless parallel connections of springs and dampers, the force across the coupling links is conserved. When the force F_1 is only applied at e_1, the response of the system can be considered as the substructure response caused by external force and coupling force. There are the following relations:

$$\mathbf{X}_{S,te1} = \mathbf{H}_{A,te1} F_1 - \mathbf{H}_{A,ta} \mathbf{F}_{c1} \tag{5}$$

$$\mathbf{X}_{S,ae1} = \mathbf{H}_{A,ae1} F_1 - \mathbf{H}_{A,aa} \mathbf{F}_{c1} \tag{6}$$

$$\mathbf{X}_{S,be1} = \mathbf{H}_{B,bb} \mathbf{F}_{c1} \tag{7}$$

$$\mathbf{F}_{c1} = \mathbf{K}_{AB} \left(\mathbf{X}_{S,ae1} - \mathbf{X}_{S,be1} \right) \tag{8}$$

where

$$\mathbf{F}_{c1} = \left[F_{c11} \;\; F_{c12} \;\; \cdots \;\; F_{c1n} \right]^T, \; \mathbf{K}_{AB} = \mathrm{diag}\left(K_1 \cdots K_n \right)$$

$\mathbf{X}_{S,te1}$ is the system response vector of DOFs *t* caused by the force applied at e_1. $\mathbf{H}_{A,te1}$ is the subsystem FRF vector between DOFs *t* and the DOF e_1, and $\mathbf{H}_{A,ta}$ the subsystem FRF matrix between DOFs *t* and DOFs *a*. $\mathbf{F}_{c1}$ is the coupling force vector caused by the force applied at e_1. $\mathbf{K}_{AB}$ is the dynamic stiffness matrix related to the coupling links.

Eq. (8) can be rewritten in the form of the difference between interface responses. That is

$$\mathbf{X}_{S,ae1} - \mathbf{X}_{S,be1} = \mathbf{K}_{AB}^{-1} \mathbf{F}_{c1} \tag{9}$$

The response difference can also be obtained in a different way by combining Eq. (6) and Eq. (7).

$$\mathbf{X}_{S,ae1} - \mathbf{X}_{S,be1} = \mathbf{H}_{A,ae1} F_1 - \mathbf{H}_{A,aa} \mathbf{F}_{c1} - \mathbf{H}_{B,bb} \mathbf{F}_{c1} \tag{10}$$

Observing Eq. (9) and Eq. (10) the following relation can be formed

$$\mathbf{K}_{AB}^{-1}\mathbf{F}_{c1} = \mathbf{H}_{A,ae1}F_1 - \mathbf{H}_{A,aa}\mathbf{F}_{c1} - \mathbf{H}_{B,bb}\mathbf{F}_{c1} \tag{11}$$

Then the coupling force can be expressed in terms of FRFs of subsystems and the dynamic stiffness of links as follows.

$$\mathbf{F}_{c1} = \left(\mathbf{H}_{A,aa} + \mathbf{H}_{B,bb} + \mathbf{K}_{AB}^{-1}\right)^{-1}\mathbf{H}_{A,ae1}F_1 = \mathbf{D}\mathbf{H}_{A,ae1}F_1 \tag{12}$$

where

$$\mathbf{D} = \left(\mathbf{H}_{A,aa} + \mathbf{H}_{B,bb} + \mathbf{K}_{AB}^{-1}\right)^{-1}$$

By substituting Eq. (12) into Eq. (5), it yields

$$\mathbf{X}_{S,te1} = \mathbf{H}_{A,te1}F_1 - \mathbf{H}_{A,ta}\mathbf{D}\mathbf{H}_{A,ae1}F_1 \tag{13}$$

When the external force is a unit force, the response vector of DOFs t is actually the corresponding FRF vector. Eq. (13) becomes as

$$\mathbf{H}_{S,te1} = \mathbf{H}_{A,te1} - \mathbf{H}_{A,ta}\mathbf{D}\mathbf{H}_{A,ae1} \tag{14}$$

When the unit force is only applied at e_m, following the same steps the similar relation can be obtained as

$$\mathbf{H}_{S,tem} = \mathbf{H}_{A,tem} - \mathbf{H}_{A,ta}\mathbf{D}\mathbf{H}_{A,aem} \tag{15}$$

Finally, the following relation can be obtained

$$\mathbf{H}_{S,te} = \mathbf{H}_{A,te} - \mathbf{H}_{A,ta}\mathbf{D}\mathbf{H}_{A,ae} \tag{16}$$

where

$$\mathbf{H}_{S,te} = \left[\mathbf{H}_{S,te1} \;\; \mathbf{H}_{S,te2} \;\; \cdots \;\; \mathbf{H}_{S,tem}\right],\; \mathbf{H}_{A,te} = \left[\mathbf{H}_{A,te1} \;\; \mathbf{H}_{A,te2} \;\; \cdots \;\; \mathbf{H}_{A,tem}\right],$$

$$\mathbf{H}_{A,ae} = \left[\mathbf{H}_{A,ae1} \;\; \mathbf{H}_{A,ae2} \;\; \cdots \;\; \mathbf{H}_{A,aem}\right]$$

Similarly, according to Eq. (7) the following relation is obtained

$$\mathbf{H}_{S,be} = \mathbf{H}_{B,bb}\mathbf{D}\mathbf{H}_{A,ae} \tag{17}$$

Eq. (16) and Eq. (17) are the main findings of this section. By changing the terms on the subscripts, different FRF relations can be obtained. For example, the FRF relations related to interface DOFs are expressed as

$$\mathbf{H}_{S,aa} = \mathbf{H}_{A,aa} - \mathbf{H}_{A,aa}\mathbf{D}\mathbf{H}_{A,aa} \tag{18}$$

$$\mathbf{H}_{S,ba} = \mathbf{H}_{B,bb}\mathbf{D}\mathbf{H}_{A,aa} \tag{19}$$

The same holds true for subsystem B. That is

$$\mathbf{H}_{S,bb} = \mathbf{H}_{B,bb} - \mathbf{H}_{B,bb}\mathbf{D}\mathbf{H}_{B,bb} \tag{20}$$

$$\mathbf{H}_{S,ab} = \mathbf{H}_{A,aa}\mathbf{D}\mathbf{H}_{B,bb} \tag{21}$$

Based on above equations, FRF relations for a multi-coordinate coupled system can be formed. For the system shown in Fig. 3, it can be written as Eq. (22). Obviously, Eq. (22) is similar to Eq. (1), and the difference is that in Eq. (22) the FRF matrices are involved.

$$\begin{bmatrix} \mathbf{H}_{S,o(a)i(a)} & \mathbf{H}_{S,o(a)c(x)} & \mathbf{H}_{S,o(a)i(b)} \\ \mathbf{H}_{S,c(x)i(a)} & \mathbf{H}_{S,c(x)c(x)} & \mathbf{H}_{S,c(x)i(b)} \\ \mathbf{H}_{S,o(b)i(a)} & \mathbf{H}_{S,o(b)c(x)} & \mathbf{H}_{S,o(b)i(b)} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{A,o(a)i(a)} & \mathbf{H}_{A,o(a)c(x)} & 0 \\ \mathbf{H}_{X,c(x)i(a)} & \mathbf{H}_{X,c(x)c(x)} & \mathbf{H}_{X,c(x)i(b)} \\ 0 & \mathbf{H}_{B,o(b)c(x)} & \mathbf{H}_{B,o(b)i(b)} \end{bmatrix}$$
$$- \begin{bmatrix} \alpha\mathbf{H}_{A,o(a)c(a)} \\ \mathbf{H}_{X,c(x)c(x)} \\ \beta\mathbf{H}_{B,o(b)c(b)} \end{bmatrix} \mathbf{D} \left[\alpha\mathbf{H}_{A,c(a)i(a)} \;\; \mathbf{H}_{X,c(x)c(x)} \;\; \beta\mathbf{H}_{B,c(b)i(b)}\right] \tag{22}$$

where

$$\alpha = \begin{cases} 1 & \text{for } x = a, X = A \\ -1 & \text{for } x = b, X = B \end{cases}, \quad \beta = \begin{cases} 1 & \text{for } x = b, X = B \\ -1 & \text{for } x = a, X = A \end{cases},$$

$$\mathbf{D} = \left(\mathbf{H}_{A,c(a)c(a)} + \mathbf{H}_{B,c(b)c(b)} + \mathbf{K}_{AB}^{-1} \right)^{-1}$$

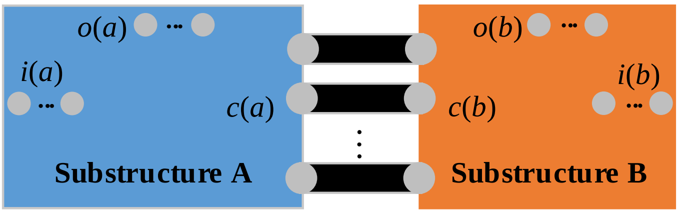

Fig. 3 A multi-coordinate coupled system.

4. THEORETICAL IMPROVEMENT OF TRADITIONAL IN-SITU DECOUPLING METHOD

In the traditional in-situ decoupling method, expressions of FRFs of substructures between internal DOFs and interface DOFs are very complicated, and too many matrix inversion operations are involved so that a lower numerical accuracy is obtained. The purpose of this section is to derive the simpler expressions for above FRFs as well as FRFs of substructures between internal DOFs.

Combining Eq. (18), Eq. (19), Eq. (20) and Eq. (21), it yields

$$\mathbf{H}_{S,aa}\mathbf{H}_{S,ba}^{-1}\mathbf{H}_{S,bb} = \mathbf{K}_{AB}^{-1} + \mathbf{H}_{A,aa}\mathbf{D}\mathbf{H}_{B,bb} = \mathbf{K}_{AB}^{-1} + \mathbf{H}_{S,ab} \tag{23}$$

It is found that the dynamic stiffness of links can be expressed in the form of FRFs of the system. This means that the dynamic stiffness of links can be determined through in-situ measurements of the system.

$$\mathbf{K}_{AB} = \left(\mathbf{H}_{S,aa}\mathbf{H}_{S,ba}^{-1}\mathbf{H}_{S,bb} - \mathbf{H}_{S,ab} \right)^{-1} \tag{24}$$

Looking back on Fig. 2, when a unit force is only applied at b_1, b_2, …, b_n, respectively, the following relationships can be obtained

$$\mathbf{H}_{S,ab} = \mathbf{H}_{A,aa}\mathbf{K}_{AB}\left(\mathbf{H}_{S,bb} - \mathbf{H}_{S,ab} \right) \tag{25}$$

$$\mathbf{H}_{S,tb} = \mathbf{H}_{A,ta}\mathbf{K}_{AB}\left(\mathbf{H}_{S,bb} - \mathbf{H}_{S,ab} \right) \tag{26}$$

Then the decoupled FRFs for subsystem A can be expressed as

$$\mathbf{H}_{A,aa} = \mathbf{H}_{S,ab}\left(\mathbf{H}_{S,bb} - \mathbf{H}_{S,ab} \right)^{-1}\left(\mathbf{H}_{S,aa}\mathbf{H}_{S,ba}^{-1}\mathbf{H}_{S,bb} - \mathbf{H}_{S,ab} \right) \tag{27}$$

$$\mathbf{H}_{A,ta} = \mathbf{H}_{S,tb}\left(\mathbf{H}_{S,bb} - \mathbf{H}_{S,ab} \right)^{-1}\left(\mathbf{H}_{S,aa}\mathbf{H}_{S,ba}^{-1}\mathbf{H}_{S,bb} - \mathbf{H}_{S,ab} \right) \tag{28}$$

From Eq. (27) and Eq. (28), it can be found that FRFs of subsystem A can be obtained solely through FRFs of the system, and expressions are not that complicated. Eq. (27) has already been derived in [11], and Eq. (28) is simpler than the corresponding expression in [11-13].

Looking back on Eq. (16), FRFs between two sets of DOFs t and e can also be expressed as follows

$$\mathbf{H}_{S,te} = \mathbf{H}_{A,te} + \mathbf{H}_{A,ta}\mathbf{K}_{AB}\left(\mathbf{H}_{S,be} - \mathbf{H}_{S,ae} \right) \tag{29}$$

By substituting Eq. (28) into Eq. (29), it yields

$$\mathbf{H}_{S,te} = \mathbf{H}_{A,te} + \mathbf{H}_{S,tb}\left(\mathbf{H}_{S,bb} - \mathbf{H}_{S,ab} \right)^{-1}\left(\mathbf{H}_{S,be} - \mathbf{H}_{S,ae} \right) \tag{30}$$

Then FRFs of substructures between internal DOFs can be obtained as

$$\mathbf{H}_{A,te} = \mathbf{H}_{S,te} - \mathbf{H}_{S,tb} \left(\mathbf{H}_{S,bb} - \mathbf{H}_{S,ab} \right)^{-1} \left(\mathbf{H}_{S,be} - \mathbf{H}_{S,ae} \right) \tag{31}$$

Eq. (28) and Eq. (31) are the main findings of this section, through which much simpler operations are required for calculating decoupled FRFs between two sets of DOFs. This also holds true for subsystem B, but for convenience, the corresponding expressions are not presented here.

Observing Eq. (31), it can be found that it is the same to Eq. (24) of reference [19], despite subscripts are different. Besides, substituting Eq. (20) of reference [19] into Eq. (22) of reference [19], an equation that is the same to Eq. (27) can be obtained for decoupled FRFs between interface DOFs. Following the same steps, a same equation to Eq. (28) can also be obtained for decoupled FRFs between interface DOFs and internal DOFs. This means Eq. (27), Eq. (28) and Eq. (31) are actually the same to Eq. (23), Eq. (21) and Eq. (24) of reference [19], respectively. Therefore, in theory the traditional in-situ decoupling method is equivalent to the virtual decoupling method of reference [19]. For the traditional in-situ decoupling method, it seems easier to calculate the dynamic stiffness of coupling links using Eq. (24). However, for the virtual decoupling method of reference [19], it is easier to calculate decoupled FRFs, since only one matrix inversion is required.

Since the traditional in-situ decoupling method [5,11-13] and the virtual decoupling method [18-20] have been validated numerically and experimentally, in this paper no numerical or experimental case will be used to illustrate and validate the benefits of the two methods. Also, no mention will be made of how measurements are to be made in practice as well as the statistical process involved.

5. CONCLUSIONS

This paper focuses on theoretical aspects of the traditional in-situ decoupling method. For the multi-coordinate coupled system, relationships between FRFs of subsystems and that of the system is further revealed. Based on the findings, the traditional in-situ decoupling method for the multi-coordinate coupled system is theoretically validated and improved. It is found that the traditional in-situ decoupling method is theoretically equivalent to the virtual decoupling method of reference [19], through which the difference between two methods are clarified. For the traditional in-situ decoupling method, it seems easier to calculate the dynamic stiffness of coupling links, while for the virtual decoupling method, it is easier to calculate decoupled FRFs. In practice, these two methods should be combined to solve engineering problems.

6. ACKNOWLEDGEMENTS

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